Estimating long-run effects in models with cross-sectional dependence using xtdcce2

Three ways to estimate long run coefficients

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Introduction

- `xtdcce2` on SSC since August 2016
- Described in *The Stata Journal* article in Vol 18, Number 3, Ditzen (2018).
- Current version 1.33 (as of 22.10.2018).
- Setting: Dynamic panel model with heterogeneous slopes and an unobserved common factor (f_t) and a heterogeneous factor loading (γ_i):

$$\begin{aligned} y_{i,t} &= \lambda_i y_{i,t-1} + \beta_i x_{i,t} + u_{i,t}, \\ u_{i,t} &= \gamma_i' f_t + e_{i,t} \\ \beta_{MG} &= \frac{1}{N}\sum_{i=1}^{N} \beta_i, \quad \lambda_{MG} = \frac{1}{N}\sum_{i=1}^{N} \lambda_i \end{aligned} \tag{1}$$

$$i = 1, ..., N \; and \; t = 1, ..., T$$

- Aim: consistent estimation of β_i and β_{MG} :
  - Large N, T = 1: Cross Section; $\hat{\beta} = \hat{\beta}_i, \; \forall \; i$
  - N=1 , Large T: Time Series; $\hat{\beta}_i$
  - Large N, Small T: Micro-Panel; $\hat{\beta} = \hat{\beta}_i, \; \forall \; i$
  - Large N, Large T: Panel Time Series; $\hat{\beta}_i$ and $\hat{\beta}_{MG}$
- If the common factors are left out, they become an omitted variable, leading to the omitted variable bias.

Introduction

- Estimation of most economic models requires heterogeneous coefficients. Examples: growth models (Lee et al., 1997), development economics (McNabb and LeMay-Boucher, 2014), productivity analysis (Eberhardt et al., 2012), consumption models (Shin et al., 1999) ,...
- Vast econometric literature on heterogeneous coefficients models (Zellner, 1962; Pesaran and Smith, 1995; Shin et al., 1999).
- Theoretical literature how to account for unobserved dependencies between cross-sectional units evolved (Pesaran, 2006; Chudik and Pesaran, 2015).

Dynamic Common Correlated Effects I

$$y_{i,t} = \lambda_i y_{i,t-1} + \beta_i x_{i,t} + u_{i,t}, \qquad (2)$$
$$u_{i,t} = \gamma_i' f_t + e_{i,t}$$

- Individual fixed effects (α_i) or deterministic time trends can be added, but are omitted in the remainder of the presentation..
- The heterogeneous coefficients are randomly distributed around a common mean, $\beta_i = \beta + v_i,\ v_i \sim IID(0, \Omega_v)$ and $\lambda_i = \lambda + \varsigma_i,\ \varsigma_i \sim IID(0, \Omega_\varsigma)$.
- f_t is an unobserved common factor and γ_i a heterogeneous factor loading.
- In a static model $\lambda_i = 0$, Pesaran (2006) shows that equation (2) can be consistently estimated by approximating the unobserved common factors with cross section averages $\bar{x}_t$ and $\bar{y}_t$ under strict exogeneity.

Dynamic Common Correlated Effects II

- In a dynamic model, the lagged dependent variable is not strictly exogenous and therefore the estimator becomes inconsistent. Chudik and Pesaran (2015) show that the estimator gains consistency if the floor of $p_T = \left[\sqrt[3]{T}\right]$ lags of the cross-sectional averages are added.
- Estimated Equation:

$$y_{i,t} = \lambda_i y_{i,t-1} + \beta_i x_{i,t} + \sum_{l=0}^{p_T} \gamma_{i,l}' \bar{\mathbf{z}}_{t-l} + \epsilon_{i,t}$$

$$\bar{\mathbf{z}}_t = (\bar{y}_t, \bar{x}_t)$$

- The Mean Group Estimates are: $\hat{\pi}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\pi}_i$ with $\hat{\pi}_i = (\hat{\lambda}_i, \hat{\beta}_i)$ and the asymptotic variance is

$$\widehat{Var}(\hat{\pi}_{MG}) = \frac{1}{N(N-1)} \sum_{i=1}^{N} (\hat{\pi}_i - \hat{\pi}_{MG})(\hat{\pi}_i - \hat{\pi}_{MG})'$$

What is new?

- This is what `xtdcce2` can do - what is new in version 1.33?
- A more general representation of eq (1) with further lags of the dependent and independent variable in the form of an ARDL(p_y, p_x) model is:

$$y_{i,t} = \sum_{l=1}^{p_y} \lambda_{l,i} y_{i,t-l} + \sum_{l=0}^{p_x} \beta_{l,i} x_{i,t-l} + u_{i,t}. \tag{3}$$

- where p_y and p_x is the lag length of y and x.
- The long run coefficient of β and the mean group coefficient are:

$$\theta_i = \frac{\sum_{l=0}^{p_x} \beta_{l,i}}{1 - \sum_{l=1}^{p_y} \lambda_{l,i}}, \quad \bar{\theta}_{MG} = \sum_{i=1}^{N} \theta_i \tag{4}$$

- `xtdcce2`, version < 1.33, is not able to estimate the sum of coefficients and their standard errors.
- How to estimate θ_i and $\bar{\theta}_{MG}$?
    - Chudik et al. (2016) propose two methods, the cross-sectionally augmented ARDL (CS-ARDL) and the cross-sectionally augmented distributed lag (CS-DL) estimator.
    - Using an error correction model (ECM).

CS-DL

- If λ_i lies within the unit circle, the general ARDL model in (3) can be re-written as a level equation:

$$y_{i,t} = \theta_i x_{i,t} + \delta_i(L)\Delta x_{i,t} + \tilde{u}_{i,t} \tag{5}$$

- and L is the lag operator.
- Idea: directly estimate the long run coefficients, by adding differences of the explanatory variables and their lags.

Details

CS-DL

- Lags of the cross-sectional averages are added to account for cross-sectional dependence.
- Together with the lags, equation (5) can be written as:

$$y_{i,t} = \theta_i x_{i,t} + \sum_{l=0}^{p_x - 1} \delta_{i,l} \Delta x_{i,t-l} + \sum_{l=0}^{p_{\bar{y}}} \gamma_{y,i,l} \bar{y}_{i,t-l} + \sum_{l=0}^{p_{\bar{x}}} \gamma_{x,i,l} \bar{x}_{i,t-l} + e_{i,t}$$

- where $p_{\bar{y}}$ and $p_{\bar{x}}$ is the number of lags of the cross-sectional averages.
- The mean group estimates are then

$$\hat{\bar{\theta}}_{MG} = \sum_{i=1}^{N} \hat{\theta}_i$$

- The variance/covariance matrix for the mean group coefficients is the same as for the "normal" (D)CCE estimator.

CS-ARDL

- Idea: Estimate the short run coefficients first and then calculate the long run coefficients.
- Equation (3) is extended by cross-sectional averages

$$y_{i,t} = \sum_{l=1}^{p_y} \lambda_{l,i} y_{i,t-l} + \sum_{l=0}^{p_x} \beta_{l,i} x_{i,t-l} + \sum_{l=0}^{p_T} \gamma_{i,l}' \bar{\mathbf{z}}_{t-l} + e_{i,t}.$$

- with $\bar{\mathbf{z}}_{t-l} = (\bar{y}_{i,t-l}, \bar{x}_{i,t-l})$
- and the long run coefficients and the mean group estimates are

$$\hat{\theta}_{CS-ARDL,i} = \frac{\sum_{l=0}^{p_x} \hat{\beta}_{l,i}}{1 - \sum_{l=1}^{p_y} \hat{\lambda}_{l,i}}, \quad \hat{\bar{\theta}}_{MG} = \sum_{i=1}^{N} \hat{\theta}_i$$

- The variance/covariance matrix for the mean group coefficients is the same as for the "normal" (D)CCE estimator.
- For the calculation of the variance/covariance matrix of the individual long run coefficients θ_i, the delta method is used. Delta Method

Error Correction Model

- Equation (3) can be transformed into an ECM[1]:

$$\Delta y_{i,t} = \phi_i \left[y_{i,t-1} - \theta_i x_{i,t} \right] - \sum_{l=1}^{p_y - 1} \lambda_{l,i} \Delta_l y_{i,t-1} - \sum_{l=1}^{p_x} \beta_{l,i} \Delta_l x_{i,t} + \sum_{l=0}^{p_T} \gamma_{i,l} \bar{z}_{i,t} + u_{i,t}$$

- where $\Delta_l =_t -_{t-l}$, for example $\Delta_3 x_{i,t} = x_{i,t} - x_{i,t-3}$ and

$$\hat{\phi}_i = -\left(1 - \sum_{l=1}^{p_y} \hat{\lambda}_{l,i}\right), \quad \hat{\theta}_i = \frac{\sum_{l=0}^{p_x} \hat{\beta}_{l,i}}{\hat{\phi}_i} \text{ and } \hat{\hat{\theta}}_{MG} = \sum_{i=1}^{N} \hat{\theta}_i$$

- For the calculation of the variance/covariance matrix of the individual long run coefficients θ_i, the delta method is used. ▸ Delta Method

[1] This function was already available in `xtdcce2` < 1.33.

xtdcce2

General Syntax

Syntax:

xtdcce2 *depvar* [*indepvars*] [*varlist2* = *varlist_iv*] [*if*]
crosssectional(varlist_cr) [, nocrosssectional pooled(*varlist_p*)
cr_lags(#) ivreg2options(*string*) e_ivreg2 ivslow lr(*varlist_lr*)
lr_options(*string*) pooledconstant noconstant reportconstant trend
pooledtrend jackknife recursive noomitted nocd fullsample
showindividual fast]

▸ More Details, ▸ Stored in e(), ▸ Bias Correction

xtdcce2

General Syntax

$$
\begin{aligned}
y_{i,t} =& \alpha_i + \sum_{l=1}^{p_y} \lambda_{l,i} y_{i,t-l} + \sum_{l=0}^{p_x} \beta_{l,i} x_{i,t-l} \\
& + \sum_{l=0}^{p_{\bar{y}}} \gamma_{y,i,l} \bar{y}_{t-l} + \sum_{l=0}^{p_{\bar{x}}} \gamma_{x,i,l} \bar{x}_{t-l} + e_{i,t}
\end{aligned}
$$

- `crosssectional`(*varlist*) specifies cross sectional means, i.e. variables in $\bar{z}_t$. These variables are partialled out.
- `cr_lags`(#) defines number of lags (p_T) of the cross sectional averages. The number of lags can be variable specific. The same order as in `cr`() applies, hence if `cr(y x)`, then `cr_lags(`$p_{\bar{y}}$ $p_{\bar{x}}$`)`.
- `pooled`(*varlist*) constraints coefficients to be homogeneous ($\beta_i = \beta, \forall\ i \in N$).
- `reportonstant` reports constant and `pooledconstant` pools it.

xtdcce2

CS-DL

- Assume an ARDL(1,2) and $p_T = (p_{\bar{y}}, p_{\bar{x}}) = (0, 2)$ such as:

$$\begin{aligned} y_{i,t} =& \lambda_i y_{i,t-1} + \beta_{0,i} x_{i,t} + \beta_{1,i} x_{i,t-1} + \beta_{2,i} x_{i,t-2} \\ & + \gamma_{y,i} \bar{y}_t + \sum_{l=0}^{2} \gamma_{x,i,l} \bar{x}_{t-l} + e_{i,t} \end{aligned}$$

- To estimate the model directly using the CS-DL estimator the following auxiliary regression is needed

$$y_{i,t} = \theta_i x_{i,t} + \delta_{0,i} \Delta x_{i,t} + \delta_{1,i} \Delta x_{i,t-1} + \gamma_{y,i} \bar{y}_t + \sum_{l=0}^{2} \gamma_{x,i,l} \bar{x}_{t-l} + \epsilon_{i,t} \qquad (6)$$

- To estimate it in `xtdcce2` the command line would be:

```
xtdcce2 y x d.x d2.x , cr(y x) cr_lags(0 2)
```

- No specific commands for the long run estimation are required.

xtdcce2

CS-DL Example

- Chudik et al. (2013) estimate the long run effect of public debt on output growth with the following equation:

$$\Delta y_{i,t} = c_i + \boldsymbol{\theta}_i' \mathbf{x}_{i,t} + \sum_{l=0}^{p_x - 1} \boldsymbol{\beta}_{i,l} \Delta \mathbf{x}_{i,t-l} + \gamma_{y,i} \Delta \bar{y}_t + \sum_{l=0}^{3} \boldsymbol{\gamma}_{x,i,l} \bar{\mathbf{x}}_{i,t-l} e_{i,t}$$

- where $y_{i,t}$ is the log of real GDP, $\mathbf{x}_{i,t} = (\Delta d_{i,t}, \pi_{i,t})'$, $d_{i,t}$ is log of debt to GDP ratio and π is the inflation rate.
- The results from Chudik et al. (2013, Table 18) with 1 lag of the explanatory variables ($p_x = 1$) in the form of an ARDL(1,1,1) and three lags of the cross sectional averages are estimated with:

```
xtdcce2133 d.y dp d.gd d.(dp d.gd) , cr(d.y dp d.gd)
cr_lags(0 3 3) fullsample
```

xtdcce2

CS-DL Example

```
. xtdcce2133 d.y dp d.gd d.(dp d.gd) , cr(d.y dp d.gd) cr_lags(0 3 3) /*
*/ fullsample
(Dynamic) Common Correlated Effects Estimator - Mean Group

Panel Variable (i): ccode                          Number of obs      =       1601
Time Variable (t): year                            Number of groups   =         40

Degrees of freedom per group:                      Obs per group (T) =         40
 without cross-sectional averages    = 35.025
 with cross-sectional averages       = 26.025
Number of                                          F(560, 1041)       =       0.90
 cross-sectional lags                0 to 3        Prob > F           =       0.93
 variables in mean group regression = 160          R-squared          =       0.33
 variables partialled out            = 400         Adj. R-squared     =      -0.04
                                                   Root MSE           =       0.03
                                                   CD Statistic       =       1.11
                                                      p-value         =     0.2667
------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 Mean Group: |
          dp |  -.0889337   .0256445    -3.47   0.001    -.1391959   -.0386715
        D.gd |  -.0865123      .0143    -6.05   0.000    -.1145398   -.0584849
        D.dp |   .0053277   .0413627     0.13   0.898    -.0757417    .0863971
       D2.gd |   .0068065   .0148306     0.46   0.646     -.022261    .0358739
------------------------------------------------------------------------------
Mean Group Variables: dp D.gd D.dp D2.gd
Cross Sectional Averaged Variables: D.y(0) dp(3) D.gd(3)
Heterogenous constant partialled out.
```

- The long run coefficients are $\hat{\theta}_{\pi,MG} = -0.0889$ and $\hat{\theta}_{\Delta d,MG} = -0.0865$.

xtdcce2

CS-DL Example

- And as an ARDL(1,3,3):

```
. xtdcce2133 d.y dp d.gd L(0/2).d.(dp d.gd) , cr(d.y dp d.gd) cr_lags(0 3 3) fullsample
(Dynamic) Common Correlated Effects Estimator - Mean Group

Panel Variable (i): ccode                          Number of obs      =       1571
Time Variable (t): year                            Number of groups   =         40

Degrees of freedom per group:                      Obs per group (T)  =         39
 without cross-sectional averages    = 30.275
 with cross-sectional averages       = 21.275
Number of                                          F(720, 851)        =       1.12
 cross-sectional lags                  0 to 3      Prob > F           =       0.06
 variables in mean group regression = 320          R-squared          =       0.49
 variables partialled out            = 400         Adj. R-squared     =       0.05
                                                   Root MSE           =       0.03
                                                   CD Statistic       =       0.73
                                                      p-value         =     0.4680
-------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+-----------------------------------------------------------------
 Mean Group: |
          dp | -.0855842   .0400845   -2.14   0.033     -.1641483    -.00702
        D.gd | -.0816583   .0196252   -4.16   0.000     -.1201231  -.0431936
        D.dp |  .0183584   .0478696    0.38   0.701     -.0754643    .112181
       LD.dp |  .0015586   .0373619    0.04   0.967     -.0716695   .0747866
      L2D.dp |  .0034012   .0294771    0.12   0.908     -.0543729   .0611752
       D2.gd |  .0045224   .0144741    0.31   0.755     -.0238463   .0328912
      LD2.gd | -.0129675   .0134553   -0.96   0.335     -.0393395   .0134045
     L2D2.gd | -.0095151   .0090813   -1.05   0.295     -.0273142    .008284
-------------------------------------------------------------------------------
Mean Group Variables: dp D.gd D.dp LD.dp L2D.dp D2.gd LD2.gd L2D2.gd
Cross Sectional Averaged Variables: D.y(0) dp(3) D.gd(3)
Heterogenous constant partialled out.
```

xtdcce2

CS-ARDL

- Assume an ARDL(1,2) and $p_T = (p_{\bar{y}}, p_{\bar{x}}) = (2, 2)$ such as:

$$y_{i,t} = \lambda_i y_{i,t-1} + \beta_{0,i} x_{i,t} + \beta_{1,i} x_{i,t-1} + \beta_{2,i} x_{i,t-2} + \sum_{l=0}^{2} \gamma_{y,i,l} \bar{y}_t + \sum_{l=0}^{2} \gamma_{x,i,l} \bar{x}_{t-l} + e_{i,t}$$

- The model is directly estimated and then the long run coefficients are calculated as:

$$\hat{\theta}_{CS-ARDL,i} = \frac{\hat{\beta}_{0,i} + \hat{\beta}_{1,i} + \hat{\beta}_{2,i}}{1 - \hat{\lambda}_i}$$

- Using `xtdcce2` the command line is:
  `xtdcce2 y , lr(L.y x L.x L2.x) lr_options(ardl) cr(y x) cr_lags(2)`
- `lr()` defines the long run variables.
- `xtdcce2` automatically detects the variables and their lags if time series operators are used. Alternatively variables can be enclosed in parenthesis, for example `lr(L.y (x lx l2x))`, with `lx = L.x` and `l2x = L2.x`.

xtdcce2

CS-ARDL Example - ARDL(1,1,1) from Chudik et al. (2013, Table 17).

```
. xtdcce2133 d.y , lr(L.d.y L.dp dp L.d.gd d.gd) lr_options(ardl) cr(d.y dp d.gd) cr_lags(3) fullsample
(Dynamic) Common Correlated Effects Estimator - (CS-ARDL)

Panel Variable (i): ccode                        Number of obs      =       1599
Time Variable (t): year                          Number of groups   =         40

Degrees of freedom per group:                    Obs per group (T)  =         40
 without cross-sectional averages    = 33.975
 with cross-sectional averages       = 21.975
Number of                                        F(720, 879)        =       0.79
 cross-sectional lags                = 3         Prob > F           =       1.00
 variables in mean group regression = 200        R-squared          =       0.39
 variables partialled out            = 520       Adj. R-squared     =      -0.11
                                                 Root MSE           =       0.03
                                                 CD Statistic       =       0.57
                                                    p-value         =     0.5690
------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 Short Run Est.|
-------------+----------------------------------------------------------------
   Mean Group:|
        LD.y |   .0475615   .0393516     1.21   0.227    -.0295662    .1246891
          dp |  -.1036032   .0402887    -2.57   0.010    -.1825676   -.0246389
        D.gd |  -.0745686   .0122305    -6.10   0.000    -.0985399   -.0505974
        L.dp |   -.019946   .0462871    -0.43   0.667    -.1106671     .070775
       LD.gd |  -.0132481   .0156115    -0.85   0.396    -.0438461    .0173498
-------------+----------------------------------------------------------------
 Long Run Est.|
-------------+----------------------------------------------------------------
   Mean Group:|
       lr_dp |  -.1639757   .0378599    -4.33   0.000    -.2381797   -.0897717
       lr_gd |  -.0873993   .0164431    -5.32   0.000    -.1196272   -.0551713
        lr_y |  -.9524385   .0393516   -24.20   0.000    -1.029566   -.8753109
------------------------------------------------------------------------------
Cross Sectional Averaged Variables: D.y dp D.gd
Long Run Variables:  lr_dp lr_gd lr_y
Cointegration variable(s): lr_y
Heterogenous constant partialled out.
```

xtdcce2

CS-ARDL Example - ARDL(3,3,3) from Chudik et al. (2013, Table 17).

```
. xtdcce2133 d.y , lr(L(1/3).(d.y) (L(0/3).dp) (L(0/3).d.gd) ) lr_options(ardl) cr(d.y dp d.gd) cr_lags(3) fullsample
(Dynamic) Common Correlated Effects Estimator -  (CS-ARDL)

Panel Variable (i): ccode                          Number of obs      =       1562
Time Variable (t): year                            Number of groups   =         40

Degrees of freedom per group:                      Obs per group (T) =         39
 without cross-sectional averages    = 27.05
 with cross-sectional averages       = 15.05
Number of                                          F(960, 602)        =       0.96
 cross-sectional lags                = 3           Prob > F           =       0.71
 variables in mean group regression = 440          R-squared          =       0.61
 variables partialled out            = 520         Adj. R-squared     =      -0.03
                                                   Root MSE           =       0.02
                                                   CD Statistic       =      -0.51
                                                      p-value         =     0.6108
-------------------------------------------------------------------------------
         D.y|     Coef.   Std. Err.       z    P>|z|     [95% Conf. Interval]
------------+------------------------------------------------------------------
 Short Run Est.|
------------+------------------------------------------------------------------
  Mean Group:|
        LD.y|  .0123738   .0349377    0.35   0.723    -.0561029   .0808506
       L2D.y| -.1395645   .0948427   -1.47   0.141    -.3254529   .0463238
       L3D.y|  -.082903   .1072901   -0.77   0.440    -.2931877   .1273817
          dp|  -.070708   .0503039   -1.41   0.160    -.1693018   .0278858
        D.gd|  -.085307   .0137595   -6.20   0.000    -.1122752  -.0583388
        L.dp| -.0312712   .0513435   -0.61   0.542    -.1319025   .0693601
       L2.dp|  .0982105   .1017365    0.97   0.334    -.1011893   .2976103
       L3.dp| -.0424631   .0581692   -0.73   0.465    -.1564726   .0715464
       LD.gd| -.0270311   .0204753   -1.32   0.187    -.0671619   .0130997
      L2D.gd| -.0114103    .012726   -0.90   0.370    -.0363528   .0135322
      L3D.gd|  .0283551   .0177666    1.60   0.110    -.0064667   .0631769
------------+------------------------------------------------------------------
 Long Run Est.|
------------+------------------------------------------------------------------
  Mean Group:|
       lr_dp| -.0795245   .0586992   -1.35   0.175    -.1945727   .0355238
       lr_gd| -.1198362   .0402251   -2.98   0.003     -.198676  -.0409965
        lr_y| -1.210094   .2005902   -6.03   0.000    -1.603243  -.8169442
-------------------------------------------------------------------------------
Cross Sectional Averaged Variables: D.y dp D.gd
Long Run Variables:  lr_dp lr_gd lr_y
Cointegration variable(s): lr_y
Heterogenous constant partialled out.
```

xtdcce2

ECM

- `xtdcce2` (also pre 1.33) can estimate a simple ECM, for example an ARDL(1,1) model, such as:

$$\Delta y_{i,t} = \phi_i \left[y_{i,t-1} - \theta_i x_{i,t-1} \right] - \beta_i \Delta x_{i,t} + \sum_{l=0}^{p_T} \gamma_{i,l} \bar{\mathbf{z}}_{i,t} + u_{i,t}$$

- Internally the following estimation is run:

$$\Delta y_{i,t} = \phi_i y_{i,t-1} + \varphi_i x_{i,t-1} + \omega_i \Delta x_{i,t} + \sum_{l=0}^{p_T} \gamma_{i,l} \bar{\mathbf{z}}_{i,t} + u_{i,t}$$

- Then the estimate of the long run coefficient is calculated as $\hat{\theta}_i = -\frac{\hat{\varphi}_i}{\hat{\phi}_i}$.
- The variance-covariance matrix is calculated using the delta method.

xtdcce2

ECM - ARDL(1,1,1)

```
. xtdcce2133 d.d.y d.(dp d.gd), lr(L.(d.y) dp d.gd ) cr(d.y dp d.gd) /*
*/ cr_lags(3) fullsample
(Dynamic) Common Correlated Effects Estimator - Mean Group

Panel Variable (i): ccode                        Number of obs      =      1599
Time Variable (t): year                          Number of groups   =        40

Degrees of freedom per group:                    Obs per group (T) =        40
 without cross-sectional averages    = 33.975
 with cross-sectional averages       = 21.975
Number of                                        F(720, 879)        =      2.84
 cross-sectional lags                = 3         Prob > F           =      0.00
 variables in mean group regression = 200        R-squared          =      0.70
 variables partialled out            = 520       Adj. R-squared     =      0.45
                                                 Root MSE           =      0.03
                                                 CD Statistic       =      0.57
                                                    p-value         =    0.5690
------------------------------------------------------------------------------
        D2.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 Short Run Est. |
-------------+----------------------------------------------------------------
  Mean Group: |
        D.dp |   .0199465   .0462873     0.43   0.667    -.0707749    .1106679
       D2.gd |   .0132482   .0156115     0.85   0.396    -.0173498    .0438463
-------------+----------------------------------------------------------------
 Long Run Est. |
-------------+----------------------------------------------------------------
  Mean Group: |
        LD.y |  -.9524386   .0393514   -24.20   0.000    -1.029566   -.8753112
          dp |  -.1639748   .0378594    -4.33   0.000    -.2381778   -.0897718
        D.gd |  -.0873991   .0164432    -5.32   0.000    -.1196271   -.0551711
------------------------------------------------------------------------------
Mean Group Variables: D.dp D2.gd
Cross Sectional Averaged Variables: D.y dp D.gd
Long Run Variables:  LD.y dp D.gd
Cointegration variable(s): LD.y
Heterogenous constant partialled out.
```

xtdcce2

ECM - ARDL(2,2,2)

```
. xtdcce2133 d.d.y d(1/2).(dp d.gd L.d.y), lr(L.(d.y) dp d.gd ) cr(d.y dp d.gd) /*
*/ cr_lags(3) fullsample
(Dynamic) Common Correlated Effects Estimator - Mean Group

Panel Variable (i): ccode                      Number of obs      =       1576
Time Variable (t): year                        Number of groups   =         40

Degrees of freedom per group:                  Obs per group (T) =         39
 without cross-sectional averages    = 29.4
 with cross-sectional averages       = 17.4
Number of                                      F(880, 696)        =       2.70
 cross-sectional lags                = 3       Prob > F           =       0.00
 variables in mean group regression = 360      R-squared          =       0.77
 variables partialled out            = 520     Adj. R-squared     =       0.49
                                               Root MSE           =       0.03
                                               CD Statistic       =      -0.30
                                                  p-value         =     0.7653
---------------------------------------------------------------------------------
        D2.y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+-------------------------------------------------------------------
 Short Run Est. |
-------------+-------------------------------------------------------------------
   Mean Group: |
        D.dp |   .0865667   .0873178    0.99   0.321     -.0845731   .2577065
       D2.dp |  -.0335994   .0412793   -0.81   0.416     -.1145054   .0473065
       D2.gd |   .0269077   .0265396    1.01   0.311     -.0251089   .0789244
       D3.gd |   .0016363   .0112576    0.15   0.884     -.0204282   .0237007
       LD2.y |   .0711921   .1122562    0.63   0.526     -.1488259   .2912102
       LD3.y |   .0098131   .0485676    0.20   0.840     -.0853776   .1050039
-------------+-------------------------------------------------------------------
 Long Run Est. |
-------------+-------------------------------------------------------------------
   Mean Group: |
        LD.y |  -1.080225   .0905837  -11.93   0.000     -1.257766  -.9026845
          dp |  -1.029103   .8762061   -1.17   0.240     -2.746435   .6882298
        D.gd |  -.4282529   .5060271   -0.85   0.397     -1.420048    .563542
---------------------------------------------------------------------------------
Mean Group Variables: D.dp D2.dp D2.gd D3.gd LD2.y LD3.y
Cross Sectional Averaged Variables: D.y dp D.gd
Long Run Variables:  LD.y dp D.gd
Cointegration variable(s): LD.y
Heterogenous constant partialled out.
```

Conclusion

`xtdcce2`...

- introduced a routine to estimate a panel model with heterogeneous slopes and dependence across cross-sectional untis by using the dynamic common correlated effects estimator.
- supports estimation of long run coefficients using three different models, using the
  - CS-DL estimator - direct estimation of the long run coefficients
  - CS-ARDL estimator - calculation of long run coefficients out of short run coefficients
  - an ECM approach
- is available on SSC (current version 1.33).
- Further developments:
  - two-step ECM.
  - Alternative calculation of standard errors for individual and mean group long run coefficients.

The Delta Method

back

- Allows calculation of an approximate probability distribution for a matrix function $a(\beta)$ based on a random vector with a known variance.
- Assume $\beta_i \rightarrow_p \beta$ and $\sqrt{n}(\beta_i - \beta) \rightarrow_d N(0, \sigma)$ and first derivate of $a(\beta)$:

$$A(\beta) \equiv \frac{\partial a(\beta)}{\partial \beta'}$$

- then the distribution of the function $a()$ is

$$\sqrt{n}\left[a(\beta_i) - a(\beta)\right] \rightarrow_d N\left(0, A(\beta)\Sigma A(\beta)'\right).$$

The Delta Method I

back

- Assume an ARDL(2,1) model with the following long run coefficients:

$$
\begin{aligned}
y_{i,t} &= \alpha_i + \lambda_{1,i} y_{i,t-1} + \lambda_{2,i} y_{i,t-2} + \beta_{0,i} x_{i,t} + \beta_{1,i} x_{i,t-1} + e_{i,t} \\
\phi_i &= -\left(1 - \lambda_{1,i} - \lambda_{2,i}\right) \\
\theta_{1,i} &= \frac{\beta_{0,i} + \beta_{1,i}}{1 - \lambda_{1,i} - \lambda_{2,i}}
\end{aligned}
$$

- Stack the short run coefficients into $\pi_i = (\lambda_{1,i}, \lambda_{2,i}, \beta_{0,i}, \beta_{1,i})$
- The vector function $a(\pi_i)$ maps the short run coefficients into a vector of the short run and long run coefficients:

  $a(\pi_i) = (\lambda_{1,i}, \lambda_{2,i}, \beta_{0,i}, \beta_{1,i}, \phi_i, \theta_{1,i})$, where $\phi_i = -1 + \lambda_{1,i} + \lambda_{2,i}$ and $\theta_{1,i} = \frac{\beta_{0,i}+\beta_{1,i}}{1-\lambda_{1,i}-\lambda_{2,i}}$.

The Delta Method II

back

- The covariance matrix is:

$$\Sigma_i = \begin{pmatrix} Var(\lambda_{1,i}) & Cov(\lambda_{1,i},\lambda_{2,i}) & Cov(\lambda_{1,i},\beta_{0,i}) & Cov(\lambda_{1,i},\beta_{1,i}) \\ & \ddots & & \\ & & \ddots & \\ & & & Var(\beta_{1,i}) \end{pmatrix}$$

- The first derivative of $a(\pi_i)$ is:

The Delta Method III

back

$$A(\pi_i) = \begin{pmatrix} \frac{\partial\lambda_{1,i}}{\partial\lambda_{1,i}} & \frac{\partial\lambda_{1,i}}{\partial\lambda_{2,1}} & \frac{\partial\lambda_{1,i}}{\partial\beta_{0,i}} & \frac{\partial\lambda_{1,i}}{\partial\beta_{1,i}} \\ \frac{\partial\lambda_{2,i}}{\partial\lambda_{1,i}} & \frac{\partial\lambda_{2,i}}{\partial\lambda_{2,i}} & \frac{\partial\lambda_{2,i}}{\partial\beta_{0,i}} & \frac{\partial\lambda_{2,i}}{\partial\beta_{1,i}} \\ \frac{\partial\beta_{0,i}}{\partial\lambda_{1,i}} & \frac{\partial\beta_{0,i}}{\partial\lambda_{2,i}} & \frac{\partial\beta_{0,i}}{\partial\beta_{0,i}} & \frac{\partial\beta_{0,i}}{\partial\beta_{1,i}} \\ \frac{\partial\beta_{1,i}}{\partial\lambda_{1,i}} & \frac{\partial\beta_{1,i}}{\partial\lambda_{2,i}} & \frac{\partial\beta_{1,i}}{\partial\beta_{0,i}} & \frac{\partial\beta_{1,i}}{\partial\beta_{1,i}} \\ \frac{\partial\phi_{i}}{\partial\lambda_{1,i}} & \frac{\partial\phi_{i}}{\partial\lambda_{2,i}} & \frac{\partial\phi_{i}}{\partial\beta_{0,i}} & \frac{\partial\phi_{i}}{\partial\beta_{1,i}} \\ \frac{\partial\theta_{1,i}}{\partial\lambda_{1,i}} & \frac{\partial\theta_{1,i}}{\partial\lambda_{2,i}} & \frac{\partial\theta_{1,i}}{\partial\beta_{0,i}} & \frac{\partial\theta_{1,i}}{\partial\beta_{1,i}} \end{pmatrix}$$

The Delta Method IV

back

- with

$$\frac{\partial \phi_i}{\partial \lambda_{1,i}} = \frac{\partial \phi_i}{\partial \lambda_{2,i}} = 1$$
$$\frac{\partial \theta_{1,i}}{\partial \beta_{0,i}} = \frac{\partial \theta_{1,i}}{\partial \beta_{1,i}} = \frac{1}{1 - \lambda_{1,i} - \lambda_{2,i}}$$
$$\frac{\partial \theta_{1,i}}{\partial \lambda_{1,i}} = \frac{\partial \theta_{1,i}}{\partial \lambda_{2,i}} = \frac{\beta_{0,i} + \beta_{1,i}}{(1 - \lambda_{1,i} - \lambda_{2,i})^2}$$

- Then $A(\pi_i)$ becomes:

The Delta Method V

back

$$A(\pi_i) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ \frac{\beta_{0,i}+\beta_{1,i}}{\left(1-\lambda_{1,i}-\lambda_{2,i}\right)^2} & \frac{\beta_{0,i}+\beta_{1,i}}{\left(1-\lambda_{1,i}-\lambda_{2,i}\right)^2} & \frac{1}{1-\lambda_{1,i}-\lambda_{2,i}} & \frac{1}{1-\lambda_{1,i}-\lambda_{2,i}} \end{pmatrix}$$

- Then the covariance matrix including the long run coefficients is

$$\Sigma_i^{lr} = A(\pi_i)\Sigma_i A(\pi_i)'$$

xtdcce2

pmg-Options

- `lr`(*varlist*) defines the variables in the long run relationship.
- `xtdcce2` estimates internally

$$\Delta y_{i,t} = \phi_i y_{i,t-1} + \gamma_i x_{i,t-1} - \beta_i \Delta x_{i,t} + \sum_{l=0}^{p_T} \gamma_{i,l} \bar{\mathbf{z}}_{i,t} + u_{i,t} \quad (7)$$

while `xtpmg` (with common factors) is based on:

$$\Delta y_{i,t} = \phi_i \left[y_{i,t-1} - \theta_i x_{i,t-1} \right] - \beta_i \Delta x_{i,t} + \sum_{l=0}^{p_T} \gamma_{i,l} \bar{\mathbf{z}}_{i,t} + u_{i,t}$$

- where $\theta_i = -\frac{\gamma_i}{\phi_i}$. θ_i is calculated and the variances calculated using the Delta method.
- `lr_option`(*string*)
  - `nodivide`, coefficients are not divided by the error correction speed of adjustment vector (i.e. estimate (7)).
  - `xtpmgnames`, coefficients names in `e(b_p_mg)` and `e(V_p_mg)` match the name convention from `xtpmg`.

xtdcce2

Test for cross sectional dependence

- `xtdcce2` package includes the `xtcd2` command, which tests for cross sectional dependence (Pesaran, 2015).
- Under the null hypothesis, the error terms are weakly cross sectional dependent.

$$H_0 : E(u_{i,t} u_{j,t}) = 0, \forall\ t \text{ and } i \neq j.$$

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij} \right)$$

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} \hat{u}_{i,t} \hat{u}_{jt}}{\left(\sum_{t=1}^{T} \hat{u}_{it}^2\right)^{1/2} \left(\sum_{t=1}^{T} \hat{u}_{jt}^2\right)^{1/2}}.$$

- Under the null the CD test statistic is asymptotically $CD \sim N(0,1)$.

Saved values ▸ back

Scalars

e(N)	number of observations	e(N_g)	number of groups
e(T)	number of time periods	e(K_mg)	number of regressors
e(N_partial)	number of variables partialled out	e(N_omitted)	number of omitted variables
e(N_pooled)	number of pooled variables	e(mss)	model sum of square
e(rss)	residual sum of squares	e(F)	F statistic
e(ll)	log-likelihood (only IV)	e(rmse)	root mean squared error
e(df_m)	model degrees of freedom	e(df_r)	residual degree of freedom
e(r2)	R-squared	e(r2_a)	R-squared adjusted
e(cd)	CD test statistic	e(cdp)	p-value of CD test statistic
e(cr_lags)	number of lags of cross sectional averages		

Scalars (unbalanced panel)

e(Tmin)	minimum time	e(Tmax)	maximum time
e(Tbar)	average time		

Macros

e(tvar)	name of time variable	e(idvar)	name of unit variable
e(depvar)	name of dependent variable	e(indepvar)	name of independent variables
e(omitted)	name of omitted variables	e(lr)	long run variables
e(pooled)	name of pooled variables	e(cmd)	command line
e(cmdline)	command line including options	e(version)	xtdcce2 version, if xtdcce2, version used
e(insts)	instruments (exogenous) variables	e(instd)	instrumented (endogenous) variables

Matrices

e(b)	coefficient vector (mean group or individual)	e(V)	variance–covariance matrix (mean group or individual)
e(bi)	coefficient vector (individual and pooled)	e(Vi)	variance–covariance matrix (individual and pooled)

Functions

e(sample)	marks estimation sample

Options

back

- pooled(*varlist*) specifies homogeneous coefficients. For these variables the estimated coefficients are constrained to be equal across all units ($\beta_i = \beta \; \forall \; i$). Variable may occur in *indepvars*. Variables in exogenous_vars(), endogenous_vars() and lr() may be pooled as well.
- crosssectional(*varlist*) defines the variables which are included in z_t and added as cross sectional averages ($\bar{z}_{t-l}$) to the equation. Variables in crosssectional() may be included in pooled(), exogenous_vars(), endogenous_vars() and lr(). Variables in crosssectional() are partialled out, the coefficients not estimated and reported. crosssectional(_all) adds adds all variables as cross sectional averages. No cross sectional averages are added if crosssectional(_none) is used, which is equivalent to nocrosssectional. crosssectional() is a required option but can be substituted by nocrosssectional.

Options I

back

- `cr_lags(#)` specifies the number of lags of the cross sectional averages. If not defined but `crosssectional()` contains *varlist*, then only contemporaneous cross sectional averages are added, but no lags. `cr_lags(0)` is equivalent to. The number of lags can be different for different variables, following the order defined in `cr()`.
- `nocrosssectional` prevents adding cross sectional averages. Results will be equivalent to the Pesaran and Smith (1995) Mean Group estimator, or if `lr(`*varlist*`)` specified to the Shin et al. (1999) Pooled Mean Group estimator.
- `xtdcce2` supports instrumental variable regression using `ivreg2`. The IV specific options are:
  - `ivreg2options` passes further options on to `ivreg2`. See `ivreg2, options` for more information.
  - `fulliv` posts all available results from `ivreg2` in `e()` with prefix *ivreg2_*.

Options II

▸ back

- `noisily` shows the output of wrapped `ivreg2` regression command.
- `ivslow` For the calculation of standard errors for pooled coefficients an auxiliary regressions is performed. In case of an IV regression, xtdcce2 runs a simple IV regression for the auxiliary regressions. this is faster. If option is used ivslow, then xtdcce2 calls ivreg2 for the auxiliary regression. This is advisable as soon as ivreg2 specific options are used.

- `xtdcce2` is able to estimate long run coefficients. Three models are supported, an error correction model, the CS-DL and CS-ARDL method. No options for the CS-DL method are necessary.
  - `lr(`*varlist*`)`: Variables to be included in the long-run cointegration vector. The first variable(s) is/are the error-correction speed of adjustment term. The default is to use the pmg model. In this case each estimated coefficient is divided by the negative of the long-run cointegration vector (the first variable). If the option ardl is used, then the long run coefficients are estimated as the sum over the coefficients relating to a variable, divided by the sum of the coefficients of the dependent variable.

Options III

back

- `lr_options(`*string*`)` Options for the long run coefficients. Options may be:
  - ★ `ardl` estimates the CS-ARDL estimator.
  - ★ `nodivide`, coefficients are not divided by the error correction speed of adjustment vector.
  - ★ `xtpmgnames`, coefficients names in `e(b_p_mg)` and `e(V_p_mg)` match the name convention from `xtpmg`.

- `noconstant` suppress constant term.
- `pooledconstant` restricts the constant to be the same across all groups ($\beta_{0,i} = \beta_0, \ \forall i$).
- `reportconstant` reports the constant. If not specified the constant is treated as a part of the cross sectional averages.
- `trend` adds a linear unit specific trend. May not be combined with `pooledtrend`.
- `pooledtrend` a linear common trend is added. May not be combined with `trend`.

Options IV

back

- jackknife applies the jackknife bias correction for small sample time series bias. May not be combined with recursive.
- recursive applies recursive mean adjustment method to correct for small sample time series bias. May not be combined with jackknife.
- nocd suppresses calculation of CD test statistic.
- nomitted suppress checks for collinearity.
- showindividual reports unit individual estimates in output.
- fast omit calculation of unit specific standard errors.
- fullsample uses entire sample available for calculation of cross sectional averages. Any observations which are lost due to lags will be included calculating the cross sectional averages (but are not included in the estimation itself).

xtdcce2

Small Sample Time Series Bias Corrections

"half panel" jackknife

$$\hat{\pi}^J_{MG} = 2\hat{\pi}_{MG} - \frac{1}{2}\left(\hat{\pi}^a_{MG} + \hat{\pi}^b_{MG}\right)$$

- where $\hat{\pi}^a_{MG}$ is the mean group estimate of the first half $(t = 1, ..., \frac{T}{2})$ of the panel and $\hat{\pi}^b_{MG}$ of the second half $(t = \frac{T}{2} + 1, ..., T)$ of the panel.

Recursive mean adjustment

$$\tilde{w}_{i,t} = w_{i,t} - \frac{1}{t-1}\sum_{s=1}^{t-1} w_{i,s} \quad \text{with} \quad w_{i,t} = (y_{i,t}, X_{i,t}).$$

- Partial mean from all variables, except the constant, removed.
- Partial mean is lagged by one period to prevent it from being infuenced by contemporaneous observations.

back

CS-DL details

$$y_{i,t} = \theta_i x_{i,t} + \delta_i(L)\Delta x_{i,t} + \tilde{u}_{i,t}$$

where (see Chudik et al. (2016, p 92))

$$\theta_i = \omega_i(1), \qquad \omega_i(L) = \frac{\beta_i(L)}{\lambda_i(L)} = \sum_{l=0}^{\infty} \omega_{i,l} L^l$$

$$\delta_i(L) = -\sum_{l=0}^{\infty} \sum_{s=l+1}^{\infty} \omega_{i,s} L^l, \qquad \lambda_i(L) = 1 - \sum_{l=1}^{p_y} \lambda_{i,l} L^l$$

$$\beta_i(L) = \sum_{l=0}^{\infty} \beta_{i,l} L^l, \qquad \tilde{u}_{i,t} = \lambda_i(L)^{-1} u_{i,t}$$

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