KU LEUVEN

`randregret`: A command for fitting random regret minimization models using Stata

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Research Centre for Operations Research and Statistics (ORSTAT)

1 Outline

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$\Rightarrow$ RRM models will (formalize and) minimize this notion of regret!

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- y_{in} is the response variable that takes the value of 1 when alternative i is chosen and 0 otherwise.

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$(x_{jm} - x_{2t})$	Travel Time	-4	0	8
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- Takeaway: We will define $R_{i \leftrightarrow j,mn}$ in terms of the attribute differences.

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- (Chorus, 2010) proposed the following attribute level regret:

$$R_{i\leftrightarrow j,mn} = \ln\left[1 + \exp\left\{\beta_m \cdot (x_{jmn} - x_{imn})\right\}\right]$$

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- β_m is the taste parameter of attribute m.

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4. Consequently, the log-likelihood will be described by:

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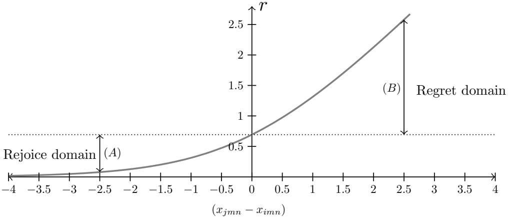
- All in all. the parameters in RUM and RRM, are expected to have the same sign, even though their interpretation is dramatically different.

2 Semi-compensatory Behavior and the Compromise Effect



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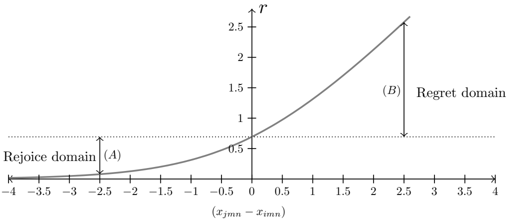

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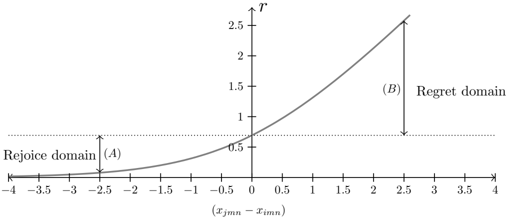

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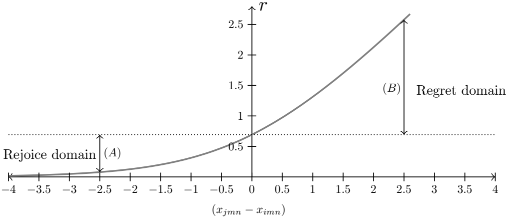

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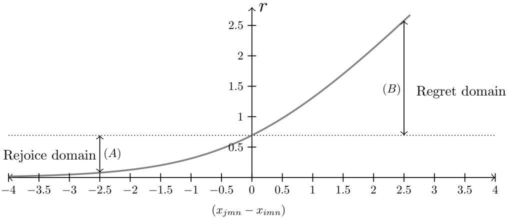

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- Linear RUM models $\Rightarrow$ fully-compensatory model.
- **Compromise Effect**: Alternatives with *"balanced"* performance in all attributes are more attractive than alternatives with a severe poor performance in one attribute.

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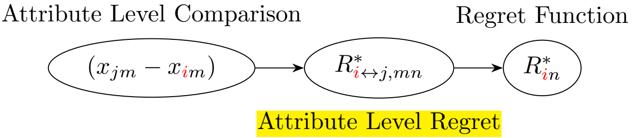

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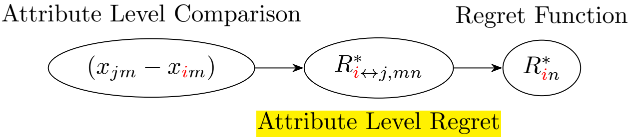

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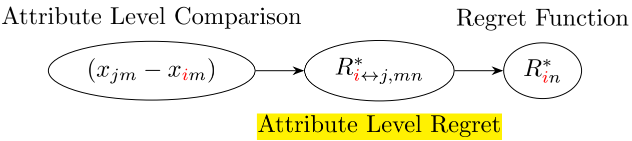

- $\Rightarrow$ all the steps described in order to obtain the log-likelihood of the model remain constant.
- All we need to do is replace the new ***attribute level regret*** from the extended model to compute the new log-likelihood.

3 Generalized RRM (Chorus, 2014)

- (Chorus, 2014) proposed a new ***attribute level regret***:

$$R_{in}^{\text{GRRM}} = \sum_{j \neq i}^{J} \sum_{m=1}^{M} R_{i \leftrightarrow j,mn}^{\text{GRRM}} = \sum_{j \neq i}^{J} \sum_{m=1}^{M} \ln \left[\gamma + \exp \left\{ \beta_m \left(x_{jmn} - x_{imn} \right) \right\} \right] \tag{4}$$

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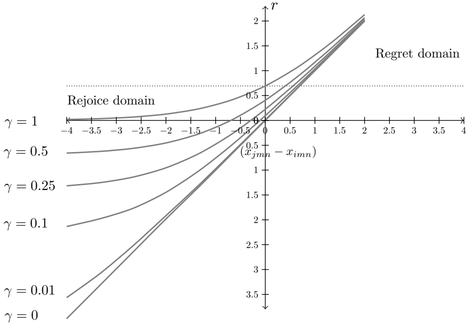
- The regret function (R_{in}^{GRRM}) (again) is just the sum of those ***attribute level regret*** ($R_{i \leftrightarrow j, mn}^{\text{GRRM}}$) across attributes.
- The new parameter (γ) alters the shape of the regret, and the degree of asymmetries between regret and rejoice.
- Model generalized the original RRM model and also the RUM model! (how?)

3 $R^{\textsf{GRRM}}_{i\leftrightarrow j,mn}$ at different values of γ conditional on $\beta_m = 1$.



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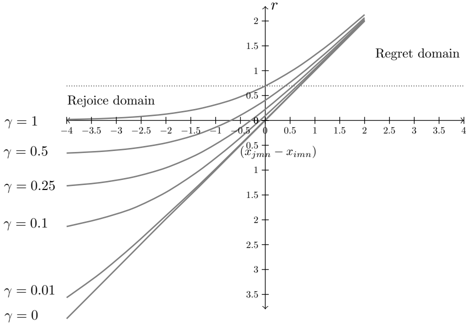

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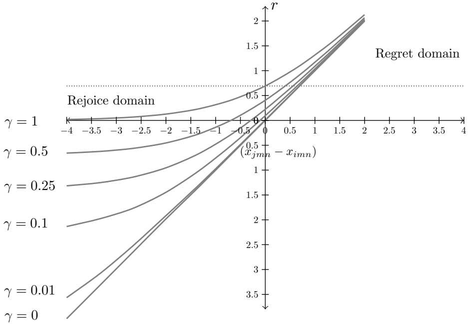

- $\gamma = 1 \Rightarrow$ Classic RRM.
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- $\gamma = 0$, no convexity $\Rightarrow$ fully compensatory behavior (RUM!).

3 μRRM (van Cranenburgh et al., 2015)

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$$R_{in}^{\mu\text{RRM}} = \sum_{j \neq i}^{J} \sum_{m=1}^{M} \mu \cdot R_{i \leftrightarrow j, mn}^{\mu\text{RRM}} = \sum_{j \neq i}^{J} \sum_{m=1}^{M} \mu \cdot \ln \left[1 + \exp \left\{ \left(\beta_m / \mu \right) \left(x_{jmn} - x_{imn} \right) \right\} \right] \tag{5}$$

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- ▶ The scale parameter is not identified in the RUM context.
- ▶ However, RRM models can describe a semi-compensatory behavior $\Rightarrow$ identification of the μ parameter.
- ▶ μ is informative of the degree of regret imposed by the model, stated otherwise, *how much semi-compensatory behavior we are observing in the decision makers choice behavior*.

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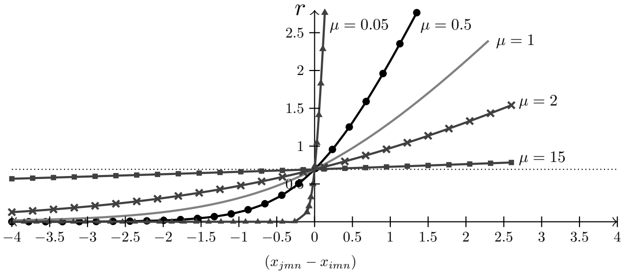

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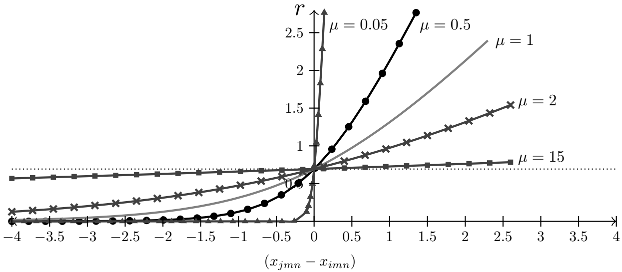

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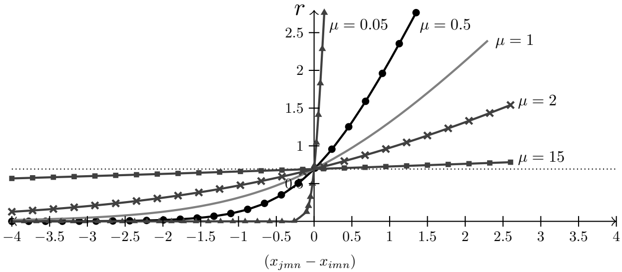

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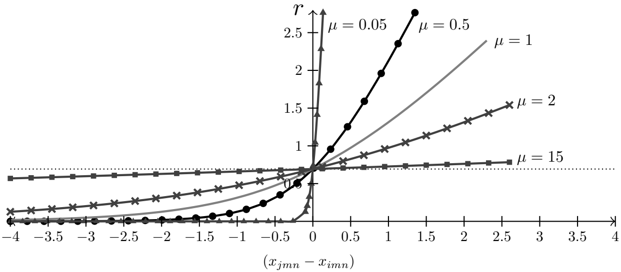

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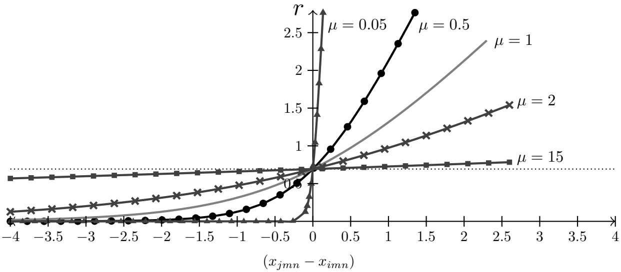

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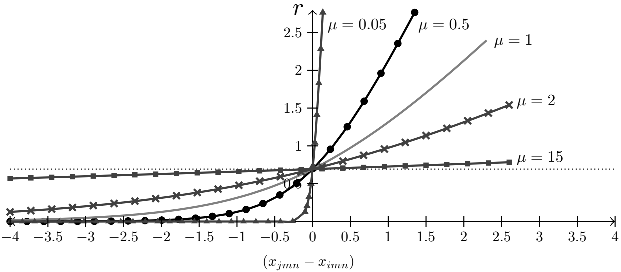

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- $\mu \to 0 \Rightarrow$ the higher the ratio (β_m/μ), $\Rightarrow$ the higher the asymmetries.
  - The model collapses into a new model: Pure RRM.

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- We need to know the sign of the attributes *a priori*!
- In some situations, this requisite is not very restrictive (e.g. price, cost).
- This model yields the strongest semi-compensatory behavior among all the RRM family

4 Outline

4 Relationships among the different models

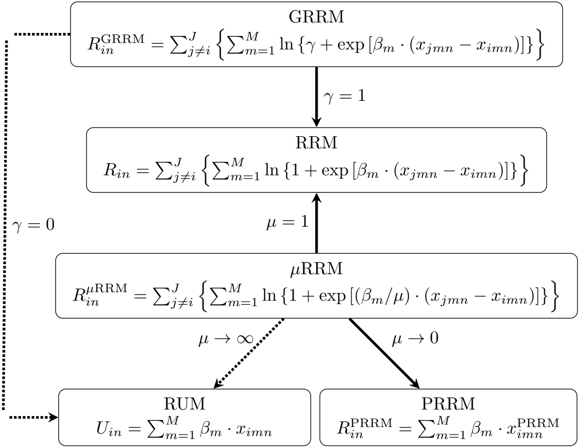

Figure: Interrelationship among the models based on parameters

4 Relationships among the different models

Table: LR test for model comparison.

Models	Hypothesis	LR statistic	Distribution under H_0
RRM v.s GRRM	$H_0 : \gamma = 1$ $H_1 : \gamma < 1$	$2\left\{\ell(\widehat{\theta}_{\mathsf{GRRM}}) - \ell(\widehat{\theta}_{\mathsf{RRM}})\right\}$	$0.5(\chi^2_0 + \chi^2_1)$
RUM v.s GRRM	$H_0 : \gamma = 0$ $H_1 : \gamma > 0$	$2\left\{\ell(\widehat{\theta}_{\mathsf{GRRM}}) - \ell(\widehat{\theta}_{\mathsf{RUM}})\right\}$	$0.5(\chi^2_0 + \chi^2_1)$
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- $\ell(.)$ represents the loglikelihood of the model, and $\widehat{\theta}_{\mathsf{RRM}}$, $\widehat{\theta}_{\mathsf{GRRM}}$, $\widehat{\theta}_{\mu\mathsf{RRM}}$, $\widehat{\theta}_{\mathsf{RUM}}$ represent the full set of parameters of the classical RRM, GRRM, μRRM and linear RUM model, respectively.

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- The fact that the two first hypotheses follow a different distribution from the traditional χ^2_1, is because we are testing a null hypothesis on the boundary of the parametric space of γ (Gutierrez et al., 2001).

5 Outline

5 Syntax

`randregret` is implemented as a Mata-based `d0 ml` evaluator. The command allows to implement four different regret functions in logit form.

`randregret` *depvar* [*indepvars*] [*if*] [*in*] *group(varname)* *alternative(varname)* *rrmfn(string)* [, `basealternative`(*string*) `noconstant` `uppermu`(#) `negative`(*varlist*) `positive`(*varlist*) `show` `notrl` `initgamma` `initmu` `robust` `cluster`(*varname*) `level`(#) *maximize_options*]

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The command `randregretpred` can be used following `randregret` to obtain predicted choice probabilities. It is also possible to recover the linear prediction of the systematic regret from equations (1), (4) (5) or (6).

`randregretpred` *newvar* [*if*] [*in*] `group(`*varname*`)` `alternatives(`*varname*`)` [, `proba` `xb`]

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  - Three unlabeled route alternatives ($J = 3$).
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2.	1	Second	0	1	27	4
3.	1	Third	1	1	35	3
4.	2	First	0	1	27	5
5.	2	Second	1	1	35	4
6.	2	Third	0	1	23	6

  - Three unlabeled route alternatives ($J = 3$).
  - Described by Travel Cost (***tc***) and Travel Time (***tt***) ($M = 2$).

- Each respondent (***id***) answered a total of 10 choice situations.

5 The Data

- Data from van Cranenburgh (2018): Stated Choice (SC) experiment.

```
. list obs altern choice id  tt tc   in 1/6, sepby(obs)
```

	obs	altern	choice	id	tt	tc
1.	1	First	0	1	23	6
2.	1	Second	0	1	27	4
3.	1	Third	1	1	35	3
4.	2	First	0	1	27	5
5.	2	Second	1	1	35	4
6.	2	Third	0	1	23	6

  - Three unlabeled route alternatives ($J = 3$).
  - Described by Travel Cost (***tc***) and Travel Time (***tt***) ($M = 2$).
- Each respondent (***id***) answered a total of 10 choice situations.
- Variable ***choice*** together with variable ***altern*** allows us identify choices.

5 Classic RRM Estimation + Cluster

```
. randregret choice  tc tt, gr(obs) alt(altern) rrmfn(classic) ///
> nocons cluster(id)
--------------------------------------------------------------------------------
Fitting Classic RRM Model
--------------------------------------------------------------------------------

initial:       log likelihood =  -1164.529
alternative:   log likelihood = -1156.5784
rescale:       log likelihood =   -1121.29
Iteration 0:   log likelihood =   -1121.29
Iteration 1:   log likelihood = -1118.4843
Iteration 2:   log likelihood = -1118.4784
Iteration 3:   log likelihood = -1118.4784
RRM: Classic Random Regret Minimization Model
Case ID variable: obs                           Number of cases    =       1060
Alternative variable: altern                    Number of obs      =       3180
                                                Wald chi2(2)       =      40.41
Log likelihood = -1118.4784                     Prob > chi2        =     0.0000
                           (Std. Err. adjusted for  106 clusters in id)
--------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+------------------------------------------------------------------
RRM          |
          tc |   -.417101    .068059    -6.13   0.000    -.5504943   -.2837078
          tt |   -.102813   .0182526    -5.63   0.000    -.1385874   -.0670386
--------------------------------------------------------------------------------
```

5 Generalized RRM Estimation + Cluster

```
. randregret choice  tc tt , gr(obs) alt(altern) rrmfn(gene) ///
> nocons  cluster(id)
-------------------------------------------------------------------------------
Fitting Classic RRM for Initial Values
-------------------------------------------------------------------------------

initial:       log likelihood =  -1164.529
alternative:   log likelihood = -1156.5784
rescale:       log likelihood =   -1121.29
Iteration 0:   log likelihood =   -1121.29
Iteration 1:   log likelihood = -1118.4843
Iteration 2:   log likelihood = -1118.4784
Iteration 3:   log likelihood = -1118.4784
-------------------------------------------------------------------------------
Fitting Conditional Logit as a Restricted Model (gamma=0) for LR test
-------------------------------------------------------------------------------
-------------------------------------------------------------------------------
Fitting Generalized RRM Model
-------------------------------------------------------------------------------

initial:       log likelihood = -1120.7001
rescale:       log likelihood = -1120.7001
rescale eq:    log likelihood = -1120.7001
Iteration 0:   log likelihood = -1120.7001
Iteration 1:   log likelihood = -1118.5366
Iteration 2:   log likelihood = -1118.3484
Iteration 3:   log likelihood = -1118.3307
Iteration 4:   log likelihood = -1118.3302
Iteration 5:   log likelihood = -1118.3302
GRRM: Generalized Random Regret Minimization Model
Case ID variable: obs                            Number of cases    =       1060
```

5 Generalized RRM Estimation + Cluster (nolog)

```
. randregret choice  tc tt , gr(obs) alt(altern) rrmfn(gene) ///
> nocons  cluster(id)  nolog
------------------------------------------------------------------------------
Fitting Classic RRM for Initial Values
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Fitting Conditional Logit as a Restricted Model (gamma=0) for LR test
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Fitting Generalized RRM Model
------------------------------------------------------------------------------

GRRM: Generalized Random Regret Minimization Model
Case ID variable: obs                          Number of cases    =       1060
Alternative variable: altern                   Number of obs      =       3180
                                               Wald chi2(2)       =      10.23
Log likelihood = -1118.3302                    Prob > chi2        =     0.0060
                                   (Std. Err. adjusted for  106 clusters in id)
------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
RRM          |
          tc |  -.3904872   .1248997    -3.13   0.002    -.6352861   -.1456884
          tt |  -.0967528   .0307009    -3.15   0.002    -.1569255     -.03658
-------------+----------------------------------------------------------------
       gamma |   .7843392   .5588736                      .0055712    .9995766
------------------------------------------------------------------------------
LR test of gamma=0: chibar2(01) = 9.41                 Prob >= chibar2 = 0.001
LR test of gamma=1: chibar2(01) = 0.30                 Prob >= chibar2 = 0.293
------------------------------------------------------------------------------
```

5 μRRM Estimation + Cluster

```
. randregret choice tc tt, gr(obs) alt(altern) rrm(mu) ///
> nocons   cluster(id)
-----------------------------------------------------------------------------
Fitting Classic RRM for Initial Values
-----------------------------------------------------------------------------

initial:       log likelihood =  -1164.529
alternative:   log likelihood = -1156.5784
rescale:       log likelihood =   -1121.29
Iteration 0:   log likelihood =   -1121.29
Iteration 1:   log likelihood = -1118.4843
Iteration 2:   log likelihood = -1118.4784
Iteration 3:   log likelihood = -1118.4784
-----------------------------------------------------------------------------
Fitting muRRM Model
-----------------------------------------------------------------------------

initial:       log likelihood = -1119.8154
rescale:       log likelihood = -1119.8154
rescale eq:    log likelihood = -1119.8154
Iteration 0:   log likelihood = -1119.8154  (not concave)
Iteration 1:   log likelihood = -1118.4346
Iteration 2:   log likelihood = -1118.3965
Iteration 3:   log likelihood = -1118.3965
muRRM: Mu-Random Regret Minimization Mode
Case ID variable: obs                             Number of cases    =       1060
Alternative variable: altern                      Number of obs      =       3180
                                                  Wald chi2(2)       =      66.95
Log likelihood = -1118.3965                       Prob > chi2        =     0.0000
                                             (Std. Err. adjusted for  106 clusters in id)
-----------------------------------------------------------------------------
```

5 μRRM Estimation + Cluster (nolog)

```
. randregret choice tc tt, gr(obs) alt(altern) rrm(mu) ///
> nocons    cluster(id) nolog
------------------------------------------------------------------------------
Fitting Classic RRM for Initial Values
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Fitting muRRM Model
------------------------------------------------------------------------------

muRRM: Mu-Random Regret Minimization Model
Case ID variable: obs                           Number of cases    =      1060
Alternative variable: altern                    Number of obs      =      3180
                                                Wald chi2(2)       =     66.95
Log likelihood = -1118.3965                     Prob > chi2        =    0.0000
                                  (Std. Err. adjusted for  106 clusters in id)
------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
RRM          |
          tc |   -.428041   .0557747    -7.67   0.000    -.5373574   -.3187246
          tt |  -.1059437   .0152902    -6.93   0.000     -.135912   -.0759754
-------------+----------------------------------------------------------------
          mu |   1.186166   .8271011                      .2464176    3.255421
------------------------------------------------------------------------------
LR test of mu=1: chi2(1) =0.16                        Prob >= chibar2 = 0.686
------------------------------------------------------------------------------
```

5 PRRM Estimation + Cluster

```
. randregret choice  , neg(tc tt)  gr(obs) alt(altern) rrmfn(pure) ///
> nocons  cluster(id)

PRRM: Pure Random Regret Minimization Model

Case ID variable: obs                          Number of cases    =      1060
Alternative variable: altern                   Number of obs      =      3180
                                               Wald chi2(2)       =     21.06
Log likelihood = -1128.3777                    Prob > chi2        =    0.0000
                                   (Std. Err. adjusted for 106 clusters in id)
```

choice	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
choice						
tc	-.285628	.0647545	-4.41	0.000	-.4125446	-.1587114
tt	-.0661575	.0169355	-3.91	0.000	-.0993505	-.0329645

```
The Pure-RRM uses a transformation of the original regressors using options
positive() and negative() as detailed in S. van Cranenburgh et. al (2015)
Afterward, randregret invokes clogit using these transormed regresors.
```

5 Prediction

```
. qui randregret choice  tc tt , gr(obs) alt(altern) rrmfn(classic) nocons nolog
. randregretpred prob,gr(obs) alt(altern) prob
. randregretpred xb ,gr(obs) alt(altern) xb
. list obs altern choice id  tt tc prob xb  in 1/6, sepby(obs)
```

	obs	altern	choice	id	tt	tc	prob	xb
1.	1	First	0	1	23	6	.22354907	3.4618503
2.	1	Second	0	1	27	4	.54655027	2.567855
3.	1	Third	1	1	35	3	.22990067	3.4338339
4.	2	First	0	1	27	5	.43840211	2.7134208
5.	2	Second	1	1	35	4	.19128045	3.5428166
6.	2	Third	0	1	23	6	.37031744	2.8821967

6 Outline

6 Source code + examples + stay in touch

- The repository with the source code is available in our Github!
- A `dofile` with the complete example listed here is also available on the repository.
- Suggestions are welcome at alvaro.gutierrezvargas@kuleuven.be.
- Also, you can consider to stay up for new features of `randregret` at `@alvarogutyerrez`.

7 Outline

8 Bibliography

Chorus, C. G. (2010). A new model of random regret minimization. *European Journal of Transport and Infrastructure Research*, 10(2):181–196.

Chorus, C. G. (2014). A generalized random regret minimization model. *Transportation Research Part B: Methodological*, 68:224 – 238.

Gutierrez, R. G., Carter, S., and Drukker, D. M. (2001). On boundary-value likelihood-ratio tests. *Stata Technical Bulletin*, 10(60).

van Cranenburgh, S. (2018). Small value-of-time experiment, netherlands. 4TU.Centre for Research Data, Dataset https://doi.org/10.4121/uuid:1ccca375-68ca-4cb6-8fc0-926712f50404.

van Cranenburgh, S., Guevara, C. A., and Chorus, C. G. (2015). New insights on random regret minimization models. *Transportation Research Part A: Policy and Practice*, 74:91 – 109.

9 Outline

8. Additional Outputs
9. Technical Details
10. Analitical Gradients

9 μRRM Estimation + Cluster (nolog) + show

```
. randregret choice tc tt, gr(obs) alt(altern) rrm(mu) ///
> nocons  show  cluster(id) nolog
------------------------------------------------------------------------------
Fitting Classic RRM for Initial Values
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Fitting muRRM Model
------------------------------------------------------------------------------

muRRM: Mu-Random Regret Minimization Model
Case ID variable: obs                           Number of cases    =      1060
Alternative variable: altern                    Number of obs      =      3180
                                                Wald chi2(2)       =     66.95
Log likelihood = -1118.3965                     Prob > chi2        =    0.0000
                                   (Std. Err. adjusted for  106 clusters in id)
------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
RRM          |
          tc |   -.428041   .0557747    -7.67   0.000    -.5373574   -.3187246
          tt |  -.1059437   .0152902    -6.93   0.000     -.135912   -.0759754
-------------+----------------------------------------------------------------
mu_star      |
       _cons |  -1.167909   .9141582    -1.28   0.201    -2.959626    .6238083
-------------+----------------------------------------------------------------
          mu |   1.186166   .8271011                      .2464176    3.255421
------------------------------------------------------------------------------
LR test of mu=1: chi2(1) =0.16                        Prob >= chibar2 = 0.686
------------------------------------------------------------------------------
```

9 Generalized RRM Estimation + Cluster + show

```
. randregret choice  tc tt , gr(obs) alt(altern) rrmfn(gene) ///
> nocons  cluster(id) show  nolog
------------------------------------------------------------------------------
Fitting Classic RRM for Initial Values
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Fitting Conditional Logit as a Restricted Model (gamma=0) for LR test
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Fitting Generalized RRM Model
------------------------------------------------------------------------------

GRRM: Generalized Random Regret Minimization Model
Case ID variable: obs                             Number of cases    =       1060
Alternative variable: altern                      Number of obs      =       3180
                                                  Wald chi2(2)       =      10.23
Log likelihood = -1118.3302                       Prob > chi2        =     0.0060
                                   (Std. Err. adjusted for  106 clusters in id)
------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
RRM          |
          tc |  -.3904872   .1248997    -3.13   0.002    -.6352861   -.1456884
          tt |  -.0967528   .0307009    -3.15   0.002    -.1569255     -.03658
-------------+----------------------------------------------------------------
gamma_star   |
       _cons |   1.291135   3.303988     0.39   0.696    -5.184563    7.766832
-------------+----------------------------------------------------------------
       gamma |   .7843392   .5588736                      .0055712    .9995766
------------------------------------------------------------------------------
LR test of gamma=0: chibar2(01) = 9.41                 Prob >= chibar2 = 0.001
LR test of gamma=1: chibar2(01) = 0.30                 Prob >= chibar2 = 0.293
------------------------------------------------------------------------------
```

10 Outline

10 Alternative Specific Constants (ASC)

- Let R^*_{in} denote a generic systematic regret of alternative i as defined in equation (1), (4), (5) or (6).
- We denote by α_i ASC of alternative i in equation (8).

$$R^*_{in} = \sum_{j \neq i}^{J} \sum_{m=1}^{M} R^*_{i \leftrightarrow j, mn} + \alpha_i \tag{8}$$

- The inclusion of the ASC serves the same purpose as in RUM models: to account for *omitted attributes for a particular alternative*.
- As usual, for identification purposes, we need to exclude one of the ASC from the model specification.

10 Robust Standard Errors

We can write our maximum-likelihood estimation equations as in equation (9). Where $\boldsymbol{\theta}$ is the full set of parameters, $\boldsymbol{S}(\boldsymbol{\theta}; y_n, \boldsymbol{x}_n) = \partial \ln L_n / \partial \boldsymbol{\theta}$ represents the score functions, $\ln L_n$ is the log likelihood of observation n, $\boldsymbol{x}_n$ is the full set of attributes, and y_n is the response variable that takes the value of 1 when alternative i is selected and 0 otherwise.

$$G(\boldsymbol{\theta}) = \sum_{n=1}^{N} \boldsymbol{S}(\boldsymbol{\theta}; y_n, \boldsymbol{x}_n) = \mathbf{0} \tag{9}$$

We can compute the robust variance estimator of $\boldsymbol{\theta}$ using equation (10), where $\boldsymbol{D} = -H^{-1}$ is the negative of the inverse of the hessian resulting from the optimization procedure, and $\boldsymbol{u}_n = \boldsymbol{S}(\widehat{\boldsymbol{\theta}}; y_n, \boldsymbol{x}_n)$ are row vectors that contains the score functions evaluated at $\widehat{\boldsymbol{\theta}}$.

$$\widehat{V}(\widehat{\boldsymbol{\theta}}) = \boldsymbol{D} \left(\frac{n}{n-1} \sum_{n=1}^{N} \boldsymbol{u}_n' \boldsymbol{u}_n \right) \boldsymbol{D} \tag{10}$$

10 Cluster Robust Standard Errors

Equation (10) is appropriate only if the observations are independent. However, when several choice situations are answered by the same individual, we can expect some degree of correlation of these choices. When such a structure is present in the data the correct cluster robust variance estimator is given by equation (11), where C_k contains the indices of all observations belonging to the same individual k for $k = 1, 2, \ldots, n_c$ with n_c the total number of different individuals present in the data set.

$$\widehat{V}(\widehat{\boldsymbol{\theta}}) = \boldsymbol{D} \left\{ \frac{n_c}{n_c - 1} \sum_{k=1}^{n_c} \left(\sum_{n \in C_k} \boldsymbol{u}_n \right)' \left(\sum_{n \in C_k} \boldsymbol{u}_n \right) \right\} \boldsymbol{D} \qquad (11)$$

Details on the analytical form of the scores by each model presented in this presentation are provided from slide number 43 on. Additionally, randregret command is able to compute corrected standard errors using the analytical form of the score functions without relying in numerical approximations.

11 Outline

8 Additional Outputs

9 Technical Details

10 Analitical Gradients

- Generic Scores Functions for RRM models
- Scores functions for the classical RRM model
- Scores functions for GRRM model
- Scores functions for μRRM model
- Scores Functions for PRRM model

11 Generic Scores Functions for RRM models

Without loss of generality, we can state that the log-likelihood of the four RRM models presented in this presentation can be represented by equation (12). In particular, when R_{in}^* is replaced by equations (1), (4), (5) or (6), we can fit respectively the classical RRM, the GRRM, the μRRM, and the PRRM model.

$$\begin{aligned}
\ln L &= \sum_{n=1}^{N} \sum_{i=1}^{J} y_{in} \ln\left(P_{in}^*\right) \\
&= \sum_{n=1}^{N} \sum_{i=1}^{J} y_{in} \ln\left(\frac{\exp\left(-R_{in}^*\right)}{\sum_{j=1}^{J} \exp\left(-R_{jn}^*\right)}\right) \\
&= -\sum_{n=1}^{N} \sum_{i=1}^{J} y_{in} R_{in}^* - \sum_{n=1}^{N} \sum_{i=1}^{J} y_{in} \ln\left(\sum_{j=1}^{J} \exp\left(-R_{jn}^*\right)\right) \qquad (12)
\end{aligned}$$

11 Generic Scores Functions for RRM models

Furthermore, any partial derivative of the log-likelihood with respect to any parameter $\theta \in \boldsymbol{\theta}$, where $\boldsymbol{\theta}$ stands for the full set of parameters of the model, can be expressed as in equation (13). The rank of $\boldsymbol{\theta}$ will depend on the particular model.

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \theta} &= -\sum_{n=1}^{N}\sum_{i=1}^{J} y_{in}\frac{\partial R^*_{in}}{\partial \theta} + \sum_{n=1}^{N}\sum_{i=1}^{J} y_{in}\left(\sum_{j=1}^{J} P_{jn}\frac{\partial R^*_{jn}}{\partial \theta}\right) \\
&= -\sum_{n=1}^{N}\sum_{i=1}^{J} (y_{in} - P_{in})\left(\frac{\partial R^*_{in}}{\partial \theta}\right) \qquad (13)
\end{aligned}
$$

In the next slides, we will list the partial derivatives, also known as scores functions, per type of parameter in each type of model. Additionally, it is crucial to notice that, in any case, we can check that $\partial R^*_{in}/\partial \alpha_i = 1$, where α_i represents the coefficient associated with the ASC of alternative i.

11 Scores functions for the classical RRM model

In order to obtain the loglikelihood of the classic RRM model we need to substitute R^*_{in} in equation (12) by equation (1). Accordingly, the set of parameters $\boldsymbol{\theta}$ is now given by $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\alpha})'$. Here $\boldsymbol{\beta}$ is a $m \times 1$ vector of alternative-specific regression coefficients and $\boldsymbol{\alpha}$ is a $(J-1) \times 1$ vector of ASC. Subsequently, the scores functions of the classical RRM model will be described as follows:

$$\begin{aligned} \frac{\partial \ln L}{\partial \boldsymbol{\theta}} &= \left(\frac{\partial \ln L}{\partial \beta_1}, \ldots, \frac{\partial \ln L}{\partial \beta_M}, \frac{\partial \ln L}{\partial \alpha_1}, \ldots, \frac{\partial \ln L}{\partial \alpha_{J-1}} \right) \\ &= \left(\frac{\partial \ln L}{\partial \boldsymbol{\beta}}, \frac{\partial \ln L}{\partial \boldsymbol{\alpha}} \right) \end{aligned}$$

Finally, to obtain the expression for $\partial \ln L / \partial \beta_m$ we need to replace equation (14) into equation (13).

$$\frac{\partial R_{in}}{\partial \beta_m} = \sum_{j \neq i}^{J} \left(\frac{\exp\left\{\beta_m \left(x_{jmn} - x_{imn}\right)\right\} \cdot \left(x_{jmn} - x_{imn}\right)}{1 + \exp\left\{\beta_m \left(x_{jmn} - x_{imn}\right)\right\}} \right) \tag{14}$$

11 Scores functions for GRRM model

The log-likelihood of the GRRM model can be constructed by replacing the term R^*_{in} in equation (12) by equation (4). Hence, the full set of parameters $\boldsymbol{\theta}$ is now given by $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\alpha}, \gamma^*)'$. Here, $\boldsymbol{\beta}$ is a $m \times 1$ vector of alternative-specific regression coefficients, $\boldsymbol{\alpha}$ is a $(J-1) \times 1$ vector of ASC and γ^* is a scalar equal to the parameter γ in the logit scale. Hence, the corresponding scores functions are described by:

$$\begin{aligned}\frac{\partial \ln L}{\partial \boldsymbol{\theta}} &= \left(\frac{\partial \ln L}{\partial \beta_1}, \ldots, \frac{\partial \ln L}{\partial \beta_M}, \frac{\partial \ln L}{\partial \alpha_1}, \ldots, \frac{\partial \ln L}{\partial \alpha_{J-1}}, \frac{\partial \ln L}{\partial \gamma^*}\right) \\ &= \left(\frac{\partial \ln L}{\partial \boldsymbol{\beta}}, \frac{\partial \ln L}{\partial \boldsymbol{\alpha}}, \frac{\partial \ln L}{\partial \gamma^*}\right)\end{aligned}$$

Additionally, in order to obtain the expression for $\partial \ln L / \partial \beta_m$ we need to replace equation (15) into equation (13).

$$\frac{\partial R^{\mathsf{GRRM}}_{in}}{\partial \beta_m} = \sum_{j \neq i}^{J} \left(\frac{\exp\left\{\beta_m \left(x_{jmn} - x_{imn}\right)\right\} \cdot \left(x_{jmn} - x_{imn}\right)}{\gamma + \exp\left\{\beta_m \left(x_{jmn} - x_{imn}\right)\right\}}\right) \qquad (15)$$

11 Scores functions for GRRM model

However, the score function of the parameter γ^* needs a slightly different treatment. As mentioned earlier, the optimization procedure does not directly fit the parameter γ, but instead, it fits the model using an ancillary parameter: $\gamma^* = \text{logit}(\gamma)$ (referred as `gamma_star` in the output when using `show` option). Hence, we model the parameter γ in the logit scale. This fact has a direct impact on the score function of parameter γ^*. Using the chain rule, we can state:

$$\frac{\partial \ln L}{\partial \gamma} = \frac{\partial \ln L}{\partial \gamma^*} \cdot \frac{\partial \gamma^*}{\partial \gamma}$$

Subsequently, solving $\partial\gamma^*/\partial\gamma$ and rearranging terms, we see in equation (16), that in order to compute the score function of the parameter γ^*, we need to adjust the partial derivative from the log-likelihood with respect to γ by a factor of $\gamma(1-\gamma)$.

$$\frac{\partial \ln L}{\partial \gamma^*} = \frac{\partial \ln L}{\partial \gamma} \cdot \gamma(1-\gamma) \tag{16}$$

11 Scores functions for GRRM model

The expression for $\partial \ln L/\partial\gamma$ can be computed replacing equation (17) into equation (13), which together with equation (16) gives us the required expression for $\partial \ln L/\partial\gamma^*$.

$$\frac{\partial R_{in}^{\mathsf{GRRM}}}{\partial\gamma} = \sum_{j\neq i}^{J}\sum_{m=1}^{M}\left(\frac{1}{\gamma + \exp\left\{\beta_m\left(x_{jmn} - x_{imn}\right)\right\}}\right) \tag{17}$$

11 Scores functions for μRRM model

The μRRM model has a log-likelihood that is a particular case of equation (13), where R^*_{in} is replaced by equation (5). Thus, the full set of parameters $\boldsymbol{\theta}$ is now described by $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\alpha}, \mu^*)'$. Here $\boldsymbol{\beta}$ is a $m \times 1$ vector of alternative-specific regression coefficients, $\boldsymbol{\alpha}$ is a $(J-1) \times 1$ vector of ASC and μ^* is a scalar equal to the μ parameter in a transformed scale. Thus, the corresponding scores functions can be represented by:

$$
\begin{aligned}
\frac{\partial \ln L}{\partial \boldsymbol{\theta}} &= \left(\frac{\partial \ln L}{\partial \beta_1}, \ldots, \frac{\partial \ln L}{\partial \beta_M}, \frac{\partial \ln L}{\partial \alpha_1}, \ldots, \frac{\partial \ln L}{\partial \alpha_{J-1}}, \frac{\partial \ln L}{\partial \mu^*} \right) \\
&= \left(\frac{\partial \ln L}{\partial \boldsymbol{\beta}}, \frac{\partial \ln L}{\partial \boldsymbol{\alpha}}, \frac{\partial \ln L}{\partial \mu^*} \right) \qquad (18)
\end{aligned}
$$

First, by replacing equation (19) back into equation (13) we can easily obtain the expression for $\partial \ln L / \partial \beta_m$.

$$
\frac{\partial R_{in}^{\mu\text{RRM}}}{\partial \beta_m} = \sum_{j \neq i}^{J} \left(\frac{\exp\left[(\beta_m/\mu) \cdot (x_{jmn} - x_{imn})\right] \cdot (x_{jmn} - x_{imn})}{\mu \cdot (1 + \exp\left[(\beta_m/\mu) \cdot (x_{jmn} - x_{imn})\right])} \right) \qquad (19)
$$

11 Scores functions for μRRM model

The μRRM model, similarly to the GRRM model, also fits the parameter μ using an unbounded ancillary parameter: $\mu^* = \ln(\mu/(M-\mu))$ (referred as `mu_star` in the output when using `show` option). Accordingly, this transformation needs to be taken into account when computing the score function of the parameter μ^*. Using the chain rule, we can state:

$$\frac{\partial \ln L}{\partial \mu} = \frac{\partial \ln L}{\partial \mu^*} \cdot \frac{\partial \mu^*}{\partial \mu}$$

Solving for $\partial\mu^*/\partial\mu$ and rearranging terms, we can see that the score function of the parameter μ^* is the same as the partial derivative of the log-likelihood with respect to μ multiplied by a factor equal to $\mu(M-\mu)/M$.

$$\frac{\partial \ln L}{\partial \mu^*} = \frac{\partial \ln L}{\partial \mu} \cdot \frac{\mu(M-\mu)}{M} \tag{20}$$

11 Scores functions for μRRM model

Finally, the expression for $\partial \ln L / \partial \mu$ can be obtained replacing equations (21) and (22) into equation (13), which together with equation (20), provides the required expression for $\partial \ln L / \partial \mu^*$.

$$\frac{\partial R_{in}^{\mu\text{RRM}}}{\partial \mu} = \sum_{j \neq i}^{J} \sum_{m=1}^{M} R_{i \leftrightarrow j,m}^{\mu\text{RRM}} + \mu \cdot \sum_{j \neq i}^{J} \sum_{m=1}^{M} \frac{\partial R_{i \leftrightarrow j,m}^{\mu\text{RRM}}}{\partial \mu} \tag{21}$$

$$\frac{\partial R_{i \leftrightarrow j,m}^{\mu\text{RRM}}}{\partial \mu} = \left(\frac{\exp\{(\beta_m/\mu) \cdot (x_{jmn} - x_{imn})\} \cdot (x_{jmn} - x_{imn}) \cdot \beta_m}{\mu^2 \cdot (1 + \exp\{(\beta_m/\mu) \cdot (x_{jmn} - x_{imn})\})} \right) \tag{22}$$

11 Scores Functions for PRRM model

We can recover the log-likelihood of the PRRM model replacing the expression R^*_{in} in equation (12) by equation (6). Thus, the full set of parameters $\boldsymbol{\theta}$ is now described by $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\alpha})'$. Here $\boldsymbol{\beta}$ is a $m \times 1$ vector of alternative-specific regression coefficients and $\boldsymbol{\alpha}$ is a $(J-1) \times 1$ vector of ASC. Consequently, the scores functions are then:

$$\begin{aligned}\frac{\partial \ln L}{\partial \boldsymbol{\theta}} &= \left(\frac{\partial \ln L}{\partial \beta_1}, \ldots, \frac{\partial \ln L}{\partial \beta_M}, \frac{\partial \ln L}{\partial \alpha_1}, \ldots, \frac{\partial \ln L}{\partial \alpha_{J-1}}\right) \\ &= \left(\frac{\partial \ln L}{\partial \boldsymbol{\beta}}, \frac{\partial \ln L}{\partial \boldsymbol{\alpha}}\right)\end{aligned}$$

Accordingly, we can obtain the expression for $\partial \ln L / \partial \beta_m$ by replacing equation (23) into equation (13).

$$\frac{\partial R^{\text{PURE}}_{in}}{\partial \beta_m} = x^{\text{PURE}}_{imn} \tag{23}$$