

# rbiprobit: Recursive bivariate probit estimation and decomposition of marginal effects

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```
net install rbiprobit, from("https://raw.githubusercontent.com/cobanomics/rbiprobit/main/")
```

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# Motivation

## Effects of Interest

### 1. What we want

- ▶ Estimate: Effect of binary or treatment variable on binary outcome variable
- ▶ Treatment variable itself is endogenous
- ▶ Unobservables may correlate with treatment and outcome equation
- ▶ Compute average treatment effect
- ▶ Compute average marginal effect for covariates

### 2. What doesn't work:

- ▶ `margins` gives incorrect treatment effect
- ▶ `margins` gives incorrect average marginal effect for covariates
- ▶ `ivprobit` inappropriate; treatment variable is binary

### 3. What we need

- ▶ Correct Estimation of a RBPM
- ▶ Considering recursive nature of the model for postestimation commands

# Contribution

## A new Stata Command

- ▶ `rbiprobit` estimates RBPMs like `biprobit` or `cmp`
- ▶ `rbiprobit` accounts for recursive nature in postestimation commands
  - ▶ `predict` and `predictnl`
  - ▶ `rbiprobit margdec`
  - ▶ `rbiprobit tmeffects`
- ▶ `rbiprobit margdec` incorporates `margins` command, enabling
  - ▶ Decomposition of average marginal effects of covariates
  - ▶ Standard errors using the delta method
- ▶ `rbiprobit tmeffects` incorporates `margins` command, enabling
  - ▶ Different definitions of treatment effects
  - ▶ Standard errors using the delta method

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## Recursive bivariate probit model

### The Model

A structural model with endogenous explanatory treatment variable  $y_2$  correlated with the unobservables

$$y_1^* = \mathbf{x}'\boldsymbol{\beta} + \alpha y_2 + \epsilon_1 \quad , y_1 = 1 \left[ y_1^* > 0 \right] \quad (1)$$

$$y_2^* = \mathbf{z}'\boldsymbol{\gamma} + \epsilon_2 \quad , y_2 = 1 \left[ y_2^* > 0 \right] \quad (2)$$

$$\text{with } \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \sim \mathcal{N} \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right]$$

- ▶ correlation between  $\epsilon_1$  and  $\epsilon_2$  induces the endogeneity
- ▶  $\mathbf{x}$  and  $\mathbf{z}$  can share some or all covariates
- ▶ Greene (2018) notes that endogenous nature of  $y_2$  can be ignored
- ▶ Han and Lee (2019): estimates are at best weakly identified if  $\mathbf{x} = \mathbf{z}$

## Recursive bivariate probit model

### Treatment Effects

#### 1. Average Treatment Effect (ATE)

$$\text{ATE} = \frac{1}{n} \sum_{i=1}^n \Phi(x'_i \beta + \alpha) - \Phi(x'_i \beta)$$

#### 2. Average Treatment Effect on the Treated (ATET)

$$\text{ATET} = \frac{1}{n_2} \sum_{i=1}^{n_2} \Phi \left( \frac{x'_i \beta + \alpha - \rho z'_i \gamma}{\sqrt{1 - \rho^2}} \right) - \Phi \left( \frac{x'_i \beta - \rho z'_i \gamma}{\sqrt{1 - \rho^2}} \right) \quad \forall y_{2i} = 1$$

#### 3. Average Treatment Effect on Conditional Probability (ATEC)

$$\text{ATEC} = \frac{1}{n} \sum_{i=1}^n \frac{\Phi_2(x'_i \beta + \alpha, z'_i \gamma, \rho)}{\Phi(z'_i \gamma)} - \frac{\Phi_2(x'_i \beta, -z'_i \gamma, -\rho)}{\Phi(-z'_i \gamma)}$$

## Decomposition of Marginal Effects

### Joint and Conditional Probabilities

- ▶ Covariate  $d$  appears in both  $x$  and  $z$
- ▶ Decomposition of total marginal effects on the probabilities (except marginal probabilities) are then
  1. Continuous Variables (see Greene, 2018)

$$\text{ME} = \frac{\partial \text{Pr}}{\partial \begin{pmatrix} x_d \\ z_d \end{pmatrix}} = \underbrace{\frac{\partial \text{Pr}}{\partial x_d}}_{\text{direct effect}} + \underbrace{\frac{\partial \text{Pr}}{\partial z_d}}_{\text{indirect effect}}$$

2. Discrete Variables (see Hasebe, 2013; Edwards et al., 2019)

$$\text{ME} = \underbrace{[\text{Pr} |_{x_d=1} - \text{Pr} |_{x_d=0}]}_{\text{direct effect}} + \underbrace{[\text{Pr} |_{z_d=1} - \text{Pr} |_{z_d=0}]}_{\text{indirect effect}}$$



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## Basic Syntax

```
rbiprobit depvar [=] [indepvars] [if] [in]  
    , endogenous(depvar_en [=] [indepvars_en] [, enopts]) [options]
```

- ▶ `depvar_en` automatically added to outcome equation as factor-variable
- ▶ `rbiprobit` implemented as an `lf1 ml` evaluator
- ▶ `depvar` and `depvar_en` have to be binary (*current version*)
- ▶ factor variables and time-series operators allowed
- ▶ `rbiprobit postestimation` available for features after estimation

## rbprobit Output

```

. webuse class10, clear
(Class of 2010 profile)

. rbprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

Univariate Probits for starting values
Comparison:      log likelihood = -2673.8688

Recursive Bivariate Probit Regression
Log likelihood = -2667.5268
Number of obs   =      2,500
Wald chi2(12)   =      964.07
Prob > chi2     =      0.0000

```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
-----						
graduate						
1.program	.3522094	.1770159	1.99	0.047	.0052646	.6991542
income	.1434782	.0142911	10.04	0.000	.1154681	.1714882
roommate						
yes	.267713	.0588568	4.55	0.000	.1523559	.3830701
hsgpagrp						
2.5-2.9	.9451679	.1357869	6.96	0.000	.6790305	1.211305
3.0-3.4	1.939513	.147325	13.16	0.000	1.650761	2.228264
3.5-4.0	6.535829	127.5038	0.05	0.959	-243.367	256.4387
_cons	-2.076232	.2181295	-9.52	0.000	-2.503758	-1.648706
-----						
program						
campus						
yes	.7465297	.0747092	9.99	0.000	.6001024	.8929569
scholar						
yes	.9007975	.0579886	15.53	0.000	.787142	1.014453
income	-.0785837	.0096477	-8.15	0.000	-.0974928	-.0596746
hsgpagrp						
2.5-2.9	.0586754	.1099653	0.53	0.594	-.1568526	.2742035
3.0-3.4	.0651845	.1152074	0.57	0.572	-.1606179	.2909869
3.5-4.0	-.0970995	.1780755	-0.55	0.586	-.4461211	.2519222
_cons	-.4441949	.1276995	-3.48	0.001	-.6944812	-.1939085
-----						
/atanrho	.4138925	.118934	3.48	0.001	.1807862	.6469988
-----						
rho	.3917727	.1006793			.178842	.5696461
-----						
Wald test of rho=0: chi2(1) = 12.1105					Prob > chi2 = 0.0005	

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## Postestimation Commands

### Predictions

```
predict [type] newvar [if] [in] [, statistic ]
```

#### ***statistic***

p11	Pr(depvar = 1, depvar_en = 1); the default
p10	Pr(depvar = 1, depvar_en = 0)
p01	Pr(depvar = 0, depvar_en = 1)
p00	Pr(depvar = 0, depvar_en = 0)
pmarg1	Pr(depvar = 1); marginal success probability for outcome eq.
pmarg2	Pr(depvar_en = 1); marginal success probability for endogenous eq.
pcond1	Pr(depvar = 1   depvar_en = 1)
pcond2	Pr(depvar_en = 1   depvar = 1)
xb1	linear prediction for outcome eq.
xb2	linear prediction for endogenous eq.
stdp1	standard error of the linear prediction for outcome eq.
stdp2	standard error of the linear prediction for endogenous eq.

## Postestimation Commands

### Margins and Treatment Effects

```
rbiprobit margdec [if] [in] [, response_options options]
```

```
rbiprobit tmeffects [if] [in] [, tmeffect(effecttype) options]
```

#### **rbiprobit margdec options**

`effect(effecttype)` specify type of effect; *effecttype* may be total, direct, or indirect; default is total

`predict(pred_opt)` estimate margins for predict, *pred\_opt* ;  
multiple predict not applicable

`dydx(varlist)` estimate marginal effect of variables in *varlist*

...

#### **rbiprobit tmeffects options**

`tmeffect(effecttype)` specify type of effect; *effecttype* may be ate, atet, or atec; default is ate

...

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## Post-Estimation: predict

### Comparison: biprobit vs. rbiprobit

```
. webuse class10, clear
(Class of 2010 profile)

. qui: rbiprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

. predict p11r, p11

. qui: biprobit (graduate = income i.roommate i.hsgpagrp i.program) ///
> (program = i.campus i.scholar income i.hsgpagrp)

. predict p11b, p11

. compare p11r p11b
```

	count	----- minimum	difference average	----- maximum
p11r<p11b	678	-.0000178	-.0000104	-1.49e-08
p11r=p11b	1			
p11r>p11b	1821	2.98e-08	.026773	.1206536
jointly defined	2500	-.0000178	.0194987	.1206536
total	2500			



## Post-Estimation: margdec

### Continuous Covariate: Total Average Marginal Effects

```
. rbiprobit margdec, dydx(income) predict(p11) effect(total)
```

```
Average marginal effects          Number of obs      =          2,500
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t.    : income
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
income	.0032146	.002856	1.13	0.260	-.0023831	.0088123

## Post-Estimation: rbiprobit margdec

### Continuous Covariate: Direct Average Marginal Effects

```
. rbiprobit margdec, dydx(income) predict(p11) effect(direct)
```

```
Average marginal effects          Number of obs      =          2,500
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t.    : income
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
income	.0207027	.0017927	11.55	0.000	.0171891	.0242163

## Post-Estimation: rbiprobit margdec

### Continuous Covariate: Indirect Average Marginal Effects

```
. rbiprobit margdec, dydx(income) predict(p11) effect(indirect)
```

```
Average marginal effects          Number of obs      =          2,500
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t.    : income
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
income	-.0174881	.00214	-8.17	0.000	-.0216825	-.0132937

# Post-Estimation: rbiprobit tmeffects

## Average Treatment Effect

```
. rbiprobit tmeffects, tmeffect(ate)
```

```
Treatment effect          Number of obs    =      2,500
Model VCE      : OIM
```

```
Expression   : Pr(graduate=1), predict(pmarg1)
Effect       : Average treatment effect
dydx w.r.t.  : 1.program
```

```
-----+-----
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]
ate	.0981233	.0476266	2.06	0.039	.0047769 .1914697

```
-----+-----
```

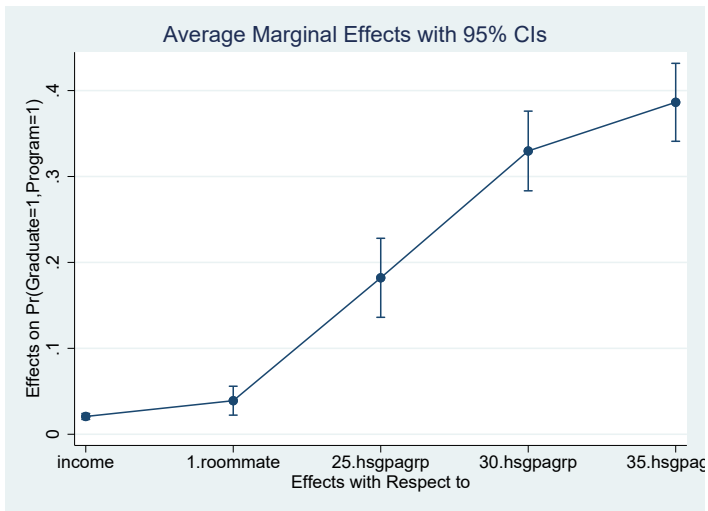
## Post-Estimation: marginsplot

Graph results from `rbiprobit margdec` and `rbiprobit tmeffects`

```
qui: rbiprobit margdec, dydx(income roommate hsgpagrp) predict(p11) effect(direct)
marginsplot
Variables that uniquely identify margins: _deriv
qui: rbiprobit tmeffects, tmeffect(ate)
marginsplot
Variables that uniquely identify margins:
```

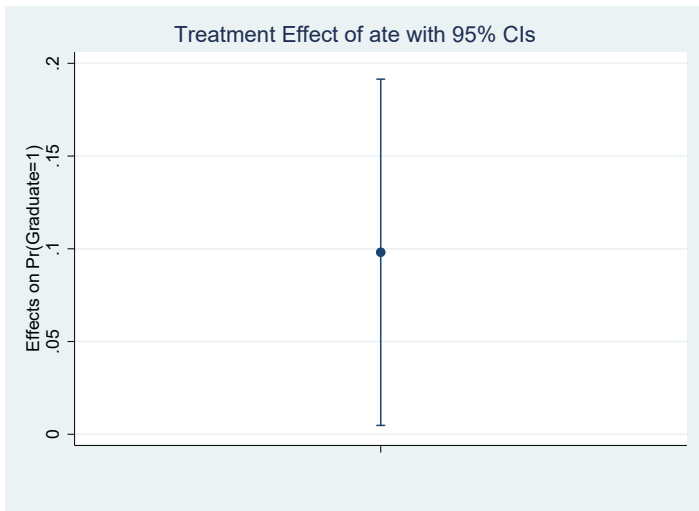
## Post-Estimation: marginsplot

Graph results from `rbiprobit margdec`



## Post-Estimation: marginsplot

Graph results from `rbiprobit tmeffects`



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# Future Additions

## Estimation and Post-Estimation Options

### 1. Estimation Options

- ▶ Weights
- ▶ Model and SE options
- ▶ Reporting options
- ▶ Maximization options

### 2. Post-Estimation Options

- ▶ Appropriate margins options (`at()`, `contrast`, etc.)
- ▶ Weights
- ▶ SE options
- ▶ Reporting options
- ▶ Maximization options

### 3. Post-Estimation Commands

- ▶ `bph1test`
- ▶ `scoregof`

# Thank you

Version 1.0.0 available

```
net install rbiprobit, from("https://raw.githubusercontent.com/cobanomics/rbiprobit/main/")
```

GitHub: [github.com/cobanomics/rbiprobit](https://github.com/cobanomics/rbiprobit)

Email: [mustafa.coban@iab.de](mailto:mustafa.coban@iab.de)

Web: [mustafacoban.de](https://mustafacoban.de)

## References

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- Han, S. and Lee, S. (2019). Estimation in a generalization of bivariate probit models with dummy endogenous regressors. *Journal of Applied Econometrics*, 34(6):994–1015.
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# Appendix

## Predictions of Interest

### 1. Joint Probabilities

$$\Pr(y_1 = 1, y_2 = 1|x, z) = \Phi_2(x'\beta + \alpha, z'\gamma, \rho)$$

$$\Pr(y_1 = 1, y_2 = 0|x, z) = \Phi_2(x'\beta, -z'\gamma, -\rho)$$

$$\Pr(y_1 = 0, y_2 = 1|x, z) = \Phi_2(-x'\beta + \alpha, z'\gamma, -\rho)$$

$$\Pr(y_1 = 0, y_2 = 0|x, z) = \Phi_2(-x'\beta, -z'\gamma, \rho)$$

### 2. Conditional Probabilities

$$\Pr(y_1 = 1|y_2 = 1, x, z) = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(z'\gamma)}$$

$$\Pr(y_2 = 1|y_1 = 1, x, z) = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(x'\beta + \alpha)}$$

# Appendix

## Predictions of Interest

### 3. Marginal Probabilities

$$\Pr(y_1 = 1|x) = \Phi(x'\beta + \alpha y_2)$$

$$\Pr(y_2 = 1|z) = \Phi(z'\gamma)$$

### 4. Unconditional Mean Function (see Blasch et al., 2019; Alrasheed, 2019)

$$\begin{aligned} E[y_1|x, z] &= \Pr(y_2 = 1|z) \cdot E[y_1|y_2 = 1, x, z] \\ &\quad + \Pr(y_2 = 0|z) \cdot E[y_1|y_2 = 0, x, z] \\ &= \Pr(y_1 = 1, y_2 = 1|x, z) + \Pr(y_1 = 1, y_2 = 0|x, z) \\ &= \Phi_2(x'\beta + \alpha, z'\gamma, \rho) + \Phi_2(x'\beta, -z'\gamma, -\rho) \end{aligned}$$

## Appendix

### Manual Changes of Dependent Variables for Predictions

```
. qui: rbiprobit graduate = income i.roommate i.hsgpagrp ///  
> , endog(program = i.campus i.scholar income i.hsgpagrp)  
  
. predict p11r, p11  
  
. qui: biprobit (graduate = income i.roommate i.hsgpagrp i.program) ///  
> (program = i.campus i.scholar income i.hsgpagrp)  
  
. replace graduate = 1  
(972 real changes made)  
  
. replace program = 1  
(1,148 real changes made)  
  
. predict p11b, p11  
  
. compare p11r p11b
```

	count	----- minimum	difference average	----- maximum
p11r<p11b	1033	-.0000178	-8.81e-06	-1.49e-08
p11r=p11b	7			
p11r>p11b	1460	2.98e-08	.0000105	.000084
jointly defined	2500	-.0000178	2.47e-06	.000084
total	2500			

# Appendix

## Incorrect Standard Errors after margins

```
. qui: rbiprobit graduate = income i.roommate i.hsgpagrp ///  
> , endog(program = i.campus i.scholar income i.hsgpagrp)
```

```
. rbiprobit margdec, dydx(income) predict(p11) effect(total)
```

```
Average marginal effects          Number of obs    =      2,500  
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)  
dy/dx w.r.t.   : income
```

```
-----  
          |          Delta-method  
          |          dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]  
-----+-----  
income |   .0032146   .002856     1.13   0.260   - .0023831   .0088123  
-----
```

```
. margins, dydx(income) predict(p11)
```

```
Average marginal effects          Number of obs    =      2,500  
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)  
dy/dx w.r.t.   : income
```

```
-----  
          |          Delta-method  
          |          dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]  
-----+-----  
income |   .0032146   .0031248     1.03   0.304   - .0029099   .0093391  
-----
```

## Appendix

### Discrete Covariate: Direct Average Marginal Effects

```
. rbiprobit margdec, dydx(hsgpagrp) predict(p11) effect(direct)
```

```
Average marginal effects          Number of obs      =          2,500  
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)  
dy/dx w.r.t.   : 25.hsgpagrp 30.hsgpagrp 35.hsgpagrp
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
hsgpagrp						
2.5-2.9	.1821001	.0234585	7.76	0.000	.1361223	.2280779
3.0-3.4	.3297082	.0236584	13.94	0.000	.2833386	.3760777
3.5-4.0	.386345	.0231423	16.69	0.000	.3409869	.4317032

Note: dy/dx for factor levels is the discrete change from the base level.



# Appendix

## Discrete Covariate: Indirect Average Marginal Effects

```
. rbiprobit margdec, dydx(hsgpagrp) predict(p11) effect(indirect)
```

```
Average marginal effects      Number of obs      =      2,500  
Model VCE      : OIM
```

```
Expression      : Pr(graduate=1,program=1), predict(p11)  
dy/dx w.r.t.   : 25.hsgpagrp 30.hsgpagrp 35.hsgpagrp
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
hsgpagrp						
2.5-2.9	.0131266	.0246783	0.53	0.595	-.035242	.0614953
3.0-3.4	.0145757	.0258243	0.56	0.572	-.036039	.0651905
3.5-4.0	-.0219075	.0402565	-0.54	0.586	-.1008087	.0569938

Note: dy/dx for factor levels is the discrete change from the base level.

# Appendix

## Average Treatment Effect on the Treated

```
. rbiprobit tmeffects, tmeffect(atet)
```

```
Treatment effect          Number of obs    =      1,352  
Model VCE      : OIM
```

```
Expression   : normal(graduate=1|program=1) - normal(graduate=1|program=0)  
Effect       : Average treatment effect on the treated  
dydx w.r.t.  : 1.program
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
atet	.1033448	.0489003	2.11	0.035	.0075019	.1991877

# Appendix

## Average Treatment Effect on the Conditional Probability

```
. rbiprobit tmeffects, tmeffect(atec)
```

```
Treatment effect                Number of obs    =      2,500  
Model VCE      : OIM
```

```
Expression   : Pr(graduate=1|program=1)-Pr(graduate=1|program=0), predict(pcond1)-  
> ict(pcond10)  
Effect      : Average treatment effect on conditional probability  
dydx w.r.t. : 1.program
```

```
-----  
          |          Delta-method  
          |          dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]  
-----+-----  
    atec |    .2765848    .0164366    16.83    0.000    .2443696    .3087999  
-----
```