A new implementation of relative distribution methods in Stata

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Outline

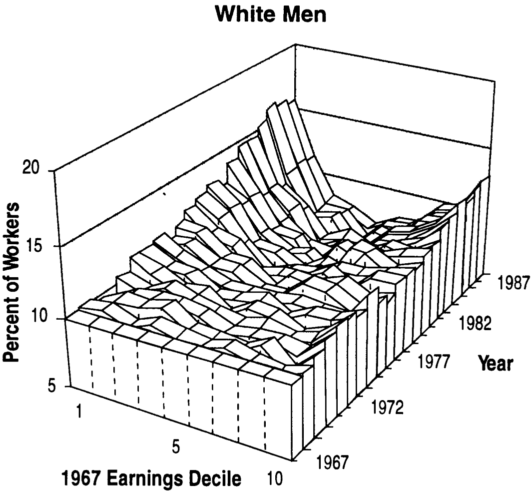
What is the “relative distribution”?

- The relative distribution is the distribution of the relative ranks that the outcomes from one distribution take on in another distribution.
- How do wages of females rank in the wage distribution of males? How are these ranks distributed?
- The method can be used to analyze differences in distributions between groups or changes in a distribution over time.
- Of interest are aspects such as the distribution function or the density function of the relative ranks, or summary statistic such as polarization or distributional divergence.
- Of interest are also counterfactual decompositions that adjust the relative distribution for differences in covariate compositions.

Example: Polarization of earnings over time

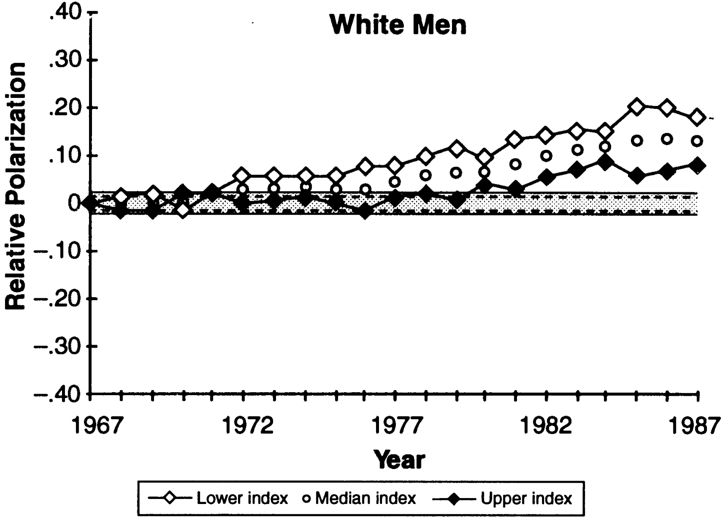
(Morris et al. 1994)

- Change in earnings of full-time, full-year workers: relative distribution of a given year compared to 1967



Example: Polarization of earnings over time
(Morris et al. 1994)

- Relative earnings polarization with respect to 1967



Some definitions

- F_Y: reference distribution (wages of males)
- F_X: comparison distribution (wages of females)
- Relative distribution

$$G(r) = F_X(F_Y^{-1}(r)), \quad r \in [0, 1]$$

- Relative density

$$g(r) = \frac{\mathrm{d}G(r)}{\mathrm{d}r} = \frac{f_X(F_Y^{-1}(r))}{f_Y(F_Y^{-1}(r))}, \quad r \in [0, 1]$$

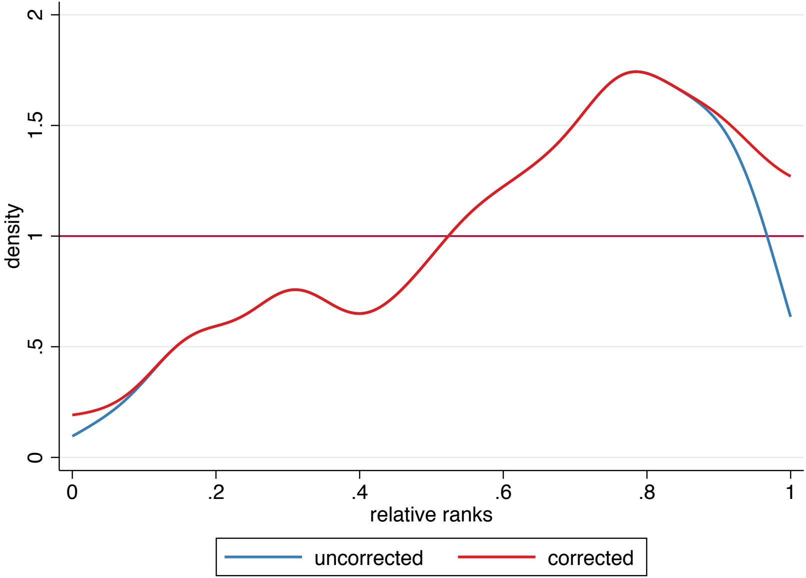
- Relative ranks

$$r_i = F_Y(X_i), \quad i \in \mathcal{X}$$

Estimation

- Estimation of the relative CDF and summary measures of the relative ranks is pretty much straightforward.
- Estimation of the PDF is more involved:
  - Standard density estimators are (severely) biased at the boundaries because relative ranks can only take on values between 0 and 1.
  - Data-driven bandwidth selection requires adjustment to take account of the two-sample nature of relative data.
  - Function `mm_density()` from `moremata` can handle both issues.
- Estimation of standard errors is *not* straightforward due to the two-sample nature of the estimation problem.
  - I use influence functions based on an analogy to GMM (also see Jann 2020a).
  - The influence functions also cover uncertainty induced by covariate balancing.
  - Advantage of influence functions: Full support for complex survey estimation.

Boundary effects



The `reldist` command

- `reldist` provides a full-blown implementation of relative distribution methods.
  - Relative CDF and PDF for continuous and discrete data.
  - Relative polarization and divergence measures.
  - Summary statistics of relative ranks such as mean and quantiles.
  - Shape and location decomposition.
  - Covariate balancing by inverse probability weighting (IPW) or entropy balancing.
  - Utility to create graphs.
  - VCE for everything, including support for `svy` (although not as prefix command; must specify option `vce(svy)`)
  - Prediction of influence functions after estimation.
- For formulas and detailed information on the command see Jann (2020b).

Estimation

Two-sample relative distribution (syntax 1)

```
reldist subcmd varname [if] [in] [weight], by(groupvar) [ options ]
```

Paired relative distribution (syntax 2)

```
reldist subcmd varname refvar [if] [in] [weight] [, options ]
```

where *subcmd* is

pdf	relative density
histogram	relative histogram
cdf	relative cumulative distribution
divergence	divergence measures
mrp	median relative polarization
summarize	summary statistics of relative ranks

Replay results

```
reldist [, noheader notable display_options ]
```

Draw graph after estimation

```
reldist graph [, graph_options ]
```

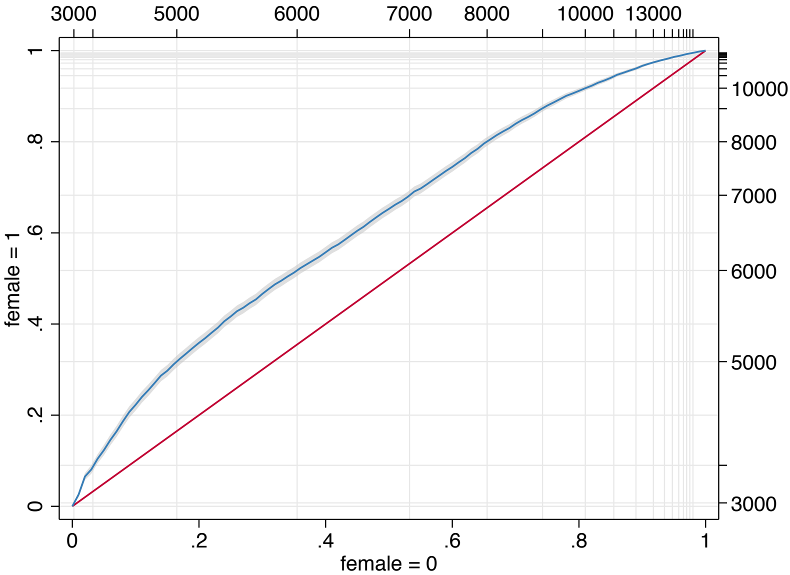
Obtain influence functions after estimation

```
predict {stub* | newvar1 newvar2 ...} [if] [in] [, scores density_options ]
```

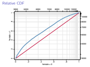
Example: Gender wage gap in Switzerland

```
. use sess16, clear
(Sample from Swiss Earnings Structure Survey 2016)

. describe

Contains data from sess16.dta
  obs:       100,000                    Sample from Swiss Earnings Structure
                                          Survey 2016
 vars:             5                    18 Nov 2020 19:02
-----------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
-----------------------------------------------------------------------------
earnings        long    %10.0g                monthly earnings in CHF (full-time
                                                equivalent)
female          byte    %8.0g                 1 = female, 0 = male
educyrs         byte    %10.0g                years of education
tenure          byte    %8.0g                 tenure (in years)
wgt             double  %10.0g                sampling weight
-----------------------------------------------------------------------------
Sorted by:

. summarize

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
    earnings |    100,000    7858.498     4249.54       2312     103998
      female |    100,000      .44628    .4971083          0          1
     educyrs |    100,000    12.67786    2.728897          7         17
      tenure |    100,000     8.57528    8.905727          0         61
         wgt |    100,000    33.13712    59.26461   8.435029   2991.433
```

Relative CDF

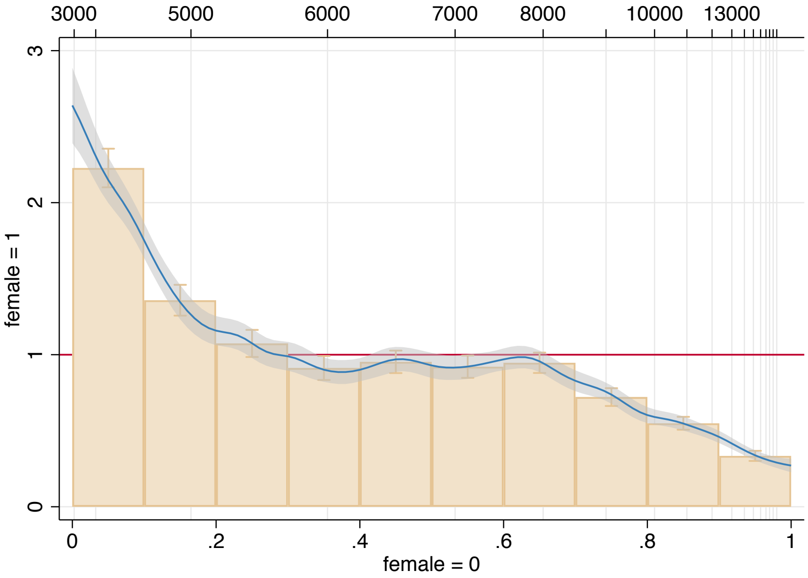


Relative CDF

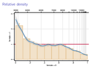


```
. reldist cdf earnings [pw=wgt], by(female) notable

Cumulative relative distribution                Number of obs     =    100,000
  F1: female = 1                                Comparison obs    =     44,628
  F0: female = 0                                Reference obs     =     55,372

. reldist graph, olab(3000(1000)20000, format(%7.0g) grid) ///
>               yolab(3000(1000)20000, format(%7.0g) grid angle(0)) ///
>               ciopts(fc(%50) lc(%0))
```

Relative density



Relative density

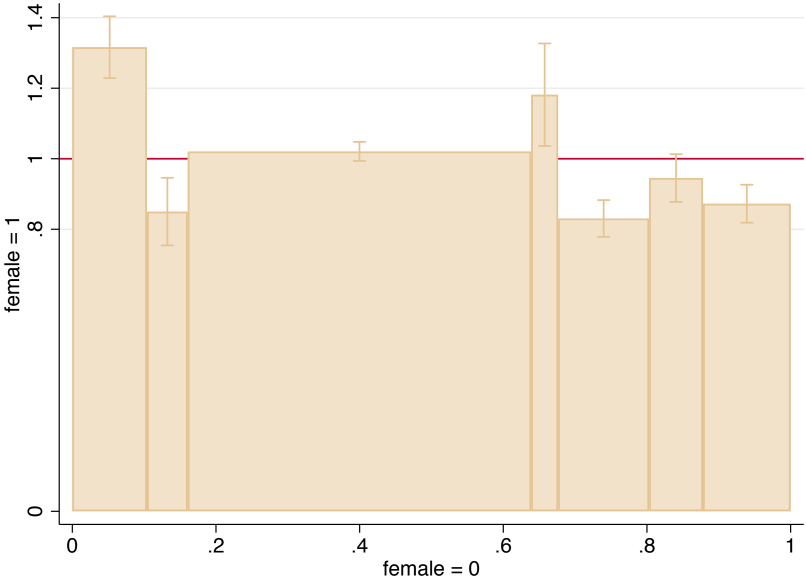


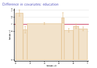
```
. reldist pdf earnings [pw=wgt], by(female) histogram notable

Relative density                                Number of obs     =    100,000
  F1: female = 1                                Comparison obs    =     44,628
  F0: female = 0                                Reference obs     =     55,372
                                                Bandwidth         =  .02515569

. reldist graph, olab(3000(1000)20000, format(%7.0g) grid) ///
>     ciopts(fc(%50) lc(%0))
```

Relative polarization

```
. reldist mrp earnings  [pw=wgt], by(female) multiplicative

Median relative polarization                    Number of obs     =    100,000
  F1: female = 1                                Comparison obs    =     44,628
  F0: female = 0                                Reference obs     =     55,372
  Adjustment: location (mult)

------------------------------------------------------------------------------
    earnings |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         MRP |  -.0465722   .0079613    -5.85   0.000    -.0621763   -.0309682
         LRP |  -.0033018   .0148662    -0.22   0.824    -.0324393    .0258358
         URP |  -.0898427   .0110417    -8.14   0.000    -.1114843   -.0682012
------------------------------------------------------------------------------
```

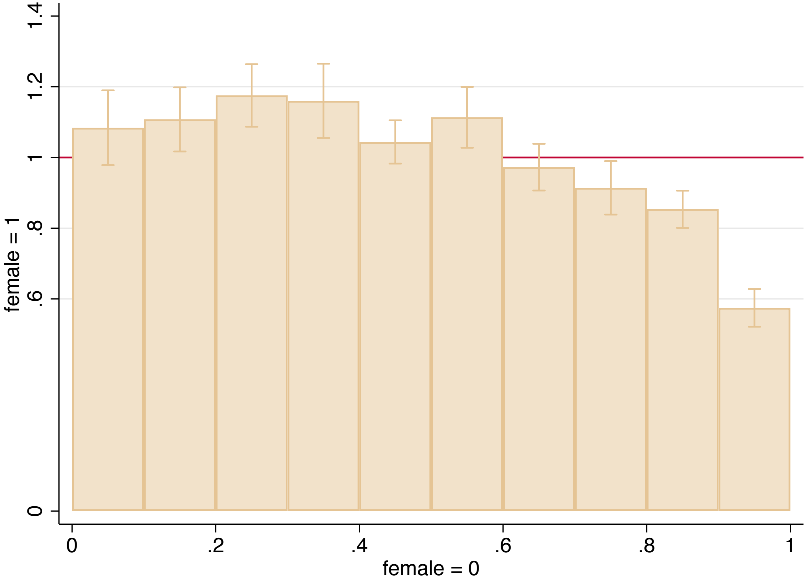
Difference in covariates: education



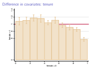


```
. reldist histogram educyrs [pw=wgt], by(female) categorical

Relative histogram                               Number of obs     =    100,000
  F1: female = 1                                 Comparison obs    =     44,628
  F0: female = 0                                 Reference obs     =     55,372

--------------------------------------------------------------
     educyrs |      Coef.   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
     educyrs |
           7 |   1.316267   .0447324      1.228592    1.403942
          11 |   .8500557   .0489017       .754209    .9459024
          12 |   1.020779   .0137853      .9937596    1.047798
          13 |   1.181543   .0741483      1.036213    1.326873
          14 |   .8305811   .0265873      .7784703    .8826918
          15 |   .9453244   .0345518      .8776033    1.013045
          17 |   .8723796   .0274635      .8185515    .9262076
--------------------------------------------------------------
(evaluation grid stored in e(at))

. reldist graph
```

Difference in covariates: tenure

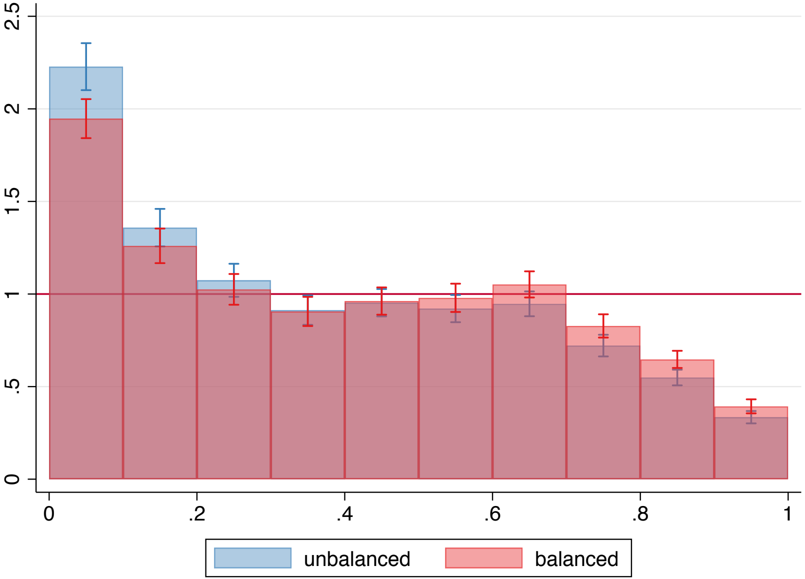


Difference in covariates: tenure

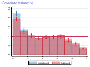



```
. reldist histogram tenure [pw=wgt], by(female)
Relative histogram                                   Number of obs     =    100,000
  F1: female = 1                                     Comparison obs    =     44,628
  F0: female = 0                                     Reference obs     =     55,372

──────────────────────────────────────────────────────────────
      tenure │      Coef.   Std. Err.      [95% Conf. Interval]
─────────────┼────────────────────────────────────────────────
          h1 │   1.084155    .053922       .9784687    1.189842
          h2 │   1.107638   .0462447       1.016999    1.198277
          h3 │   1.175377   .0450791       1.087022    1.263731
          h4 │   1.160171    .053622       1.055073     1.26527
          h5 │    1.04392   .0311894       .9827894    1.105051
          h6 │   1.113525    .043905       1.027472    1.199578
          h7 │   .9726401   .0337204       .9065484    1.038732
          h8 │   .9141628   .0385788       .8385488    .9897768
          h9 │   .8535668   .0268357       .8009691    .9061645
         h10 │   .5748437   .0272384       .5214568    .6282306
──────────────────────────────────────────────────────────────
(evaluation grid stored in e(at))

. reldist graph
```

Covariate balancing



Covariate balancing



```
. reldist histogram earnings [pw=wgt], by(female)
  (output omitted)
. estimates store unbalanced
. reldist histogram earnings [pw=wgt], by(female) ///
>     balance(eb:i.educyrs c.tenure##c.tenure)
Relative histogram                              Number of obs     =    100,000
  F1: female = 1                                Comparison obs    =     44,628
  F0: female = 0                                Reference obs     =     55,372
  Balancing of F1
      method = eb
      i.educyrs tenure c.tenure#c.tenure

------------------------------------------------------------------
    earnings |      Coef.   Std. Err.      [95% Conf. Interval]
-------------+----------------------------------------------------
          h1 |   1.947315   .0537421      1.841982    2.052649
          h2 |    1.26018    .047665      1.166757    1.353602
          h3 |   1.025128   .0424014       .942022    1.108235
          h4 |   .9059489   .0401832      .8271904    .9847074
          h5 |   .9619829   .0375585      .8883687    1.035597
          h6 |   .9794557   .0389595      .9030956    1.055816
          h7 |   1.051987   .0360187      .9813911    1.122584
          h8 |   .8276325   .0320939      .7647288    .8905361
          h9 |   .6469504   .0236423      .6006119     .693289
         h10 |   .3934189    .019337      .3555186    .4313191
------------------------------------------------------------------
(evaluation grid stored in e(at))
. estimates store balanced
. coefplot unbalanced balanced, at nooffset citop cirecast(rcap) ///
>     recast(bar) barwidth(0.1) color(%50) ylabel(0(.5)2.5) yline(1)
```

Covariate balancing

```
. reldist summarize earnings [pw=wgt], by(female) stat(mean med)
Relative ranks                              Number of obs     =    100,000
  F1: female = 1                            Comparison obs    =     44,628
  F0: female = 0                            Reference obs     =     55,372

------------------------------------------------------------------
    earnings |      Coef.   Std. Err.      [95% Conf. Interval]
-------------+----------------------------------------------------
        mean |   .3756438   .0034384      .3689046     .382383
      median |   .3348484   .0066729      .3217696    .3479273
------------------------------------------------------------------

. reldist summarize earnings [pw=wgt], by(female) stat(mean med) ///
>     balance(eb:i.educyrs c.tenure##c.tenure)
Relative ranks                              Number of obs     =    100,000
  F1: female = 1                            Comparison obs    =     44,628
  F0: female = 0                            Reference obs     =     55,372
  Balancing of F1
      method = eb
      i.educyrs tenure c.tenure#c.tenure

------------------------------------------------------------------
    earnings |      Coef.   Std. Err.      [95% Conf. Interval]
-------------+----------------------------------------------------
        mean |   .4040611   .0027288      .3987127    .4094096
      median |   .3854737   .0057611      .3741821    .3967654
------------------------------------------------------------------
```

Analysis of univariate distributions

- After deriving the equations and implementing `reldist`, I realized that I had all the building blocks in front of me for putting together a general command for the analysis of (univariate) distributions (summary statistics, density, quantile function, inequality measures, etc.).
- This may not seem very exciting.
- After all, many official (`mean`, `proportion`, `ci`, `summarize`, `tabstat`, `pctile`, `cumul`, `kdensity`, `histogram`, etc.) and user-written commands (`catplot`, `cdfplot`, `distplot`, `fre`, `kdens`, `lorenz`, `pshare`, `glcurve`, `svylorenz`, `robstat`, etc.) are available.

Analysis of univariate distributions

- But it is!
- All these statistics can be combined in a general framework based on influence functions. This means that you get `svy`-compatible standard errors for everything (as well as covariances between any kind of statistic).
- Covariate balancing/standardization can easily be integrated in a general way.
- RIFs (recentered influence functions) are available for everything and can be used in further analysis, e.g. in RIF regressions or RIF decompositions.

Estimation

Scalar summary statistics

dstat [**su**mmarize] [(*stats*)] *varlist* [(*stats*) *varlist* ...] [*if*] [*in*] [*weight*] [, *options*]

Distribution functions

dstat *subcmd* *varlist* [*if*] [*in*] [*weight*] [, *options*]

where *subcmd* is

density	density function
histogram	histogram
proportion	proportions or totals
cdf	cumulative distribution
quantile	quantile function
lorenz	lorenz curve
share	percentile shares

varlist may contain factor variables; see fvvarlist.
fweights, **pweight**s, and **iweight**s are allowed; see weight.

Postestimation

Replay results

dstat [, *reporting_options*]

Draw graph

dstat **gr**aph [, *graph_options*]

Obtain (recentered) influence functions

predict {*stub*** | *newvar1* *newvar2* ...} [*if*] [*in*] [, *predict_options*]

stats	Description
Points in the distribution	
quantile(*p*)	*p*/100-quantile; *p* in [0,100]
p(*p*)	alias for **quantile()**
density(*x*)	kernel density at value *x*
hist(*x1*,*x2*)	histogram density of data within (*x1*,*x2*]
cdf(*x*)	cumulative distribution (CDF) at value *x*
cdfm(*x*)	mid-adjusted CDF at value *x*
prop(*x*)	proportion of data equal to value *x*
prop(*x1*,*x2*)	proportion of data within [*x1*,*x2*]
pct(*x*)	percent of data equal to value *x*
pct(*x1*,*x2*)	percent of data within [*x1*,*x2*]
freq(*x*)	frequency of data equal to value *x*
freq(*x1*,*x2*)	frequency of data within [*x1*,*x2*]
Location measures	
mean	arithmetic mean
gmean	geometric mean (data must be positive)
hmean	harmonic mean (data must be positive)
trim[(*alpha*)]	*alpha* trimmed mean; *alpha* in [0,50]; default is *alpha*=25
winsor[(*alpha*)]	*alpha* winsorized mean; *alpha* in [0,50]; default is *alpha*=25
median	median; equal to **q50**
huber[(*p*)]	Huber M estimate with gaussian efficiency *p* in [63.7,99.9]; default is *p*=95
biweight[(*p*)]	biweight M estimate with gaussian efficiency *p* in [.01,99.9]; default is *p*=95
hl	Hodges-Lehmann location measure (Hodges and Lehmann 1963)
Scale measures	
sd[(*df*)]	standard deviation; default is *df*=1
variance[(*df*)]	variance; default is *df*=1
mse[(*t*[,*df*])]	mean squared error from target *t*; default is *t*=0 and *df*=0
smse[(*t*[,*df*])]	square-root of mean squared error; default is *t*=0 and *df*=0
iqr[(*p1*,*p2*)]	interquantile range; default is **iqr(25,75)** (interquartile range)
iqrn	rescaled interquartile range
mad[(*l*[,*t*])]	median (or mean if *l*!=0) absolute deviation from the median (or mean if *t*!=0)
madn[(*l*[,*t*])]	normalized MAD; equal to 1/invnormal(0.75) * **mad** or sqrt(pi/2) * **mad**
mae[(*l*[,*t*])]	median (or mean if *l*!=0) absolute error from target *t*; default is *t*=0
maen[(*l*[,*t*])]	normalized MAE; equal to 1/invnormal(0.75) * **mae** or sqrt(pi/2) * **mae**
md	mean absolute pairwise difference; equal to 2 * **mean** * **gini**
mdn	normalized mean absolute pairwise difference; equal to sqrt(pi)/2 * **md**
mscale[(*bp*)]	M estimate of scale with breakdown point *bp* in [1,50]; default is *bp*=50
qn	Qn scale coefficient (Rousseeuw and Croux 1993)
Skewness measures	
skewness	skewness
qskew[(*alpha*)]	quantile skewness measure (Hinkley 1975); *alpha* in [0,50]; default is *alpha*=25
mc	medcouple (Brys et al. 2004)

Kurtosis measures

kurtosis	kurtosis
qw[(*alpha*)]	quantile tail weight measure; *alpha* in [0,50]; default is *alpha*=25
lqw[(*alpha*)]	left quantile tail weight measure; *alpha* in [0,50]; default is *alpha*=25
rqw[(*alpha*)]	right quantile tail weight measure; *alpha* in [0,50]; default is *alpha*=25
lmc	left medcouple tail weight measure (Brys et al. 2006)
rmc	right medcouple tail weight measure (Brys et al. 2006)

Inequality measures

gini[(*df*)]	Gini coefficient; *df* applies small-sample adjustment; default is *df*=0
agini[(*df*)]	absolute Gini coefficient
mld	mean log deviation; equal to **ge(0)**
theil	Theil index; equal to **ge(1)**
cv[(*df*)]	coefficient of variation; default is *df*=1; **cv(0)**=sqrt(2***ge(1)**)
ge[(*alpha*)]	generalized entropy (Shorrocks 1980) with parameter *alpha*
atkinson[(*epsilon*)]	Atkinson index with parameter *epsilon*>=0; default is *epsilon*=1
lvar[(*df*)]	logarithmic variance; *df* applies small-sample adjustment; default is *df*=1
vlog[(*df*)]	variance of logarithm; *df* applies small-sample adjustment; default is *df*=1
top[(*p*)]	outcome share of top *p* percent; default is *p*=10
bottom[(*p*)]	outcome share of bottom *p* percent; default is *p*=40
mid[(*p1*,*p2*)]	outcome share of mid *p1* to *p2* percent; default is *p1*=40 and *p2*=90
palma	palma ratio; equal to **top/bottom** or **sratio(40,90)**
qratio[(*p1*,*p2*)]	quantile ratio q(*p2*)/q(*p1*); default is *p1*=10 and *p2*=90
sratio[(*u1*,*l2*)]	percentile share ratio; default is *u1*=10 and *l2*=90
sratio[(*l1*,*u1*,*l2*,*u2*)]	percentile share ratio; default is *l1*=0, *u1*=10, *l2*=90, *u2*=100
***lorenz**(*p*)	Lorenz ordinate, *p* in [0,100]; prefix * is empty for default, **g** for generalized, **t** for total, **a** for absolute, **e** for equality gap
***share**(*p1*,*p2*)	percentile share, *p1* and *p2* in [0,100]; prefix * is empty for default, **d** for density, **g** for generalized, **t** for total, **a** for average

Concentration measures

gci(*zvar*[,*df*])	Gini concentration index; *zvar* specifies the sort variable; *df* applies small-sample adjustment; default is *df*=0
gci[(*df*)]	**gci** using sort variable from option **zvar()**
aci(*zvar*[,*df*])	absolute Gini concentration index; *zvar* and *df* are as for **gci**
aci[(*df*)]	**aci** using sort variable from option **zvar()**
***ccurve**(*p*[,*zvar*])	concentration curve ordinate, *p* in [0,100]; prefix * is empty for default, **g** for generalized, **t** for total, **a** for absolute, **e** for equality gap
***cshare**(*p1*,*p2*[,*zvar*])	concentration share, *p1* and *p2* in [0,100]; prefix * is empty for default, **d** for density, **g** for generalized, **t** for total, **a** for average

Poverty measures

watts[(*pline*)]	Watts index (see, e.g., Saisana 2014); *pline* specifies the poverty line(s) > 0; *pline* can be *varname* or #; the default is as set by option **pline()**
fgt[(*a*[,*pline*])]	Foster–Greer–Thorbecke index with *a*>=0 (Foster et al. 1984, 2010); default is *a*=0 (headcount ratio); *pline* specifies the poverty line(s) > 0; *pline* can be *varname* or #; the default is as set by option **pline()**

Example

```
. dstat (mean gmean med sd Gini MLD Theil Palma) earnings [pw=wgt], over(female)
Summary statistics                    Number of obs     =    100,000

            0: female = 0
            1: female = 1
```

earnings	Coef.	Std. Err.	[95% Conf.	Interval]
0				
mean	7964.767	32.99754	7900.093	8029.442
gmean	7231.028	23.98644	7184.015	7278.041
med	6803	27.13438	6749.817	6856.183
sd	4539.07	102.4153	4338.337	4739.803
Gini	.2433624	.0019915	.239459	.2472657
MLD	.0966465	.001718	.0932792	.1000138
Theil	.1137077	.0027248	.1083671	.1190484
Palma	.8660138	.00943	.8475311	.8844965
1				
mean	6515.329	24.85582	6466.611	6564.046
gmean	6082.104	18.92963	6045.003	6119.206
med	5893	26.14387	5841.758	5944.242
sd	2897.98	78.86047	2743.415	3052.546
Gini	.2061163	.001989	.2022179	.2100147
MLD	.0688069	.0015461	.0657765	.0718373
Theil	.076873	.0023677	.0722324	.0815136
Palma	.7110416	.0084675	.6944454	.7276379

Installation

- `reldist` requires the latest version of `moremata`. To install both packages, type

  ```
  . ssc install reldist, replace
  . ssc install moremata, replace
  ```

  Or install from GitHub: http://github.com/benjann/reldist

- `dstat` should become available on GitHub and SSC soon; check http://github.com/benjann/dstat in some weeks.

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