Cointegrating VAR and Probability Forecasting: An applicacion for small open economies

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Outline

- Why Cointegrating VAR models
- Why Probability Forecasting
- VEC and Cointegrating VAR Models
    - Theoretical comments
    - Case study: Uruguay
- Probability Forecasting
    - Some theoretical comments
    - Case study: Uruguay

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Why Cointegrating VAR

- -vec- command: The default (Johansen normalization)

```
.   webuse lutkepohl
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)

.   vec linvest lconsumption lincome, lags(4) rank(2) noetable

Vector error-correction model

Sample:  1961q1 - 1982q4                          No. of obs      =         88

Identification:  beta is exactly identified

                 Johansen normalization restrictions imposed
```

beta	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_ce1						
linvestment	1	.	.	.	.	.
lconsumption	0	(omitted)				
lincome	-.863381	.0468879	-18.41	0.000	-.9552796	-.7714825
_cons	.034525	.	.	.	.	.
_ce2						
linvestment	8.67e-19	.	.	.	.	.
lconsumption	1	.	.	.	.	.
lincome	-.9670451	.0045147	-214.20	0.000	-.9758938	-.9581964
_cons	-.1447284	.	.	.	.	.

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- -vec- command: Theoretical long-run relationship

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.   constraint define 1 [_ce1]linvest =1
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.   constraint define 3 [_ce2]lconsumption=1
.   constraint define 4 [_ce2]lincome =-.9
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>               bconstraints(1/4) nolog nocnsreport

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linvestment	1	.	.	.	.	.
lconsumption	-.1172448	.0480998	-2.44	0.015	-.2115187	-.022971
lincome	-.75	.	.	.	.	.
_cons	.0514938	.	.	.	.	.
_ce2						
linvestment	-.0776541	.0068539	-11.33	0.000	-.0910876	-.0642206
lconsumption	1	.	.	.	.	.
lincome	-.9	.	.	.	.	.
_cons	-.1474094	.	.	.	.	.

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Why Cointegrating VAR

```
.   vec linvest lconsumption lincome, ...
```

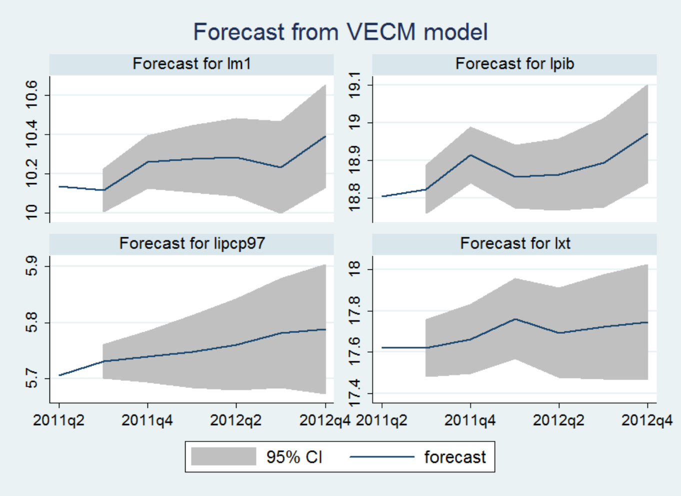
*** Output Omitted ***

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_linvestment						
_ce1						
L1.	-.1040128	.0790621	-1.32	0.188	-.2589717	.0509461
_ce2						
L1.	.6979509	.4424682	1.58	0.115	-.169271	1.565173
linvestment						
LD.	-.2246769	.1290886	-1.74	0.082	-.4776859	.028332
L2D.	-.1279608	.1286637	-0.99	0.320	-.3801371	.1242155
L3D.	.0304474	.126218	0.24	0.809	-.2169353	.2778301
lconsumption						
LD.	.0546931	.6438309	0.08	0.932	-1.207192	1.316578
L2D.	.3857924	.6440095	0.60	0.549	-.8764429	1.648028
L3D.	-.0152393	.5789952	-0.03	0.979	-1.150049	1.11957
lincome						
LD.	1.059818	.6446678	1.64	0.100	-.2037081	2.323343
L2D.	.7647925	.5900074	1.30	0.195	-.3916007	1.921186
L3D.	.7128608	.5460636	1.31	0.192	-.3574042	1.783126
_cons	.0007756	.0211156	0.04	0.971	-.0406103	.0421615

*** Output Omitted ***

Why Probability Forecasting

- Uncertainty expressed in terms of confidence intervals



Why Probability Forecasting

- Uncertainty expressed in terms of simulations for single and composed events

Scenarios for Inflation and change in GDP

Event			Year	Proportion	Std. Err.
Inflation < 8.8			2011	.49	.05024184
GDP change > 7.5			2011	.06	.02386833
Inflation > 4	&	Inflation < 9	2012	.11	.0314466
Inflation < 7.8	&	GDP change > 5.9	2012	.36	.04824182

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VECM and Cointegrating VAR Models

Cointegrating VAR Models - Theoretical comments

- Based on the vector error correction (VEC) model specification.
- The specification assumes that economic theory characterizes the long-run behavior.
- The short-run fluctuations represent deviations from the equilibrium
- The short-run and long-run economic concepts are linked to the statistical concept of stationarity

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Cointegrating VAR Models - Theoretical comments

Reduced form for a VEC model

$$\Delta y_t = \gamma_0 + \gamma_1 * t - \alpha\beta y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \mu_t \qquad (1)$$

Where:

- y_t : $I(1)$ Endogenous variables
- $\alpha\beta$: Matrices containing the long-run adjustment coefficients and cointegrating relationships
- Γ_i : Matrix with coefficients associated to short-run dynamic effects
- $\gamma_0\gamma_1$: Vectors with coefficients associated to the intercepts and trends
- μ_t : Innovations

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- Johansen (1988) FIML estimation identifies α *and* β by imposing r^2 atheoretical restrictions

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Cointegrating VAR Models - Theoretical comments

- Garrat et al. (2006) describe the Cointegrating VAR approach:
  - Use economic theory to impose restrictions to identify α *and* β.
  - Exact identification is not necessarily achieved by the theoretical restrictions.
  - 'Test whether the overidentifying restrictions are valid'.

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Case study: Uruguay

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- Small open economy with soy, livestock, leather, and rice as the main export products.
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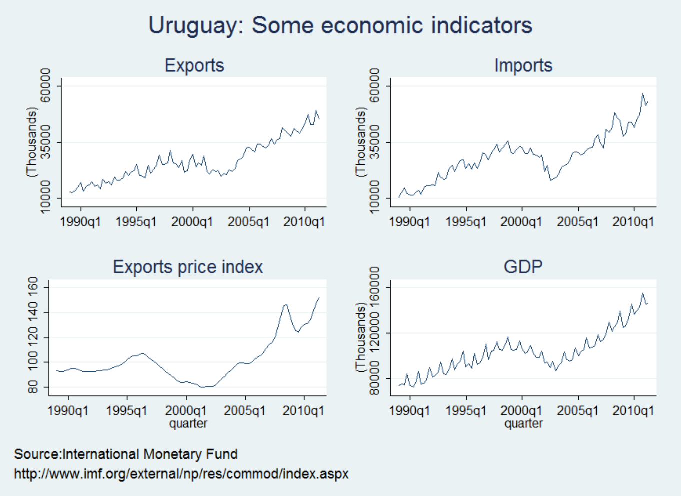
Uruguay: Some economic indicators
Exports
(Thousands)
10000
35000
60000
1990q1
1995q1
2000q1
2005q1
2010q1
Imports
(Thousands)
10000
35000
60000
1990q1
1995q1
2000q1
2005q1
2010q1
Exports price index
80
100
120
140
160
1990q1
1995q1
2000q1
2005q1
2010q1
quarter
GDP
(Thousands)
80000
120000
160000
1990q1
1995q1
2000q1
2005q1
2010q1
quarter
Source:International Monetary Fund
http://www.imf.org/external/np/res/commod/index.aspx

Variables in the cointegrating VAR

- m1 : Currency and demand deposits
- pib : Gross domestic product (GDP)
- tipp906bn : Interest rate.
- tcpn : Exchange rate.
- ipcp97 : Consumer price index $(1997 = 100)$:
- mt : Imports
- xt : Exports
- ipex : Exports price index.

Long-run cointegrating relationships

$$\_ce1: \quad \texttt{lm1} = \beta_{11} \times \texttt{lpib} + \beta_{12} \times \texttt{ltipp906bn} + \beta_{10}$$

$$\_ce2: \quad \texttt{lmt} = \beta_{21} \times \texttt{lpib} + \beta_{22} \times \texttt{ltcpn} + \beta_{20}$$

$$\_ce3: \quad \texttt{lxt} = \beta_{31} \times \texttt{lipcp97} + \beta_{32} \times \texttt{ltcpn} + \beta_{33} \times \texttt{lipex} + \beta_{30}$$

Long-run adjustment restrictions

$$\texttt{[D\_lipex]}\alpha_{\_ce1} = 0$$

$$\texttt{[D\_lipex]}\alpha_{\_ce2} = 0$$

$$\texttt{[D\_lipex]}\alpha_{\_ce3} = 0$$

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** Restrictions on long-run cointegrating equations (bconstraints) **

    * Restriction on long run equation for lm1 *

        constraint 1 [_ce1]lm1=1
        constraint 2 [_ce1]lipcp97=0
        constraint 3 [_ce1]ltcpn=0
        constraint 4 [_ce1]lmt=0
        constraint 5 [_ce1]lxt=0
        constraint 6 [_ce1]lipex=0

    * Restrictions on long run equation for lmt *

        constraint  7 [_ce2]lm1=0
        constraint  8 [_ce2]ltipp906bn=0
        constraint  9 [_ce2]lipcp97=0
        constraint 10 [_ce2]lmt=1
        constraint 11 [_ce2]lxt=0
        constraint 12 [_ce2]lipex=0

    * Restrictions on  long run equation for lxt *

        constraint 13 [_ce3]lm1=0
        constraint 14 [_ce3]lpib=0
        constraint 15 [_ce3]ltipp906bn=0
        constraint 16 [_ce3]lmt=0
        constraint 17 [_ce3]lxt=1
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```

```
* Restrictions on long-run adjustment coefficients (aconstraints) *

      constraint 18 [D_lipex]l._ce1=0
      constraint 19 [D_lipex]l._ce2=0
      constraint 20 [D_lipex]l._ce3=0


*** VEC model specification ***

      vec lm1 lpib ltipp906bn lipcp97 ltcpn lmt lxt lipex, ///
          bconstraints(1/17)                              ///
          aconstraints(18/20)                             ///
          lags(4) rank(3) noetable                        ///
          ltolerance(1e-7) tolerance(1e-4)
```

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```

Cointegrating Equations

```
Sample:  1990q1 - 2011q2                          No. of obs      =         86
                                                  AIC             = -27.58711
Log likelihood =  1414.246                        HQIC            = -24.96839
Det(Sigma_ml)  =  7.19e-25                        SBIC            = -21.08023
Cointegrating equations
Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  2     170.89   0.0000
_ce2                  2   115.2276   0.0000
_ce3                  3   111.1596   0.0000
-------------------------------------------

Identification:  beta is overidentified
```

beta	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
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lm1	1	.	.	.	.	.
lpib	-2.388095	.2667403	-8.95	0.000	-2.910897	-1.865294
ltipp906bn	.2394962	.0401545	5.96	0.000	.1607948	.3181976
lipcp97	0	(omitted)				
ltcpn	0	(omitted)				
lmt	0	(omitted)				
lxt	0	(omitted)				
lipex	0	(omitted)				
_cons	34.28762	.	.	.	.	.

Cointegrating Equations

beta	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_ce2						
lm1	0	(omitted)				
lpib	-1.190284	.1176426	-10.12	0.000	-1.420859	-.9597084
ltipp906bn	0	(omitted)				
lipcp97	0	(omitted)				
ltcpn	.1906142	.0365307	5.22	0.000	.1190154	.262213
lmt	1	.	.	.	.	.
lxt	0	(omitted)				
lipex	0	(omitted)				
_cons	4.087814	.	.	.	.	.
_ce3						
lm1	0	(omitted)				
lpib	0	(omitted)				
ltipp906bn	0	(omitted)				
lipcp97	268.7157	28.66662	9.37	0.000	212.5301	324.9012
ltcpn	-130.17	27.2982	-4.77	0.000	-183.6735	-76.66648
lmt	0	(omitted)				
lxt	1	.	.	.	.	.
lipex	-337.4693	45.183	-7.47	0.000	-426.0264	-248.9123
_cons	470.4702	.	.	.	.	.

Long-run cointegrating relationships

$$\texttt{lm1} = 2.39 \times \texttt{lpib} - 0.24 \times \texttt{ltipp906bn} - 34.29$$

$$\texttt{lmt} = 1.19 \times \texttt{lpib} - 0.19 \times \texttt{ltcpn} - 4.09$$

$$\texttt{lxt} = -268.7 \times \texttt{lipcp97} + 130.2 \times \texttt{ltcpn} + 337.5 \times \texttt{lipex} - 470.5$$

Probability Forecasting

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- Obtain the probability that a single or joint event occurs, conditional on the information contained in the estimation sample.
- We could define the event in terms of the levels of one or more variables, for one or more future time periods.
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- This methodology can be applied to a wide spectrum of models. Our focus here is on the predictions from a cointegrating VAR model.
- In general, forecasting based on econometric models are subject to:
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- Future uncertainty with parametric approach
    - Let's consider a standard linear regression model:

$$y_t = x_{t-1} * \beta + \mu_t \qquad ; \quad \mu \sim N(0, \sigma^2)$$

    - For σ^2 known we could simulate $y_{T+1}^{(s)}$:

$$y_{T+1}^{(s)} = x_t * \hat{\beta} + \mu_{T+1}^{(s)} \quad ; \quad s = 1, 2, ..., S$$

Where:

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- Computations for VAR cointegrating models
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$$\Delta y_t = \gamma_0 + \gamma_1 * t - \alpha\beta y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \mu_t$$

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- Simulated errors are drawn from in sample residuals.
- The Choleski decomposition for the estimated Var-Cov matrix of the error term is used in a two-stage procedure combined with the simulated errors.

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Sample code for Probability forecasting after VEC

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** VEC model **
    vec lm1 lpib ltipp906bn lipcp97 ltcpn lmt lxt lipex,      ///
       bconstraints(1/17) aconstraint(18/20) lags(4) rank(3) ...

** Residuals for VEC ecuations **
    foreach x of varlist lm1 lmt lxt lpib ... lipex {
        predict double res_`x´ if e(sample),resid equat(D_`x´)
    }

** Transform residuals for simulation - Garrat et al. **
    matrix sigma=e(omega)
    matrix P=cholesky(sigma)
    mkmat res_lm1 res_lmt res_lxt res_lpib ... res_lipex    ///
          if tin(1990q1,2011q2),matrix(res)
    matrix invP_res=inv(P)*res´
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  - Generate dynamic predictions (from Cointegrating VAR) for the forecasting period.
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- Scenarios for Export Prices

Interannual Change in Exports Prices

	Scenario	
Quarter	Inertial	Moderate Impact
2011Q3	17.26	16.40
2011Q4	13.63	9.90
2012Q1	10.28	2.70
2012Q2	7.44	-2.00
2012Q3	6.04	-3.60
2012Q4	7.41	-2.40

Case study: Uruguay

- Probability forecasting for GDP

Scenarios for Change in GDP

Event	Year	Proportion	Std. Err.
Change in GDP < 6	2011	.72	.04512609
Change in GDP > 7.5	2011	.06	.02386833
Change in GDP > 5.5 and < 7.5	2011	.33	.04725816
Change in GDP < 4.2	2012	.41	.04943111
Change in GDP > 5.9	2012	.33	.04725816
Change in GDP > 4.2 and < 5.9	2012	.26	.0440844

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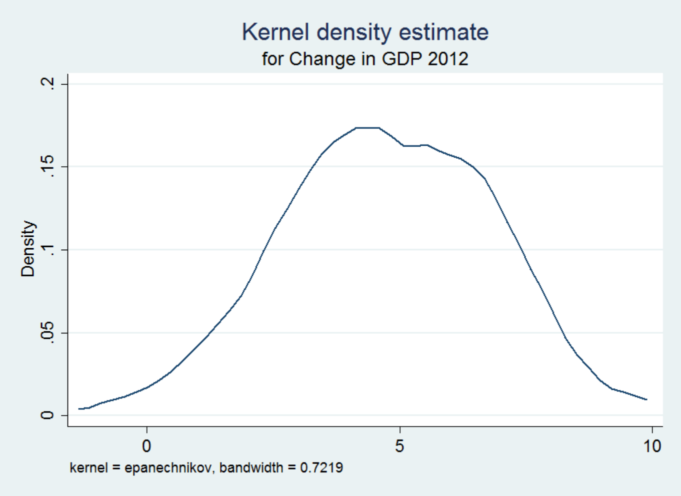

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- Johansen, S. (1988) 'Statistical analysis of cointegration vectors' Journal of Economic Dynamics and Control, 12, 231-254.
- International Monetary Fund: http://www.imf.org/external/np/res/commod/index.aspx

Questions - Comments