

# Applications of Generalized Structural Equation Modeling for Enhanced Credit Risk Management <sup>1</sup>

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<sup>1</sup> The views expressed are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. Any errors or omissions are the responsibility of the authors. The authors thank Gerald Rama for outstanding assistance on this project. Corresponding author: [jose.canals-cerda@phil.frb.org](mailto:jose.canals-cerda@phil.frb.org).

## MOTIVATION OF THIS PRESENTATION:

That the GSEM framework holds great potential for the analysis of risk in consumer credit portfolios.

The GSEM framework can assist the risk management profession on the development of a holistic approach to model building that can simplify and enhance each step of the model building process.

We illustrate the potential of GSEM with two empirical examples.

What topics are we going to cover in this presentation?

- I. We introduce the “workhorse” loss projection framework typical in the risk management of consumer finance portfolios.
- II. We review the empirical literature and highlight areas where GSEM can have an impact.
- III. We introduce the data that we use in our empirical examples.
- IV. We present examples of empirical applications of GSEM.
- V. We discuss results from the empirical implementation of GSEM.
- VI. We conclude with some final thoughts.

## Consumer finance portfolios and associated “stress” loss rates.<sup>2</sup>

USA TOTAL As of 2020:Q1	# accounts (millions)	\$ balance (Trillions)
MORTGAGE LOANS	81.1	9.7
HOME EQUITY LOANS	14.82	0.39
AUTO LOANS	116.43	1.35
CREDIT CARD LOANS	511.41	0.89
STUDENT LOANS		1.54
OTHER		0.43
<b>TOTAL CONSUMER DEBT</b>	<b>14.3</b>	

### PROJECTED PORTFOLIO LOSSES FOR CCAR BANKS IN THE 2020 STRESS TEST

Projected loan losses, by type of loan, 2020:Q1–2022:Q1		
Loan type	Billions of dollars	Portfolio loss rates (percent) <sup>1</sup>
Loan losses	432.5	6.3
First-lien mortgages, domestic	19.4	1.5
Junior liens and HELOCs, domestic	7.4	3.1
Commercial and industrial <sup>2</sup>	114.0	7.2
Commercial real estate, domestic	47.6	6.3
Credit cards	144.0	17.1
Other consumer <sup>3</sup>	48.4	6.5
Other loans <sup>4</sup>	51.7	3.6

<sup>1</sup> Average loan balances used to calculate portfolio loss rates exclude loans held for sale and loans held for investment under the fair-value option, and are calculated over nine quarters.

<sup>2</sup> Commercial and industrial loans include small- and medium-enterprise loans and corporate cards.

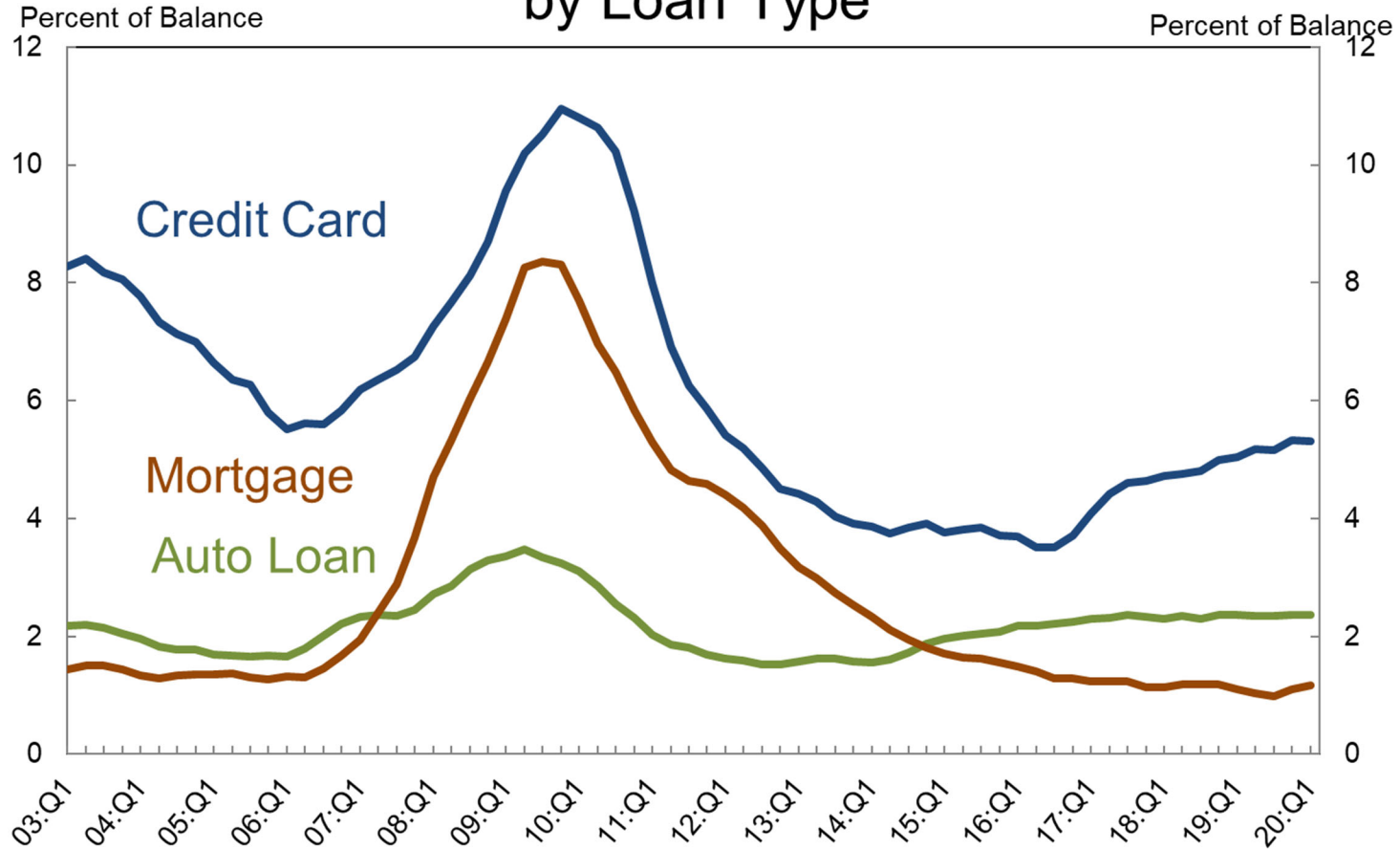
<sup>3</sup> Other consumer loans include student loans and automobile loans.

<sup>4</sup> Other loans include international real estate loans.

<sup>2</sup> <https://www.newyorkfed.org/microeconomics/hhdc/background.html>  
<https://www.federalreserve.gov/publications/files/2020-dfast-results-20200625.pdf>

# Consumer finance loans, performance over the business cycle.

## Transition into Serious Delinquency (90+) by Loan Type



Source: New York Fed Consumer Credit Panel/Equifax

Note: 4 Quarter Moving Sum  
Student loan data are not reported prior to 2004 due to uneven reporting

The “workhorse” loss projection framework in consumer finance.

A FINANCE COMPANY EXPERIENCES A LOSS ON A LOAN WHEN:

1. The loan defaults (D)
2. The loan collateral (C) is less than the exposure at default (EAD), or unpaid remaining balance on the loan.

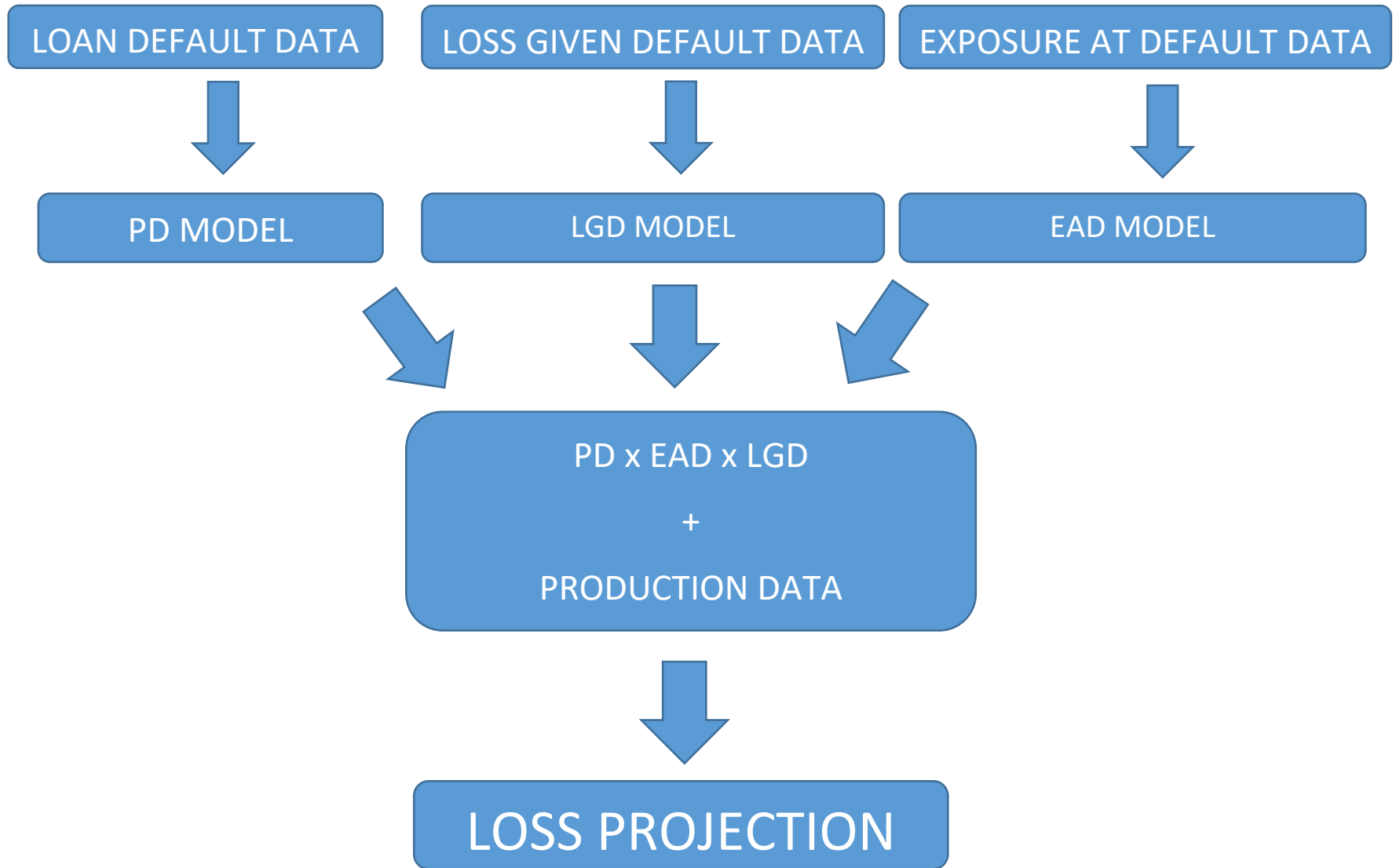
When (1) and (2) occur, the bank experiences a loss (L), with a resulting loss rate, or loss given default (LGD), equal to

$$\text{LGD} = L/\text{UPB} \text{ or } L/\text{EAD}$$

$$\text{Expected loss} = \text{Prob. Default} \times \text{EAD} \times \text{LGD}$$

This is a common parametrization, but not the only one!

A closer look at the standard loss projection framework.



Publicly circulated studies in consumer finance have embraced a piecemeal approach to model building, rather than a holistic approach.

<b>CREDIT RISK<sup>3</sup></b>	PD	LGD	LOSS
Deng, Y., & Gabriel, S. (2006). Risk-Based Pricing and the Enhancement of Mortgage Credit Availability among Underserved and Higher Credit-Risk Populations.	yes	No	No
Kristopher S. Gerardi, A. Lehnert, S. M. Sherlund, P. Willen (2009). Making Sense of the Subprime Crisis Brookings Papers on Economic Activity 39(2 (Fall)):69-159.	yes	No	No
Anthony Pennington-Cross (2003). Credit History and the Performance of Prime and Nonprime Mortgages.			imputed
Jason Thomas, Robert Van Order (2018) <i>“Fannie Mae and Freddie Mac: Risk Taking and the Option to Change Strategy”</i>	yes	No	No
<b>CECL<sup>4</sup></b>	yes	yes	yes
Chae, Sarah, Robert Sarama, Cindy Vojtech and James Wang. (2018) <i>“The Impact of the Current Expected Credit Loss Standard (CECL) on the Timing and Comparability of Reserves.”</i>	yes	imputed	imputed
DeRitis, Christian and Mark Zandi. (2018) <i>“Gauging CECL Cyclicity.”</i>	yes	imputed	imputed
<b>STRESS TESTING, REGULATIONS AND ACCOUNTING STANDARDS</b>			
W. Scott Frame, Kristopher Gerardi, and Paul S. Willen (2015). The Failure of Supervisory Stress Testing: Fannie Mae, Freddie Mac, and OFHEO.	yes	imputed	imputed
The Basel II framework advanced approach	yes	yes	yes
Federal Housing Finance Agency, NPR (2018). Enterprise Capital Requirements. <sup>5</sup>	imputed	imputed	imputed
Regulatory Stress Tests	yes	yes	yes
CECL <sup>6</sup>	na	na	yes

<sup>3</sup> Many other papers have tackled the problem of loan default/prepayment, including Deng (1997), Ambrose and Capone (2000), Deng, Quigley, and Van Order (2000), Calhoun and Deng (2002), Pennington-Cross (2003), Deng, Pavlov, and Yang (2005), Clapp, Deng, and An (2006), and Pennington-Cross and Chomsisengphet (2007).

<sup>4</sup> Chae et al. considers a simple imputation of LGD= 0.3. Similarly, DeRitis and Zandi considers LGD= 0.35.

<sup>5</sup> Federal Register, Vol. 83, No. 137, Tuesday, July 17, 2018, Proposed Rules.

<sup>6</sup> CECL considers a principles based rule framework and is agnostic about loss projection methodology, although guidance on best practices is emerging.



## The dangers of piecemeal model development.<sup>7 8</sup>



September 2003: The Spanish government approved the purchase of four S-80A submarines.

May 2013: Navantia announced that a serious weight imbalance design flaw had been identified. “a ‘misplaced decimal’ point caused the designers to overshoot the submarine’s planned 2,300-ton displacement by 70 to 125 tons.”

A team was hired from General Dynamics for 14 million euros. It concluded that the easiest way to fix the buoyancy issue was to lengthen the S-80 from 71 to 81 meters, which also increased the weight from 2,300 to 3,300 tons submerged!

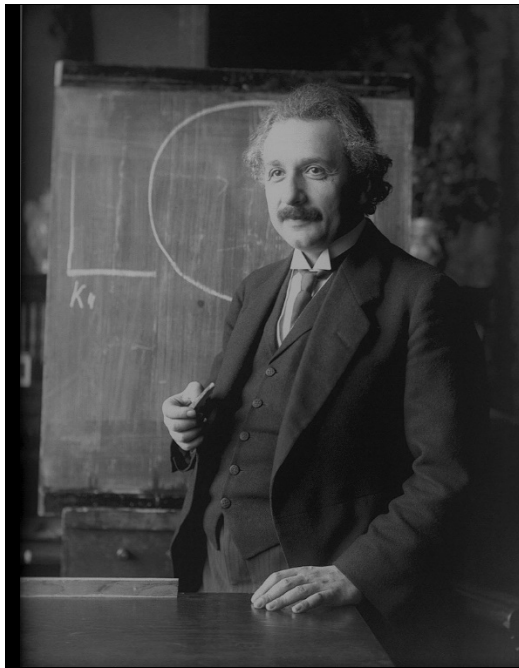
The now eighty-one-meter long S-80 Plus submarines won’t fit in the seventy-eight-meter-long docks at Cartagena, apparently necessitating a €16 million expansion project.

<sup>7</sup> [https://en.wikipedia.org/wiki/File:Tramontana\\_S74.jpg](https://en.wikipedia.org/wiki/File:Tramontana_S74.jpg)

Note, the picture is from a tramontane submarine rather than an S80-Plus Class submarine, currently in construction.

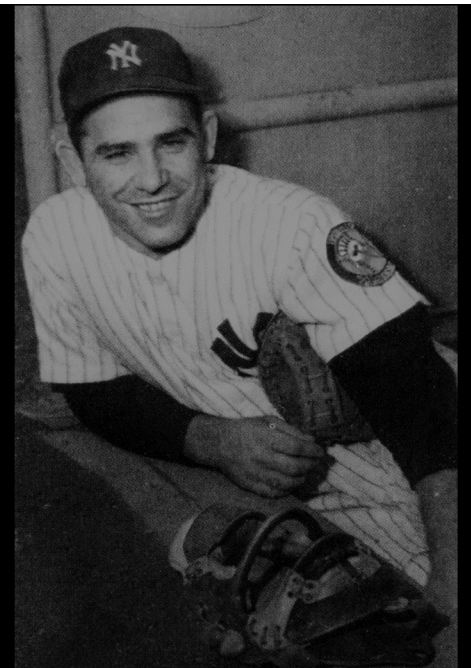
<sup>8</sup> <https://nationalinterest.org/blog/buzz/spain%E2%80%99s-billion-dollar-ethanol-powered-s-80-super-submarines-are-too-big-fit-their-docks>

Brilliant minds think alike.



In theory, theory and practice  
are the same. In practice, they  
are not. *Albert Einstein*

In theory, there is no difference  
between theory and practice. But in  
practice, there is. *Fogi Berra*



GSEM CAN BE INSTRUMENTAL WHEN APPLYING A WHOLISTIC APPROACH TO MODEL BUILDING.

### A MODEL OF PREPAY/DEFAULT/LOSS:

Consider a portfolio of loans characterized by a vector of loan characteristics  $Z_i$  and outcomes:

default (0), prepay (1), still active (2) AND loss ( $l$ ) if default

- Default can be represented in the form of a multinomial logit probability conditional on a set of risk drivers  $X_{it} = (Z_i, M_{it})$  where  $Z_i$  represents characteristics of the loan at observation time  $t$  and  $M_{it}$  represents a set of macroeconomic risk drivers specific to a specific time interval.

$$p_{it} = \frac{\exp(X_{it}\beta_i)}{\sum_{j=1}^2 \exp(X_{ij}\beta_j)} \quad i = 1,2 \quad p_{i0} = \frac{1}{\sum_{j=1}^2 \exp(X_{ij}\beta_j)}$$

- Loss given default can be represented by a simple linear specification:  $lgd_t = X_t\delta$

We can use this model to project,

$$\begin{array}{ll} \text{Default probability:} & \widehat{p}_{0t} \\ \text{Prepay probability:} & \widehat{p}_{1t} \\ \text{Expected loss:} & \widehat{p}_{0t} \cdot \widehat{lgd}_t \end{array}$$

# MODEL 1: EMPIRICAL IMPLEMENTATION OF A BENCHMARK MODEL OF PREPAY/DEFAULT/LOSS OVER A 9-QUARTER PERIOD.

```
gsem (lgd_9q <- `...') (0b.out_9q 1.out_9q 2.out_9q <- `...')
```

Generalized structural equation model	Number of obs	=	4,509,622
Response : lgd_upto9q	Number of obs	=	22,914
Family : Gaussian			
Link : identity			
Response : reason_exit_9q_typ~1	Number of obs	=	4,509,622
Base outcome : 0			
Family : multinomial			
Link : logit			

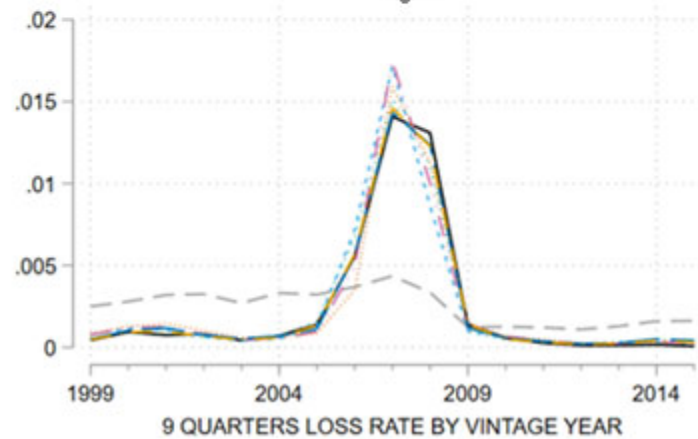
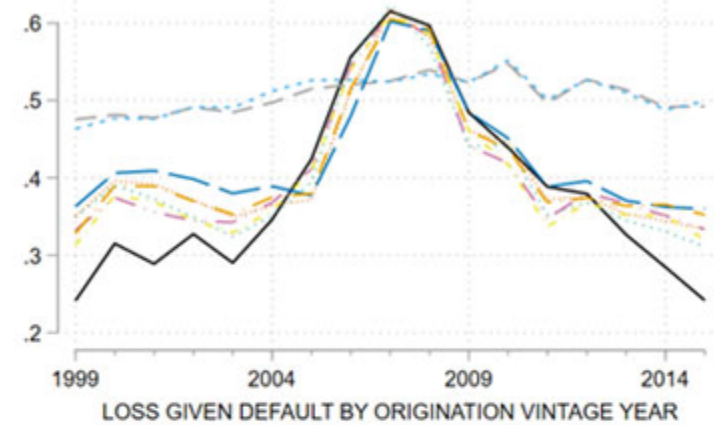
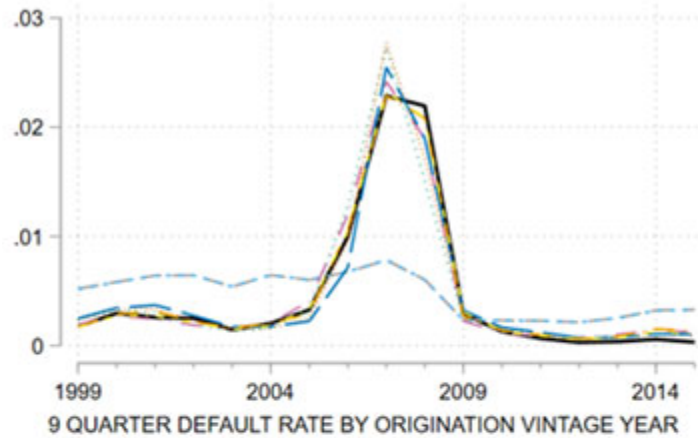
Typical output ... for a very simple model specification.

```

-----
                |      Coef.   Std. Err.   z   P>|z|   [95% Conf. Interval]
-----+-----
lgd_upto9q
  o_d_fico_b
    1 |      .0033537   .0053809    0.62   0.533   - .0071928   .0139001
    2 |      .0115168   .0056724    2.03   0.042    .000399   .0226345
    3 |     -.0186205   .0058764   -3.17   0.002   - .0301381   -.007103
  o_d_cltv_b
    1 |     -.0157983   .0056197   -2.81   0.005   - .0268127   -.0047839
    2 |     -.1637341   .0053727  -30.48   0.000   - .1742643   -.1532038
    3 |     -.2758061   .005835   -47.27   0.000   - .2872425   -.2643697
    _cons |      .6246028   .0048044  130.01   0.000    .6151864   .6340192
-----+-----
0.r~q_type_1 | (base outcome)
-----+-----
1.r~q_type_1
  o_d_fico_b
    1 |      .4611256   .0178318   25.86   0.000    .4261759   .4960752
    2 |      .9064634   .0187568   48.33   0.000    .8697009   .943226
    3 |      1.762915   .0195608   90.12   0.000    1.724577   1.801253
  o_d_cltv_b
    1 |     -.5529025   .0184509  -29.97   0.000   - .5890656   -.5167395
    2 |     -1.033925   .0178432  -57.95   0.000   -1.068897   -.9989535
    3 |     -.8636082   .0194302  -44.45   0.000   - .9016907   -.8255256
    _cons |      3.718394   .0164882  225.52   0.000    3.686078   3.75071
-----+-----
2.r~q_type_1
  o_d_fico_b
    1 |      .....
    2 |      .....
    3 |      .....
    _cons |      .....
-----+-----
var(e.lgd~9q) |      .0942719   .0008807                .0925614   .0960141
-----

```

The overarching goal is the projection of losses ... GSEM estimation can offer a wholistic view on the task.



## EMPIRICAL EXAMPLES ... THE DATA

I employ a publicly available mortgage panel dataset of loans originated between 1999 and 2015, including their historical performance information.

This dataset is available from Freddie Mac, which is making available loan-level credit performance data on a portion of fully amortizing fixed-rate mortgages that the company purchased or guaranteed as part of a larger effort to increase transparency.<sup>9</sup>

The dataset covers approximately 22.5 million fixed-rate mortgages originated between January 1, 1999, and September 30, 2015.

Our sample represents a 25% random sample of the overall data.

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<sup>9</sup> Comprehensive information about the dataset described in this section, including access to the overall dataset, is available from [http://www.freddiemac.com/news/finance/sf\\_loanlevel\\_dataset.html](http://www.freddiemac.com/news/finance/sf_loanlevel_dataset.html). Much of the data description in this section is extracted directly from the information provided at this website.

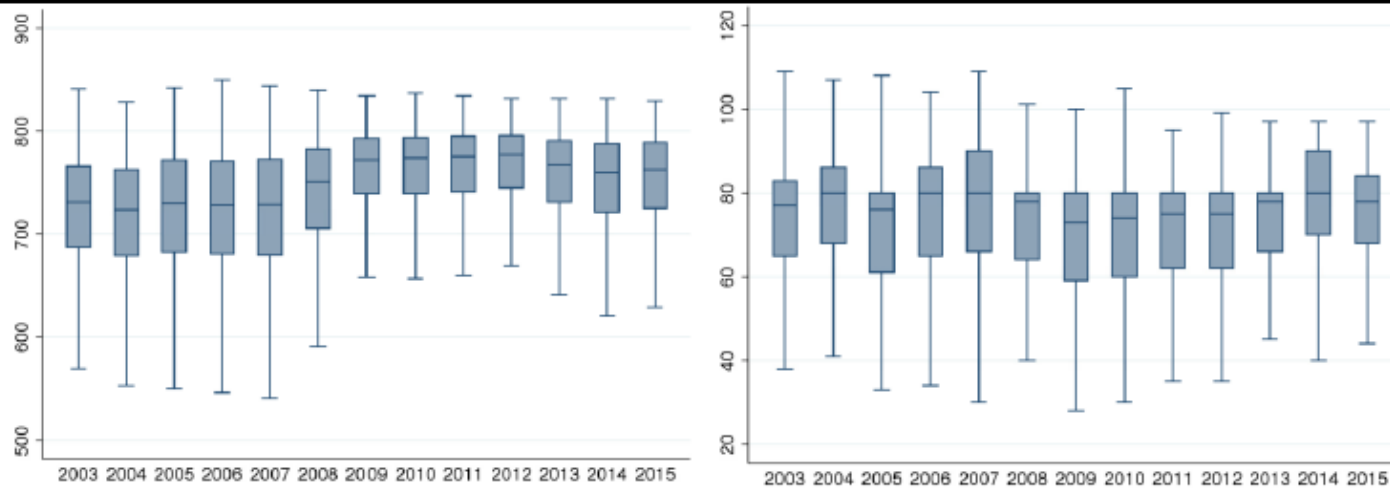


**Table 2: Relevant Variable Definitions**

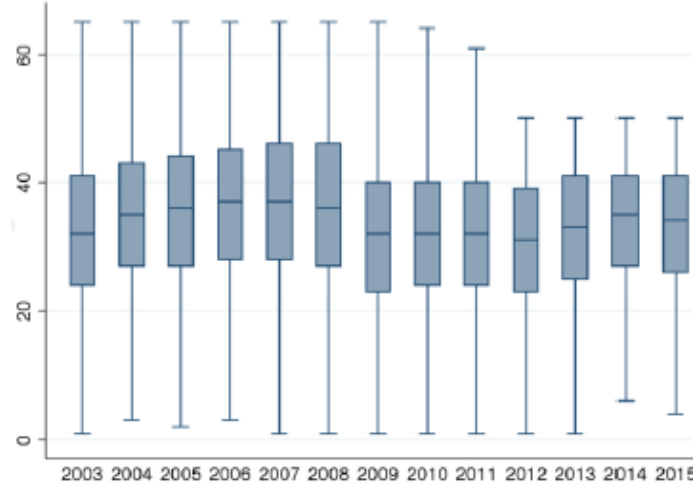
<i>Acyclical variables</i>	
<b>Account age</b>	Categorical controls for account age in years
<b>FICO score</b>	Categorical controls for credit score range at origination for the following ranges: up to 580, 580–620, 620–650, 650–680, 680–720, 720–760, 760–900
<b>Debt-to-income</b>	Categorical controls for debt-to-income at origination for the following ranges: less than 20, 20–30, 30–35, 35–40, 40–45, more than 45
<b>LTV</b>	Categorical controls for LTV at origination for the following ranges: less than 75%, 75–80%, 80–85%, 85–90%, 90–95%, 95–100%, 100–105%, 105–110%, more than 110%
<b>Interest rate spread</b>	Interest rate spread at origination, measured with respect to the 10-year Treasury note ratio
<b>Borrowers</b>	Categorical control for number of borrowers
<b>Purpose</b>	Categorical control for loan purpose
<b>Loan balance</b>	Categorical controls for loan balance range at origination for the following ranges: less than 75K, 75–100K, 100–150K, 150–250K, 250–325K, more than 325K
<b>Occupancy type</b>	Categorical control for occupancy type
<b>First-time buyer</b>	First-time buyer dummy.
<b>Judiciary</b>	Dummy for judiciary state
<i>Cyclical variables – updated account information</i>	
<b>Delinquency history</b>	Specific risk drivers derived from delinquency history
<b>Highest del. in the past 12 months</b>	Highest delinquency history over the past 12 months
<b>Delinquency status</b>	Updated delinquency status at observation time
<b>Equity ratio</b>	Categorical controls for updated equity ratio using appraisal at origination combined with a price index updated history and the updated loan amount
<i>Cyclical variables – macroeconomic risk drivers</i>	
<b>Interest rate spread</b>	Updated interest rate spread, measured with respect to the 10-year treasury Note
<b>House price index change</b>	Updated 12-month home price index change
<b>Unemployment</b>	Updated unemployment rate
<b>Unemployment change</b>	Updated change in unemployment rate



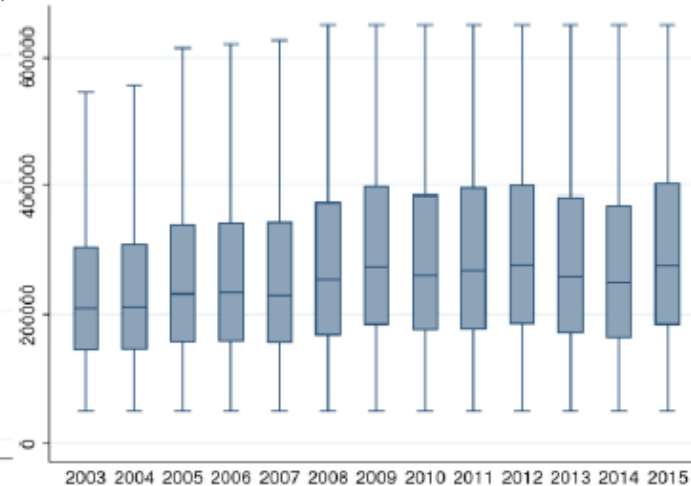
**Figure 2: Distribution of Origination Risk Drivers Across Origination Years**



*Origination Credit Score*



*Origination Combined LTV*



*Origination Debt-to-Income*

*Origination Appraisal Amount*

Note: For each origination year, the table presents loan characteristics at origination for that particular year.

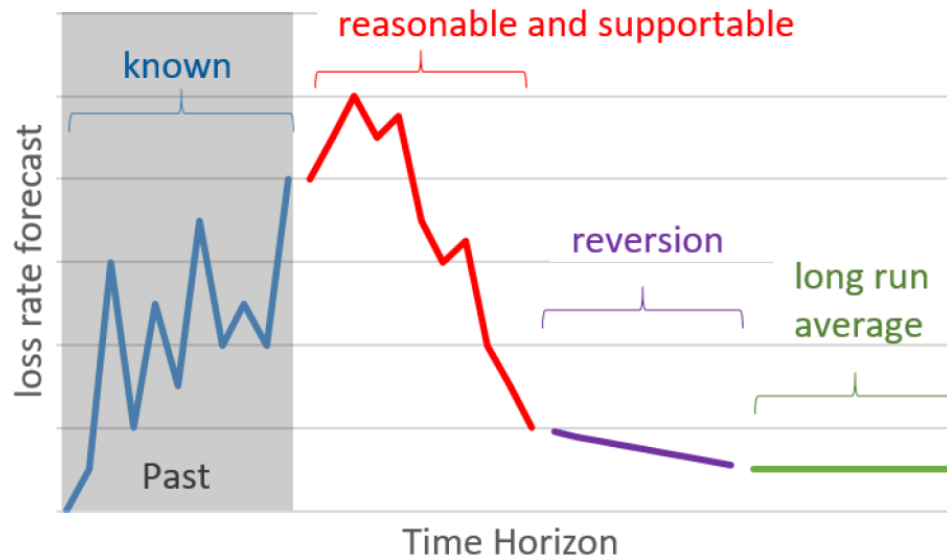
## SPECIFIC USE CASE APPLICATIONS:

1. STRESS TESTS: Financial institutions regularly conduct stress tests of their consumer finance portfolios in order to ascertain the potential for significant financial loss under “tail loss” economic conditions. In recent years, it has become typical industry practice to project loss over a 9-quarter period.
2. The allowance for loan and lease losses (ALLL): is an estimate of uncollectible amounts used to reduce the book value of loans and leases to the amount that a bank expects to collect.

<b>Net loans</b>	376
Gross Loans	381
Loan Loss Allowances	-5

The novel allowance framework requires an organization to measure all expected credit losses for financial assets held at the reporting date based on historical experience, current conditions, and reasonable and supportable forecasts<sup>1011</sup>

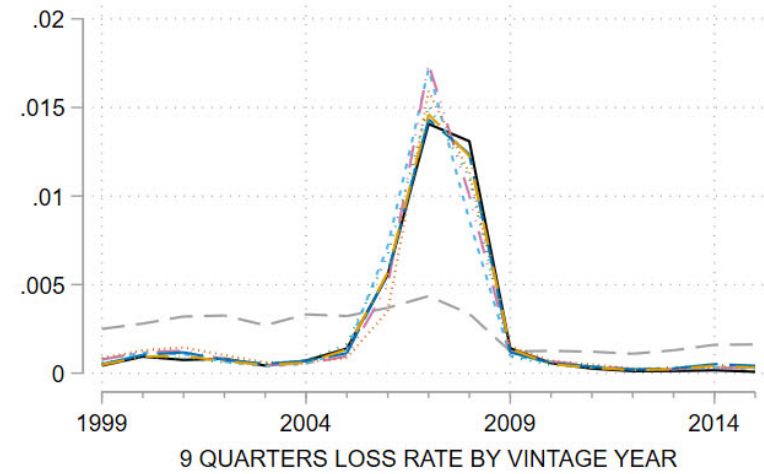
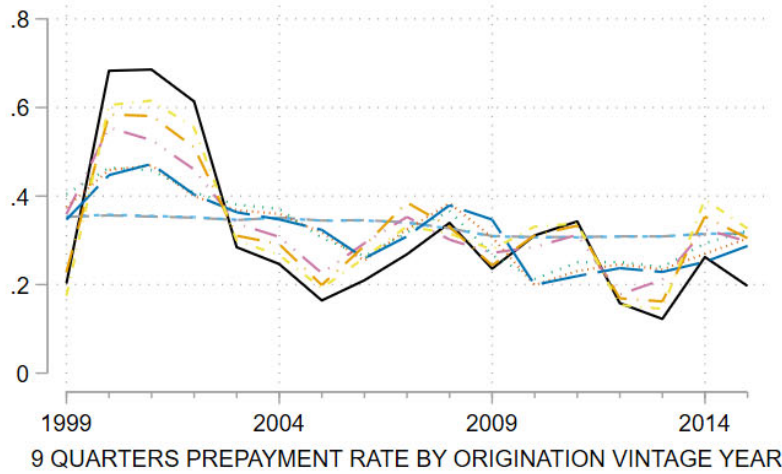
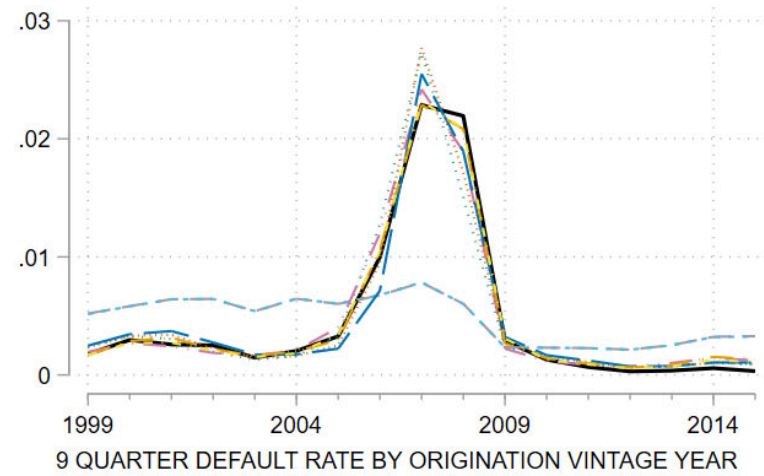
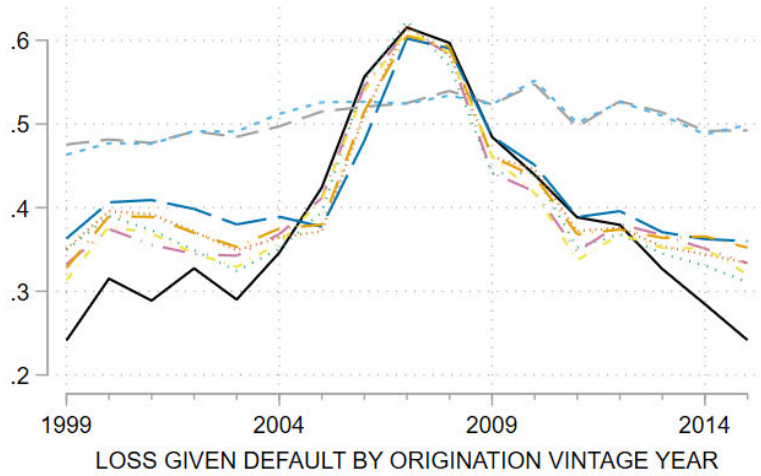
*Loss Rate Forecast Overview*



<sup>10</sup> Figure on the left is Figure 2 in Loudis and Ranish (2019).

<sup>11</sup> In June 16 2016 FASB issued the “Accounting Standards Update No. 2016-13” an important component this standard was a novel allowance framework, the “Current Expected Credit Loss” or CECL.

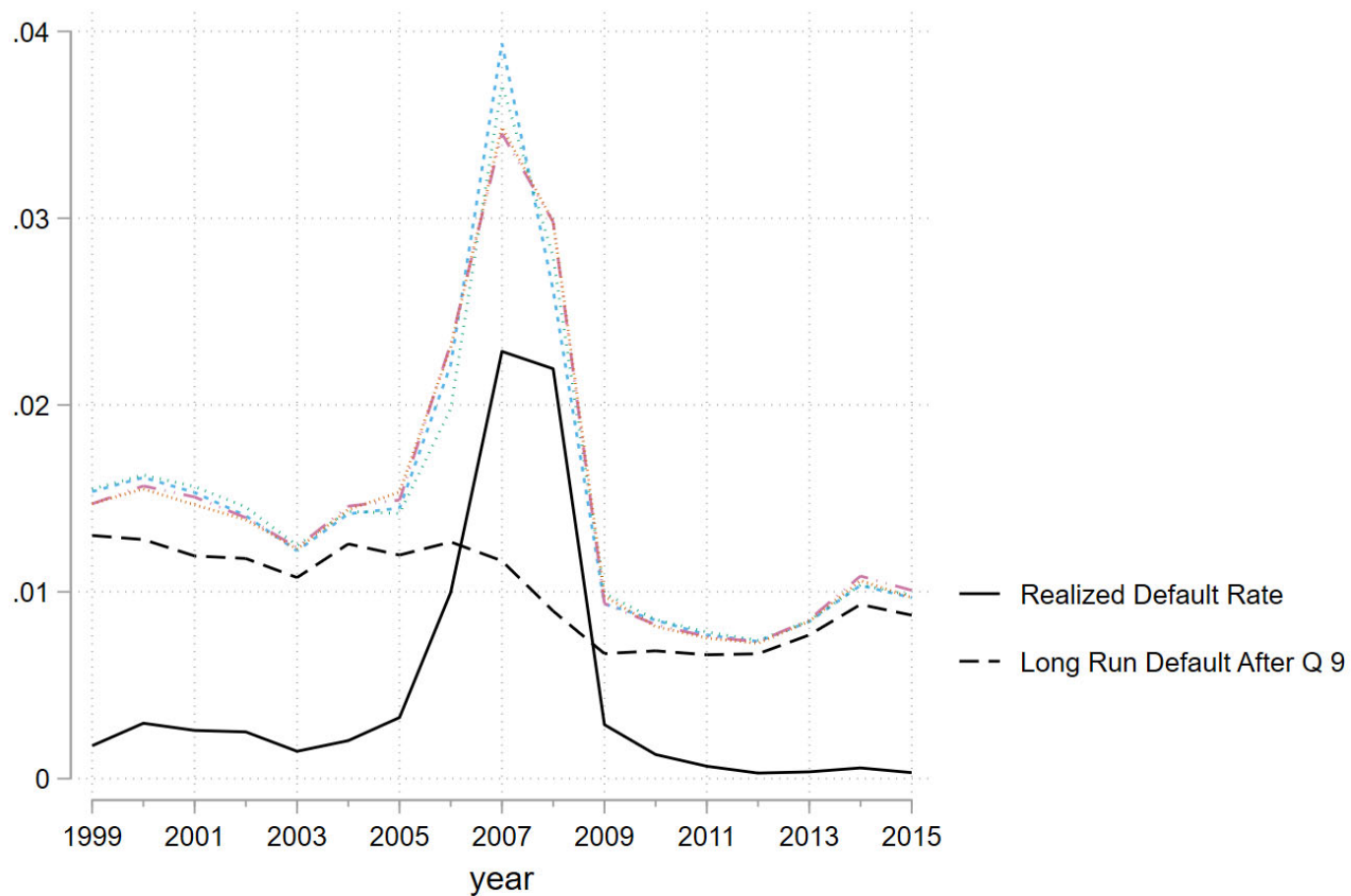
# MODEL 1: EMPIRICAL RESULTS OF A BENCHMARK 9Q MODEL OF PREPAY/DEFAULT/LOSS.



Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

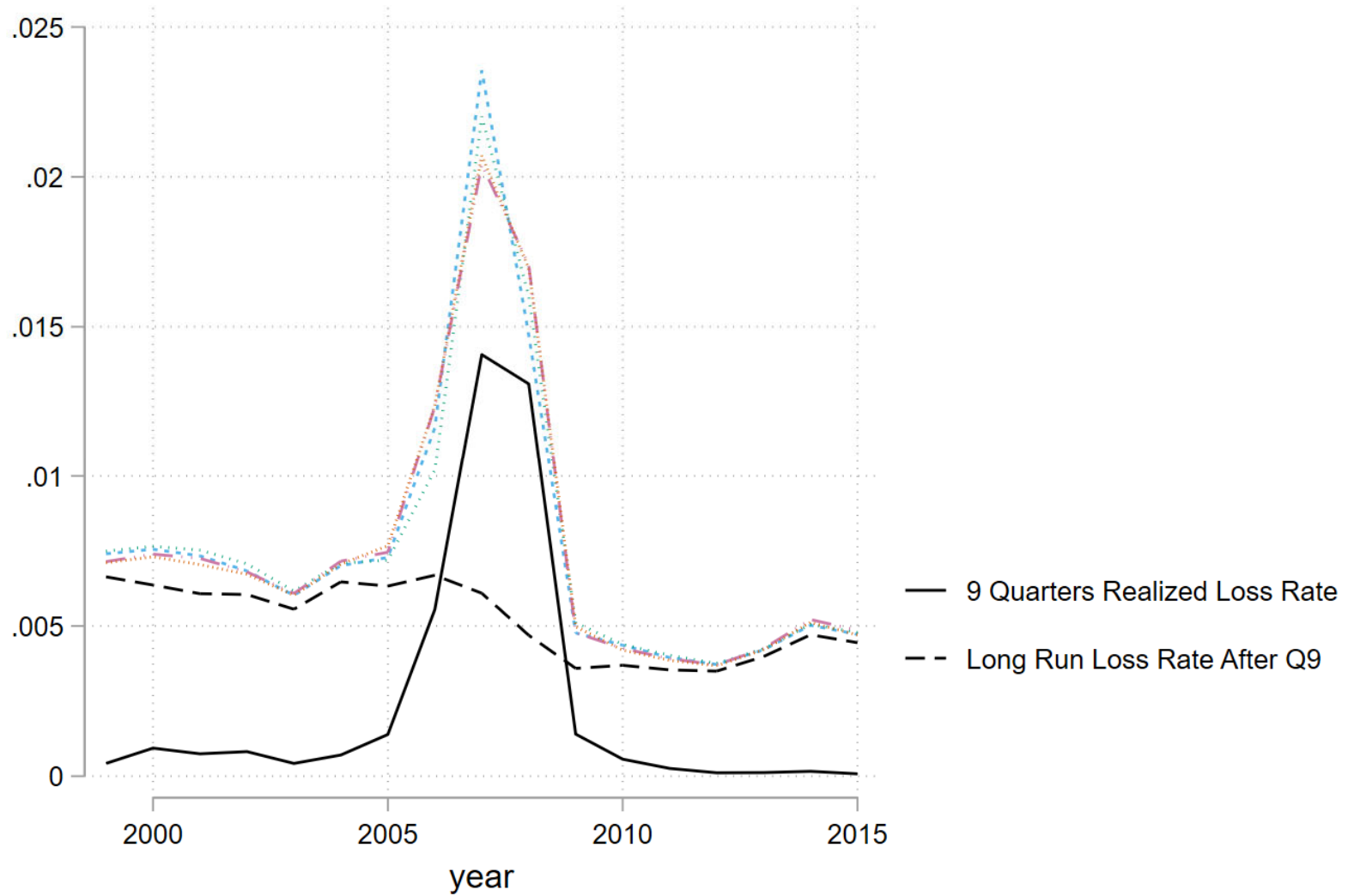
# MODEL 1: ALL AGGREGATED RESULTS EXPANDING THE BENCHMARK MODEL BEYOND 9 QUARTERS.

```
gsem (lgd_9q <- `...`) (0b.out_9q 1.out_9q 2.out_9q <- `...`) ///  
(lgd_9to20q <- `...`) (0b.def_9to20 1.def_9to20 <- `...`)
```



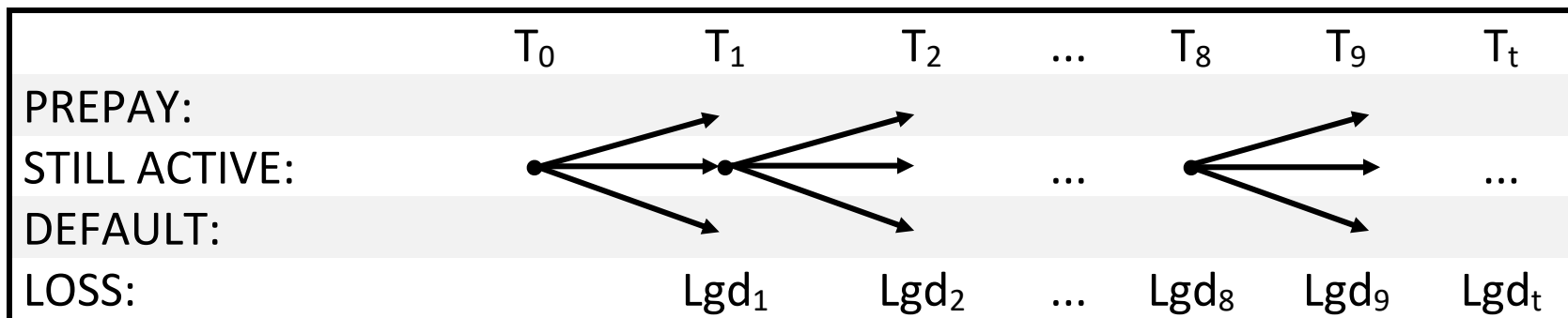
Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

# MODEL 1: ALLL AGGREGATED RESULTS (LOSS RATE).



Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

## MODEL 2: EMPIRICAL IMPLEMENTATION OF A QUARTERLY MODEL OF PREPAY/DEFAULT/LOSS:



```
gsem (lgd_q1 <- `...`) (0b.out_q1 1.out_q1 2.out_q1 <- `...`) ///
    ... ///
    (lgd_q9 <- `...`) (0b.out_q9 1.out_q9 2.out_q9 <- `...`)
```

## MODEL 2: EXPANDING THE QUARTERLY MODEL BEYOND 9 QUARTERS.

```
gsem (lgd_q1 <- `...`) (0b.out_q1 1.out_q1 2.out_q1 <- `...`) ///
    ... ///
    (lgd_q9 <- `...`) (0b.out_q9 1.out_q9 2.out_q9 <- `...`) ///
    (lgd_9to20q <- `...`) (0b.def_9to20 1.def_9to20 <- `...`)
```

## GSEM OUTPUT FOR THE QUARTERLY MODEL.

```
Generalized structural equation model      Number of obs   =  4,434,522

Response      : lgd1                      Number of obs   =      867
Family        : Gaussian
Link          : identity

Response      : mlogit_9q_out1            Number of obs   =  4,434,522
Base outcome  : 2
Family        : multinomial
Link         : logit

Response      : lgd2                      Number of obs   =      1,132
Family        : Gaussian
Link         : identity

Response      : mlogit_9q_out2            Number of obs   =  4,381,984
Base outcome  : 2
Family        : multinomial
Link         : logit

.....

Response      : lgd9                      Number of obs   =      3,951
Family        : Gaussian
Link         : identity

Response      : mlogit_9q_out9            Number of obs   =  3,096,979
Base outcome  : 2
Family        : multinomial
Link         : logit
```



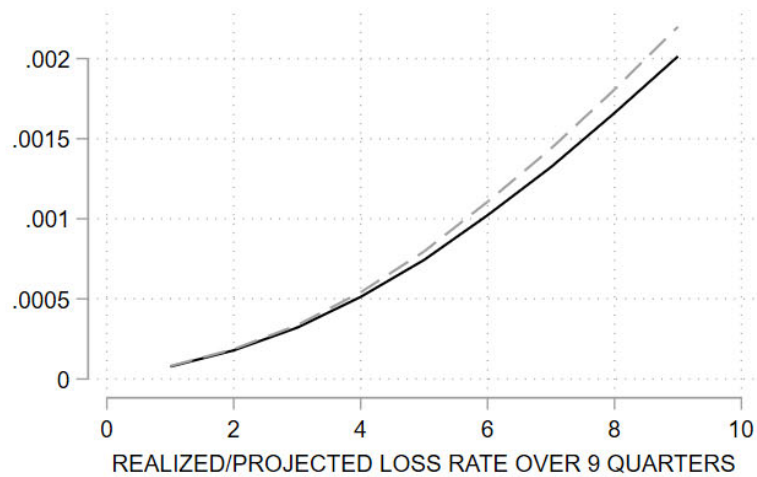
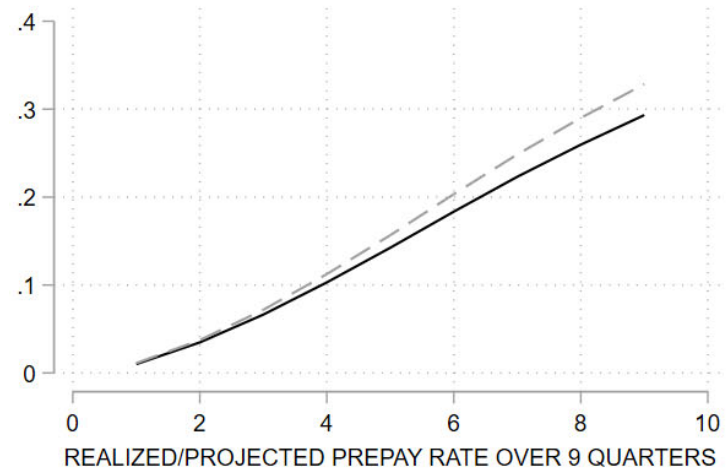
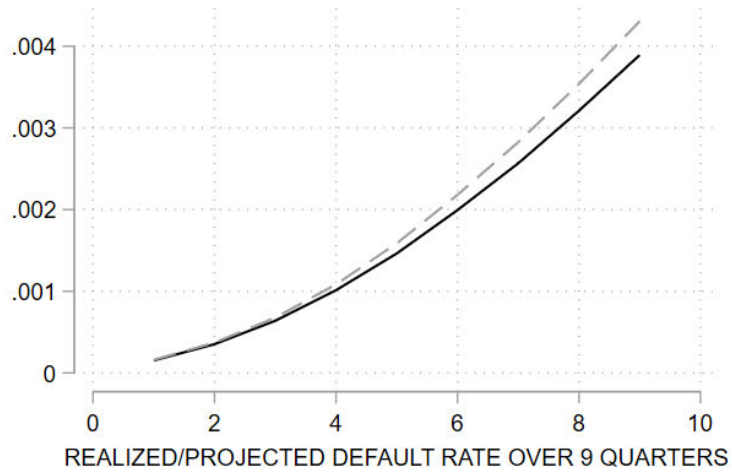
Typical output ... for a very simple model specification.

```

-----+-----
                |      Coef.   Std. Err.      z    P>|z|      [95% Conf. Interval]
-----+-----
lgd_upto9q |
  o_d_fico_b |
    1 |      .0033537   .0053809     0.62   0.533    - .0071928   .0139001
    2 |      .0115168   .0056724     2.03   0.042     .000399   .0226345
    3 |     -.0186205   .0058764    -3.17   0.002    - .0301381  -.007103
  o_d_cltv_b |
    1 |     -.0157983   .0056197    -2.81   0.005    - .0268127  -.0047839
    2 |     -.1637341   .0053727   -30.48   0.000    - .1742643  -.1532038
    3 |     -.2758061   .005835   -47.27   0.000    - .2872425  -.2643697
    _cons |      .6246028   .0048044   130.01   0.000     .6151864   .6340192
-----+-----
0.r~q_type_1 | (base outcome)
-----+-----
1.r~q_type_1 |
  o_d_fico_b |
    1 |      .4611256   .0178318    25.86   0.000     .4261759   .4960752
    2 |      .9064634   .0187568    48.33   0.000     .8697009   .943226
    3 |      1.762915   .0195608    90.12   0.000     1.724577   1.801253
  o_d_cltv_b |
    1 |     -.5529025   .0184509   -29.97   0.000    - .5890656  -.5167395
    2 |     -1.033925   .0178432   -57.95   0.000    -1.068897  -.9989535
    3 |     -.8636082   .0194302   -44.45   0.000    - .9016907  -.8255256
    _cons |      3.718394   .0164882   225.52   0.000     3.686078   3.75071
-----+-----
2.r~q_type_1 |
  o_d_fico_b |
    1 |      .....
    2 |      .....
    3 |      .....
    _cons |      .....
-----+-----
var(e.lgd~9q) |      .0942719   .0008807                .0925614   .0960141
-----+-----

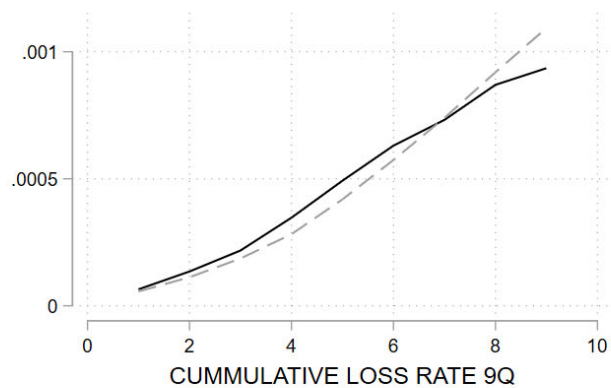
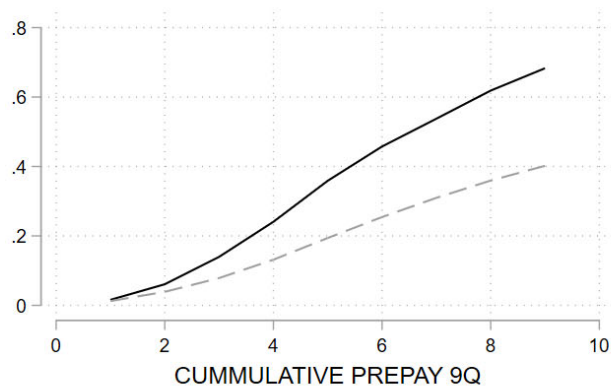
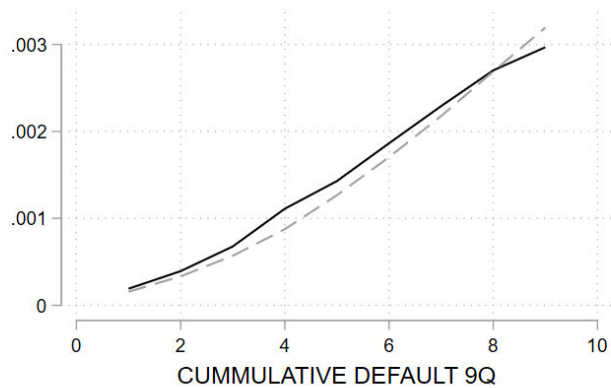
```

## MODEL 2: EMPIRICAL IMPLEMENTATION OF A QUARTERLY MODEL OF PREPAY/DEFAULT/LOSS



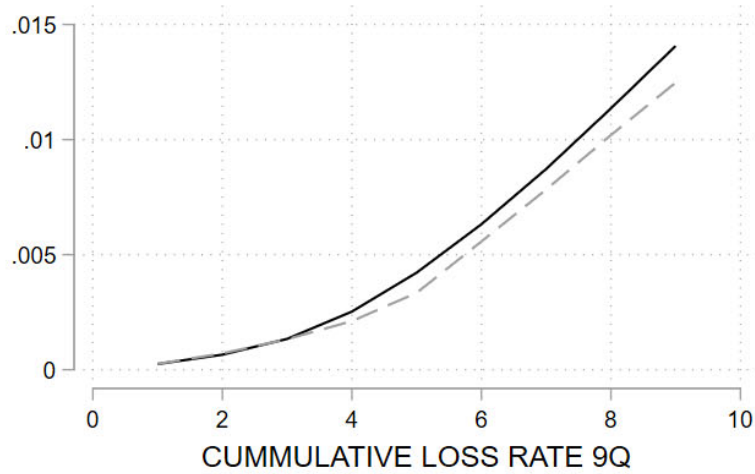
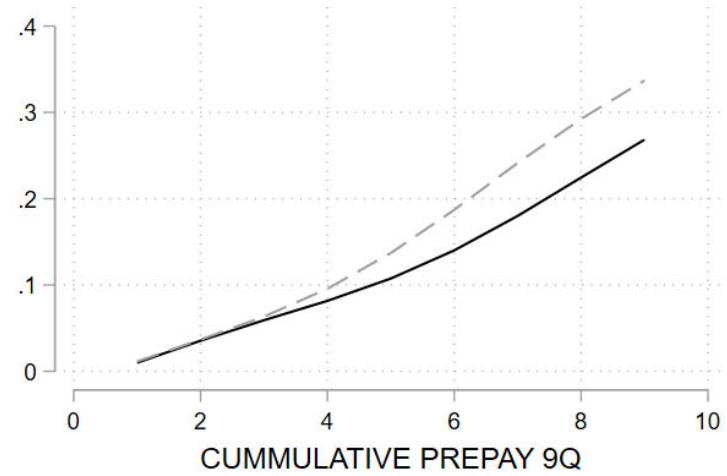
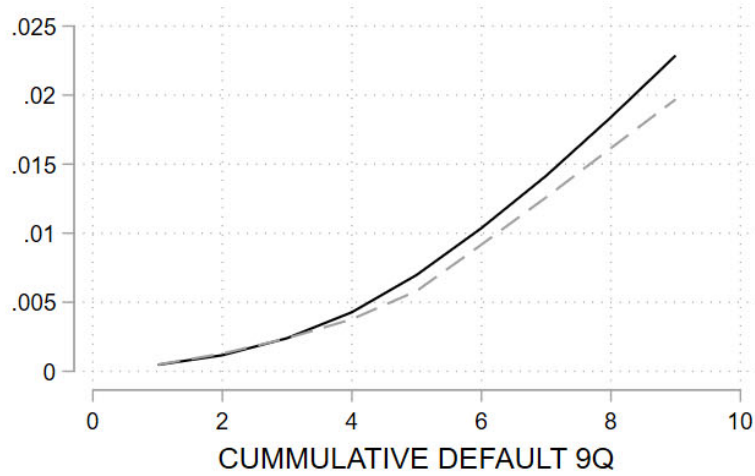
Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

## MODEL 2: QUARTERLY MODEL OF PREPAY/DEFAULT/LOSS, YEAR 2000 VINTAGE



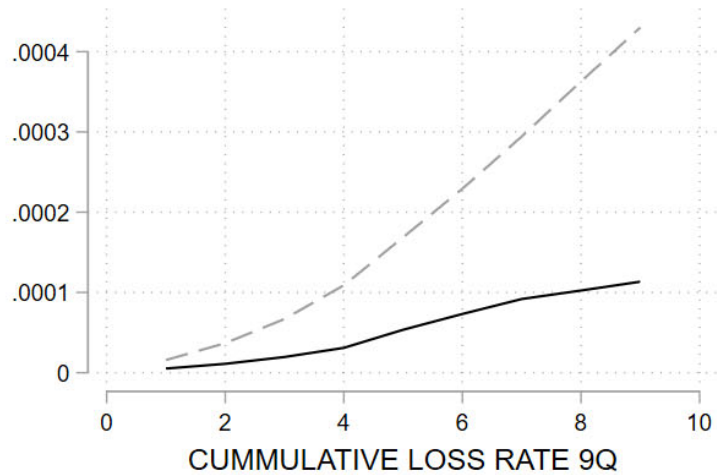
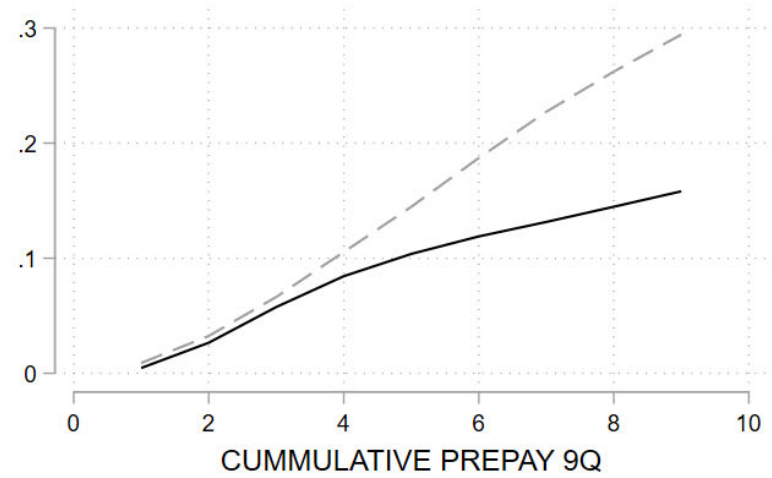
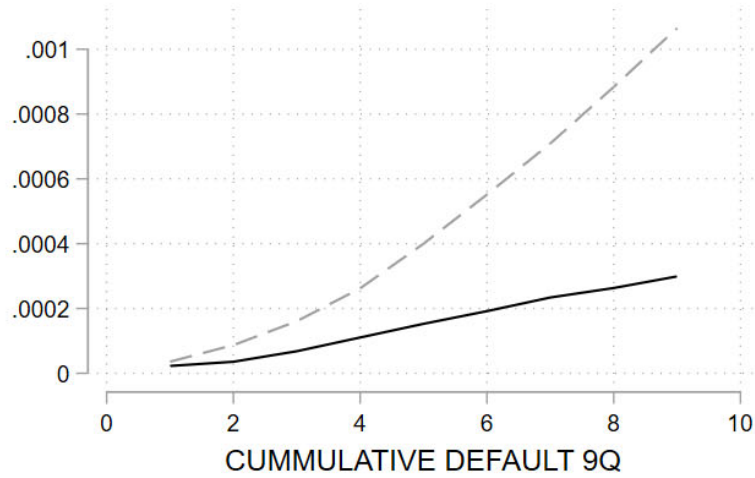
Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

## MODEL 2: QUARTERLY MODEL OF PREPAY/DEFAULT/LOSS, YEAR 2007 VINTAGE



Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

## MODEL 2: QUARTERLY MODEL OF PREPAY/DEFAULT/LOSS, YEAR 2012 VINTAGE



Note: realized outcome displayed on solid black, additional lines represent predicted outcomes for different model specifications.

## How GSEM can enhance the modeling framework in risk management:

From a technical perspective,

- Simplify the process of model building.
- Expand the set of available custom model alternatives: improve our ability to use latent variables to analyze non-standard model structures and linkages across estimation equations.
- Easily perform complex global hypothesis tests.
- Other ...?

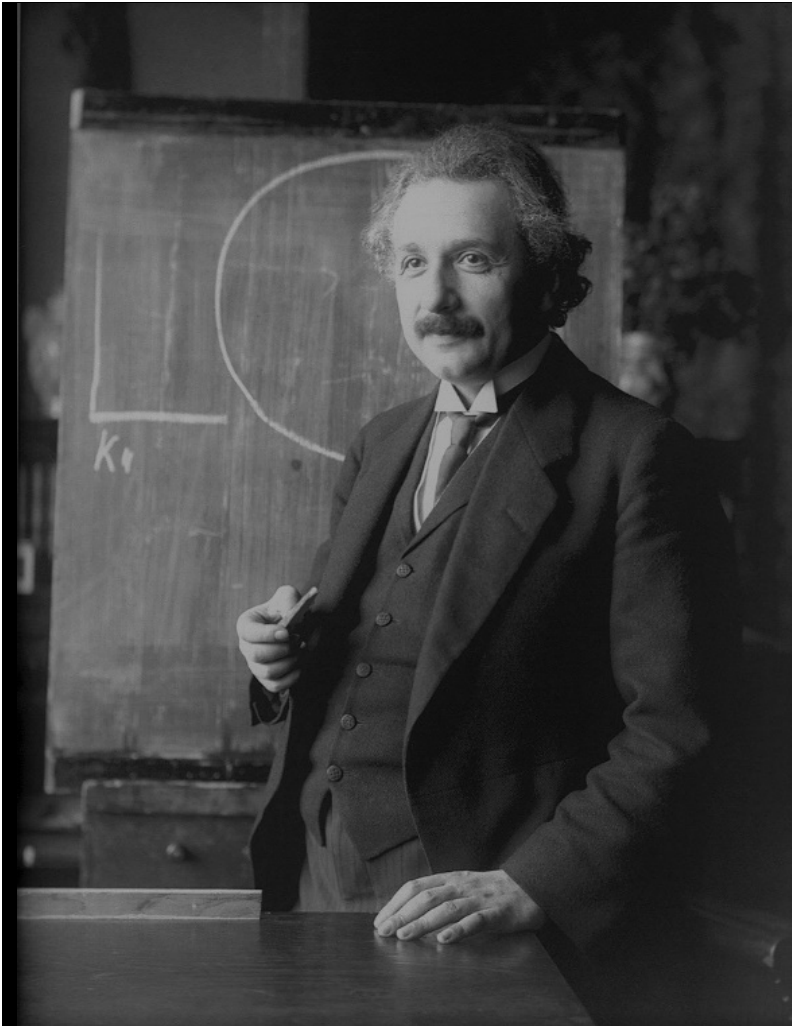
From a practical perspective,

- Streamline model development where the different components of a larger model can be easily combined into a coherent framework.
- Create a coherent framework where models can coexist: challenger models, benchmark models, models in production vs next generation of models in development, etc.
- Streamline the use of data, with a single dataset attending multiple goals.
- Simplified model documentation, validation, audit and implementation/production, as well as ongoing monitoring and redevelopment.
- Reduce the risk of errors and simplify the analysis of errors, i.e. reduce model risk.

Some areas where GSEM can improve:

- Simplify the use of the builder and improve the automatically generated code.
- Make the syntax more intuitive and flexible.
- Enhance the menu interface in addition to the graphical interface.
- Improve optimization.

FINAL THOUGHTS ...



A clever person solves a problem.  
A wise person avoids it.

*Albert Einstein*