


Computing score functions numerically using Mata

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1 Outline

- 1 Introduction
- 2 The `m1` command
- 3 Linear-form Restriction
- 4 The Problem
- 5 Robust Variance Covariance Matrix: A very brief review
- 6 The Solution
- 7 Conclusions

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- ▶ In particular: we will focus on models where the log-likelihood function **does not meet the linear-form restrictions**, which can be fitted using the **d-family of evaluators**.
- ▶ The minimum requirement to implement a model using the `m1` command is to write its log-likelihood function (i.e., **d0 evaluator**).
- ▶ Faster methods can be implemented depending on what we provide the `m1` command with:
 - **d0 evaluator** = **Log-likelihood**
 - **d1 evaluator** = Log-likelihood + Gradient
 - **d2 evaluator** = Log-likelihood + Gradient + Hessian

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```
. list in 1/3, sep(1)
```

	y	x1	x2
1.	-1.09811	-.3591099	.3387246
2.	-1.742268	.1902105	-1.498368
3.	1.273768	-1.602709	1.034604

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. list in 1/6 , sep(3)

	id	altern-e	x1	x2	choice
1.	1	1	-1.666827	-1.969941	0
2.	1	2	.5580259	-.2189879	0
3.	1	3	1.054737	1.894969	1
4.	2	1	-1.913301	-.1506114	0
5.	2	2	-.1818884	-.2132395	1
6.	2	3	1.19467	-.6775483	0

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- ▶ In other words, if the model uses data in long format, it probably does not meet the restriction.

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. qui clogit choice x1 x2 , gr(id) nolog
. matrix b_clogit = e(b)
. MyClogit choice x1 x2 , gr(id) nolog
MyClogit                                     Number of obs   =       300
                                           Wald chi2(2)     =       38.72
Log likelihood = -53.10466                  Prob > chi2      =       0.0000
```

choice	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	.5233348	.1771384	2.95	0.003	.1761499 .8705197
x2	1.922775	.3146272	6.11	0.000	1.306117 2.539433

```
. matrix b_MyClogit = e(b)
. di mreldif(b_MyClogit, b_clogit)
2.308e-08
```

- We also check that the estimates from our program are numerically equivalent to Stata's `clogit` command.

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- ▶ Hence, we are in  trouble  !

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- ▶ $\mathbf{u}_n = \mathbf{S}(\widehat{\beta}; y_n, \mathbf{x}_n)$ are row vectors that contains the score functions evaluated at $\widehat{\beta}$.
- ▶ Hence, \mathbf{u}_n is the only object that is missing in order to compute $\widehat{V}(\widehat{\beta})$.

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 - First (and the most obvious one), the developer needs to derive the score functions by hand (using pencil and paper + calculus).
 - Second, after knowing the algebraic expression, it has to be coded on Stata or Mata.
 - 2 **Another possible solution:** **Numerically** approximate the score functions, using what we already have coded: **the log-likelihood function**.
- ▶ **⚠ SPOILER ALERT ⚠:**
- Our solution will consist in:
 - 1 (Numerically) approximate the vector u_n .
 - 2 Compute $\widehat{V}(\widehat{\beta})$ using it.

6 The Solution [2]: Collecting everything we need

- ▶ First, we provide Mata with everything we need to compute the log-likelihood contribution of each individual.

```
. // We create relevant matrices on Stata to push them to Mata afterwards.
. matrix b = e(b) // Maximum Likelihood estimates
. matrix W = e(V) // Non-robust variance-covariance matrix
. // We initialize Mata
. mata:
----- mata (type end to exit) -----
: // Invoking Stata matrices
: betas = st_matrix("b") // Calls from Stata the matrix "b"
: W = st_matrix("W") // Calls from Stata the matrix "W"
:
: // Invoking Stata Variables
: st_view(X = ., ., "x1 x2") // View of all regressors x1 and x2
: st_view(Y = ., ., "choice") // View of response variable "choice"
: XY = (Y,X) // Generates XY matrix for future usage.
:
: // Extracting information about the id of individuals.
: st_view(panvar = ., ., "id") // View of individuals id
: paninfo = panelsetup(panvar, 1) // Sets up panel processing
: N = panelstats(paninfo)[1] // Number of Individuals
: end
```

6 The Solution [3]: Writing our log-likelihood function

- ▶ Second, we will create a void function, `LL_d()`, that resembles our log-likelihood function.

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```
. mata:
----- mata (type end to exit) -----
: // Creating the function we will invoke using Mata's deriv().
: void LL_d(real rowvector b , // 1ST ARGUMENT: Maximum likelihood estimates
>         real matrix   XY , // 2ND ARGUMENT: Convariates + dependent variable
>         real scalar lnf) // Output:      Log-likelihood contribution
> {
>   Y = XY[.,1] // Extract variable Y
>   X = XY[., (2:::cols(XY))] // Extract the regressors (x1 and x2)
>   U = rowsum(b:*X) // Observed Utility
>   P = exp(U):/colsum(exp(U )) // Multinomial Probability
>   lnf = colsum(Y:*ln(P)) // Individual contribution to the log-likelihood
> }
: end
-----
```

6 The Solution [3]: Writing our log-likelihood function

- ▶ Second, we will create a void function, `LL_d()`, that resembles our log-likelihood function.
- ▶ We will invoke it later when using Mata's `deriv()` function.

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>   U = rowsum(b:*X)      // Observed Utility
>   P = exp(U):/colsum(exp(U)) // Multinomial Probability
>   lnf = colsum(Y:*ln(P)) // Individual contribution to the log-likelihood
> }
: end
-----
```

- ▶ As you can see, this resembles exactly our log-likelihood.

$$\ln L = \sum_{n=1}^N \sum_{i=1}^J y_{in} \ln(P_{in}) = \sum_{n=1}^N \sum_{i=1}^J y_{in} \ln \left(\frac{\exp(\beta' x_{in})}{\sum_{j=1}^J \exp(\beta' x_{in})} \right)$$

6 The Solution [4]: Score function of the first individual

- ▶ Third, to begin with, we will illustrate how to compute the score function of the first individual using `deriv()`:

```
. mata:
----- mata (type end to exit) -----
: D =deriv_init() // Init deriv() `struct` and call it "D"
: deriv_init_evaluator(D, &LL_d()) // We provide the `struct` D with function LL_d()
: deriv_init_evaluortype(D, "d") // Set that deriv() must returns a scalar
: deriv_init_params(D, betas) // Provide D with beta estimates (deriv at)
: xy_n = panelsubmatrix(XY, 1, paninfo) // Extract first individual's X and Y
: xy_n
      1          2          3
1  [ 0 -1.666826963 -1.969941497
2  [ 0 .5580258965 - .218987897
3  [ 1 1.054736972 1.894969106

: deriv_init_argument(D, 1, xy_n) // provide D with X and Y of the first individual
: score_fn= deriv(D, 1) // <--- Perform the numerical derivation!
: score_fn // Display it
      1          2
1  [ .006871893 .0281603095

: end
-----
```

6 The Solution [4]: Score function of the first individual

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```
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----- mata (type end to exit) -----
: D = deriv_init() // Init deriv() `struct` and call it "D"
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: deriv_init_evaluortype(D,"d") // Set that deriv() must returns a scalar
: deriv_init_params(D, betas) // Provide D with beta estimates (deriv at)
: xy_n = panelsubmatrix(XY, 1, paninfo) // Extract first individual's X and Y
: xy_n
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      1          2
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: end
-----
```

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```
. mata:
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: D = deriv_init() // Init deriv() `struct` and call it "D"
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: deriv_init_evaluortype(D,"d") // Set that deriv() must returns a scalar
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: xy_n = panelsubmatrix(XY, 1, paninfo) // Extract first individual's X and Y
: xy_n
      1          2          3
1  0 -1.666826963 -1.969941497
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: deriv_init_argument(D, 1, xy_n) // provide D with X and Y of the first individual
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: score_fn // Display it
      1          2
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: end
```


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```
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: deriv_init_evaluator(D, &LL_d()) // We provide the `struct` D with function LL_d()
: deriv_init_evaluortype(D,"d") // Set that deriv() must returns a scalar
: deriv_init_params(D, betas) // Provide D with beta estimates (deriv at)
: xy_n = panelsubmatrix(XY, 1, paninfo) // Extract first individual's X and Y
: xy_n
      1          2          3
1  0 -1.666826963 -1.969941497
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: deriv_init_argument(D, 1, xy_n) // provide D with X and Y of the first individual
: score_fn= deriv(D, 1) // <--- Perform the numerical derivation!
: score_fn // Display it
      1          2
1  .006871893 .0281603095

: end
-----
```

6 The Solution [4]: Score function of the first individual

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```
. mata:
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: D = deriv_init()           // Init deriv() `struct` and call it "D"
: deriv_init_evaluator(D, &LL_d()) // We provide the `struct` D with function LL_d()
: deriv_init_evaluatoretype(D,"d") // Set that deriv() must returns a scalar
: deriv_init_params(D, betas) // Provide D with beta estimates (deriv at)
: xy_n = panelsubmatrix(XY, 1, paninfo) // Extract first individual's X and Y
: xy_n
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: score_fn                        // Display it
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: end
```

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- ▶ Third, to begin with, we will illustrate how to compute the score function of the first individual using `deriv()`:

```
. mata:
----- mata (type end to exit) -----
: D = deriv_init() // Init deriv() 'struct' and call it "D"
: deriv_init_evaluator(D, &LL_d()) // We provide the 'struct' D with function LL_d()
: deriv_init_evaluatoretype(D,"d") // Set that deriv() must returns a scalar
: deriv_init_params(D, betas) // Provide D with beta estimates (deriv at)
: xy_n = panelsubmatrix(XY, 1, paninfo) // Extract first individual's X and Y
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: score_fn= deriv(D, 1) // <--- Perform the numerical derivation!
: score_fn // Display it
      1          2
1  [ .006871893 .0281603095 ]

: end
-----
```

6 The Solution [5]: Score functions of the entire sample

- ▶ Now that we know how to perform the derivative of a function we can apply it to the whole sample (e.g., to all the individuals in the sample):

```
. mata:
----- mata (type end to exit) -----
: D = deriv_init() // Init deriv() `struct`
: deriv_init_evaluator(D, &LL_d()) // `struct` D is provided with the pointer LL_d()
: deriv_init_evaluatoretype(D, "d") // Set that deriv() must returns a scalar
: score_fn = J(0, cols(betas),.) // Vector length 0xcols(betas)
: for(n=1; n <= N; ++n) { // Looping over n individuals
>     xy_n = panelsubmatrix(XY, n, paninfo) // Extract submatrix of individual n
>     deriv_init_params(D, betas) // provide D with beta estimates
>     deriv_init_argument(D, 1, xy_n) // provide D with attributes values
>     score_fn = score_fn \ deriv(D, 1) // Collect score functions from each individual
> }
: score_fn[1..4,] // display the score functions of the first 4 individuals
      1          2
1      .006871893      .0281603095
2     -.1607972318      .1576732297
3     -.0730075944      .1282049291
4      .0035216089      .0050014822

: // Finally, we save the score functions as S just for a handy matrix multiplication afterwards.
: S = score_fn
: end
```

6 The Solution [6]: Obtaining the robust correction

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- ▶ $\mathbf{W} = -H^{-1}$ is the negative of the inverse of the hessian (Object W).
- ▶ $\mathbf{u}_n = \mathcal{S}(\widehat{\beta}; y_n, \mathbf{x}_n)$ are row vectors that contains the score functions evaluated at $\widehat{\beta}$ (Object S).

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- ▶ Accordingly, it is as simple as:

```
. mata:
----- mata (type end to exit) -----
: meat = (N/(N-1)) * S' * S // Some people call this part the "meat".
: V_robust_approx= W * meat * W // Approximated robust variance-covariance matrix.
: st_matrix("V_robust_approx", V_robust_approx) // Save robust matrix into a Stata Matrix.
: end
```

6 The Solution [7]: Checking our approximation

- ▶ Using `V_robust_approx` we can check how far are our numerically approximated robust covariance matrices compared with Stata's `clogit`.

```
. clogit choice x* ,gr(id) robust nolog
Conditional (fixed-effects) logistic regression

                                Number of obs   =          300
                                Wald chi2(2)      =          42.15
                                Prob > chi2       =          0.0000
                                Pseudo R2        =          0.5166
Log pseudolikelihood = -53.10466
                                (Std. Err. adjusted for clustering on id)
```

choice	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
x1	.5233348	.1587735	3.30	0.001	.2121444	.8345252
x2	1.922775	.3334521	5.77	0.000	1.26922	2.576329

```
. mat V_robust_clogit = e(V)
. mat li V_robust_approx
symmetric V_robust_approx[2,2]
      c1      c2
r1 .02520903
r2 .00291664 .11119032
. mat li V_robust_clogit
symmetric V_robust_clogit[2,2]
      choice:      choice:
          x1      x2
choice:x1 .02520903
choice:x2 .00291664 .11119031
. display mrelidif(V_robust_approx, V_robust_clogit)
7.734e-09
```

7 Outline

- ① Introduction
- ② The `m1` command
- ③ Linear-form Restriction
- ④ The Problem
- ⑤ Robust Variance Covariance Matrix: A very brief review
- ⑥ The Solution
- ⑦ Conclusions


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 - [Gould \(2007\)](#) for a better understanding about `-struct-` type objects.
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 - [Gould \(2001\)](#) for more insights about *Statistical Software Certification*.

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10 Outline

⑨ MyClogit

⑩ MyLikelihood_LL.mata

10 MyClogit.ado

```
program MyClogit
  version 12
  if replay() {
    if ("`e(cmd)'" != "MyClogit") error 301
    Replay `0'
  }
  else Estimate `0'
end

program Estimate, eclass sortpreserve
  syntax varlist(fv) [if] [in] , GGroup(varname) ///
    [TECHnique(passthru) noLog ROBUST ]
  local mlopts `technique'
  if ("`technique'" == "technique(bhhh)") {
    di in red "technique(bhhh) is not allowed."
    exit 498
  }
  gettoken lhs rhs : varlist
  marksample touse
  markout `touse' `group'
  global MY_panel = "`group'"
  ml model d0 MyLikelihood_LL() ///
    (MyClogit: `lhs' = `rhs', nocons) ///
    if `touse', missing first `log' ///
    title("MyClogit") `robust' maximize
  // Show model
  ereturn local cmd MyClogit
  Replay , level(`level')
  ereturn local cmdline ```0'""
end

program Replay
  syntax [, Level(cilevel) ]
  ml display , level(`level')
end

// include mata functions from MyLikelihood_LL.mata
findfile "MyLikelihood_LL.mata"
do "`r(fn)'"
```

11 Outline

9 MyClogit

10 MyLikelihood_LL.mata

11 MyLikelihood_LL.mata

```
mata:
  void MyLikelihood_LL(transmorphic scalar M, real scalar todo,
    real rowvector b, real scalar lnf,
    real rowvector g, real matrix H)
{
  // variables declaration
  real matrix panvar
  real matrix paninfo
  real scalar npanels
  real scalar n
  real matrix Y
  real matrix X
  real matrix x_n
  real matrix y_n
  Y = moptimize_util_depvar(M, 1) // Response Variable
  X = moptimize_init_eq_indepvars(M,1) // Attributes
  id_beta_eq=moptimize_util_eq_indices(M,1) // id parameters
  betas= b[id_beta_eq] // parameters
  st_view(panvar = ., ., st_global("MY_panel"))
  paninfo = panelsetup(panvar, 1)
  npanels = panelstats(paninfo)[1]
  lnfj = J(npanels, 1, 0) // object to store loglikelihood
  for(n=1; n <= npanels; ++n) {
    x_n = panelsubmatrix(X, n, paninfo)
    y_n = panelsubmatrix(Y, n, paninfo)
    U_n =exp(rowsum(betas :* x_n) // Linear utility
    p_i = colsum(U_n:* y_n) / colsum(U_n) // Probability of each alternative
    lnfj[n] = ln(p_i) // Add contribution to the likelihood
  }
  lnf = moptimize_util_sum(M, lnfj)
}
end
```