Marginal Unit Interpretation of Unconditional Quantile Regression and Recentered Influence Functions using Centered Regression

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The Issue

- Firpo, Fortin and Lemieux's (2009) Econometrica paper "Unconditional Quantile Regressions" (UQR): we are able to interpret UQR coefficients more intuitively – unit's outcome variable quantile is **not** conditional on explanatory variables, just outcome itself.
- FFL use Recentered Influence Functions (RIF) and **standard linear regression**. This opens up a whole new class of estimators
- We can now easily look at specific quantiles in **unconditional distribution** of outcome variables
- We can also look at distribution summary statistics based **on those unconditional distributions** of outcome variables
- UQR identifies how **very small location shifts** in the distribution of explanatory variables affect the statistic of interest.
- **But ...**

The Issue

- **But...** UQR cannot be used to identify distributional treatment effects. Why?
  1. Standard RIF-regressions are only linear approximations of nonlinear functionals;
  2. The implicit inter-relationship within binary variables sets are usually ignored;
  3. Marginal effects are estimated assuming a 1 unit change in the independent variables (going in a discrete jump from 0 to 1 in the case of dummies), which is **too large for a small location shift**
- For dummies: small locational shifts should be interpreted as a change in the **percent-point proportion** of individuals in a particular group along with a decline in similar magnitude of other groups, not jumping from one extreme to another (e.g., not **all** male vs **all** female, but from 48% to 49% male)
- We have an answer for this...

The Issue

- The Issue: How do we achieve these **small locational shifts** as required in RIF regressions?
- We use a mix of Restricted Least Squares, implemented through linear combinations of the estimated coefficients and continuous variable centering to arrive at the desired interpretability
- This combines RLS from Haisken-DeNew and Schmidt (1997) and Rios-Avila (2020) into a new Stata post-estimation command “creg”, or “centered (linear) regression”.
- You first run the linear regression and then in a second step, the coefficients are transformed to provide appropriate RIF interpretation

Contribution

- Through centering, we can cleanly **identify the constant** as the **unconditional mean**, also the **functional** (quantile, statistic of interest, etc)
- All dummy coefficients are in the appropriate interpretation and magnitude for RIF regression as **infinitesimally small changes**: percent point changes
- This corresponds exactly to $RIF = v(F) + IF$, where $v(F) = E(Y)^{Unconditional}$
- Can also obtain magnitude of Unconditional Treatment Effects, relative to the magnitude of the outcome variable: divide through by constant using non-linear combinations of coefficients (but maintain correct standard errors)
- We provide a post-estimation tool “creg.ado” for Stata that achieves this Run after **reg**, **xtreg**, **rifhdreg**, etc
- This is straightforward to deal with standard dummy variables. But, how to deal with complex interactions with the dummy variables? Marginals!

Contribution

- We cannot use Stata's margins command (based on *Delta Method*), as we can get **either** marginals, **or** the unconditional constant, but **not both** at the same time. We need everything all at once. What to do?
- Using linear combinations of coefficients, we allow for complicated forms of Stata's **##** interactions:
  1. dummy interactions: *i.dummy***##***i.dummy* ...**##***i.dummy*
  2. dummy/continuous interactions: *i.dummy***##***c.continuous*
  3. continuous polynomials: *c.continuous***##***c.continuous* ...**##***c.continuous*
- First we calculate marginal effects by hand using linear combinations, then to run RIF simulation for dummy and continuous variables; divide by _cons
- *This is the first method that we are aware of, of estimating and presenting the unconditional partial effects (UPEs) appropriate to RIF-based estimators. Also, these UPEs can be displayed relative to E(Y) allowing comparison.*

Unconditional Quantile Regression

Fernando Rios-Avila (2019) on Unconditional Quantile Regression:

- When comparing distributional statistics, one requires a minimum of one of the following items:
  1. Unit record data: $Y = y_1, y_2, y_3, ..., y_N$
  2. The CDF cumulative distribution function $F(Y)$ or F_Y
  3. The PDF probability density function $f(Y)$ or f_Y
- Once any one of these three pieces is obtained, any distributional statistic $v()$ can be easily estimated
- *Differences* across two groups can be obtained straightforwardly
- $\Delta v = v(G_y) - v(F_y)$, where Δv is the change in v when $F_y \rightarrow G_y$

Influence Functions and RIFs

- Influence Functions (IF) can be thought as a generalisation of the above experiment
- The IF represents the re-scaled effect that a change in the distribution from $F_y \rightarrow G_y$ has on statistic v, when the change is *infinitesimally* small:
- $G_Y^{y_i} = (1 - \varepsilon)F_Y + \varepsilon 1_{y_i}$
- $IF(y_i, v(F_Y)) = \lim_{\varepsilon \rightarrow 0} \dfrac{v(G_Y^{y_i}) - v(F_Y)}{\varepsilon}$
- Firpo, Fortin, Lemieux (2009) make the point that the **Recentered Influence Function** (RIF) is: $\mathbf{RIF}(y_i, v(F_Y)) = v(F_Y) + IF(y_i, v(F_Y))$

Influence Functions and RIFs

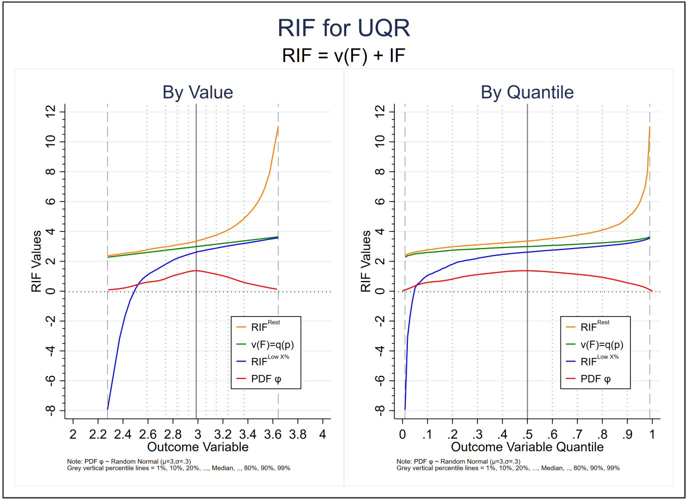
- The RIF is the contribution of y_i to the statistic $v()$
- **Heckley, Gerdtham, Kjellsson (2015)**: "The IF captures the (limiting) influence of an individual observation on the **functional** $v(F)$. Calculating an IF yields an influence function value for **each** individual in the sample"
- We can now define the RIF. The RIF is obtained from the IF by adding back the original functional $v(F)$
- "An important property of an IF is that its expectation is **zero** (e.g. an observation equal to the mean has no influence on the mean). The minor transformation of the IF into a RIF **re-centers** the IF so that its expectation is **equal** to the original distributional statistic $v(F)$ (Firpo et al., 2009)... **This characteristic implies that the mean value of the RIF is equal to the statistic.**"

How this relates to Unconditional Quantile and Centering

- The RIF for Unconditional Quantile Regression
- $RIF(y_i, q_Y(p)) = q_Y(p) + \dfrac{p - 1(y \leq q_Y(p))}{f_Y(q_Y(p))}$
- Another way of putting this for the two cases $(y \leq ? > q_Y(p))$

- $RIF^{High}(y_i, q_Y(p)) = q_Y(p) + \dfrac{p}{f_Y(q_Y(p))}$, if $(y > q_Y(p))$
- $RIF^{Low}(y_i, q_Y(p)) = q_Y(p) - \dfrac{(1-p)}{f_Y(q_Y(p))}$, if $(y \leq q_Y(p))$

Unconditional Quantile RIF: 2 Unique Values



Characteristics of the RIF/IF

- The RIF contains the functional $v(F_Y)$ and the *IF*
  $\textbf{RIF}(y_i, v(F_Y)) = v(F_Y) + IF(y_i, v(F_Y))$

- The expectation of the RIF is simply the functional
  $E(\textbf{RIF}(y_i, v(F_Y))) = v(F_Y)$

- **The expectation of the *IF* itself is zero.** If observations influence the PDF/CDF in one area positively (add mass), there must be another area that experiences a decrease (drops mass). The sum of the positives must equal the sum of the negatives.
  $E(IF(y_i, v(F_Y))) = 0 \leftarrow$ (this is important!)

- Standard variance:
  $var(v(F_Y)) = E(IF(y_i, v(F_Y))^2)$

Problems with Interpreting the Coefficients

- As FFL (2009) state, we want to use Unconditional Quantile Regression to estimate **unconditional partial effects** (UPE) of **small** changes in the distribution of the independent variables X on the distributional statistic v, be it a **quantile** or **distributional statistic**
- Continuous variables: an additional increment of one unit, **all good**
- **Rios-Avila (2020)**: Categorical variables are "more challenging": **Problem!**
- "RIF and standard RIF regressions are meant to estimate the impact of **small changes** in the distribution of the independent variables, one **should not interpret** the coefficients of categorical variables as changes from 0 to 1
- (Standard dummy interpretation) implies a large change in the distribution of the categorical variable, from 0% of observations being classified in that group to 100% being classified in that group
- May introduce a large bias on the predicted *UPE*."

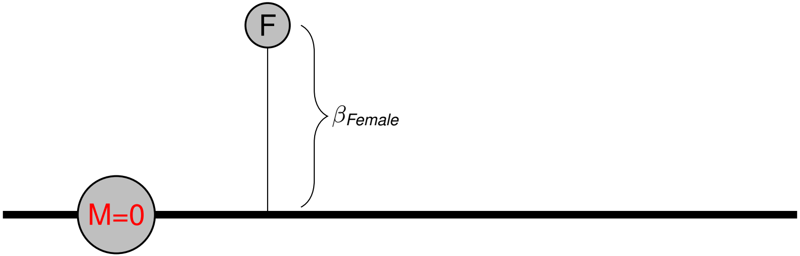
Interpreting the Coefficients

- Thus if you run the regression (see Rios-Avila (2020, p66)):
- $RIF_{UQR}(Y;p) = \alpha + \beta male + \varepsilon$
- Then you are capturing in β the 100% "effect" of going from full female to full male, at the p^{th} quantile
- But: the *RIF* is **only** appropriate to capture changes on the **margin**, **not** 100% mega changes!
- Thus it makes more sense to talk about an **incremental** change in the **share** of males
- E.g., the **share** of males moves from $\overline{male}_{actual} = 0.49$ to $\overline{male}_{CF} = 0.50$, or say 1 *% point* increase in the share of males
- Then: the *UPE* or %Δ is $\dfrac{\Delta X}{\Delta Y} = \dfrac{\hat{\beta} \cdot (0.50 - 0.49)}{\overline{RIF}_{UQR}(Y)}$, but there is more...

Simple Dummy Sets for (RHS) Explanatory Variables

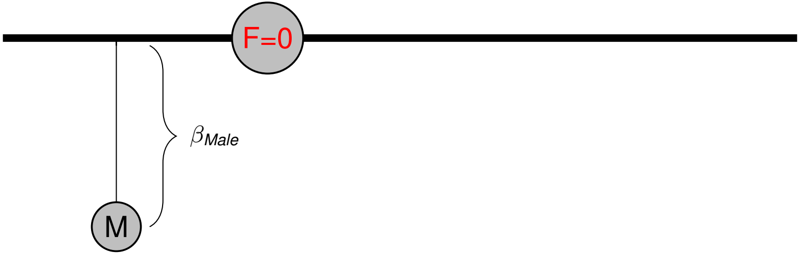
- $Y_i = \alpha + \beta_{Female} Female_i + \gamma X_i + \epsilon_i$
- There are **only two** categories, e.g.:
  (1) Yes and No, or
  (2) Male and Female
- If a dummy set is simple, then I can chose one category to be the "reference" and then that "effect" becomes zero (e.g. male)
- The "other" category becomes the coefficient to estimate (e.g. female)
- The other coefficient is the effect, relative to the reference (e.g. female in comparison to male)

Simple Dummy Sets: [Males] and Females



- $Y_i = \alpha + \beta_{Female} Female_i + \gamma X_i + \epsilon_i$
- We have defined Males to be the "reference"
- Effect of Males is zero (0) by definition: $\beta_{Male} = 0$
- Effect of Females is given by the bar (positive): $\hat{\beta}_{Female}$
- $\hat{\beta}_{Female} - \hat{\beta}_{Male} = \hat{\beta}_{Female} - 0 = \hat{\beta}_{Female}$

Simple Dummy Sets: Males and [Females]

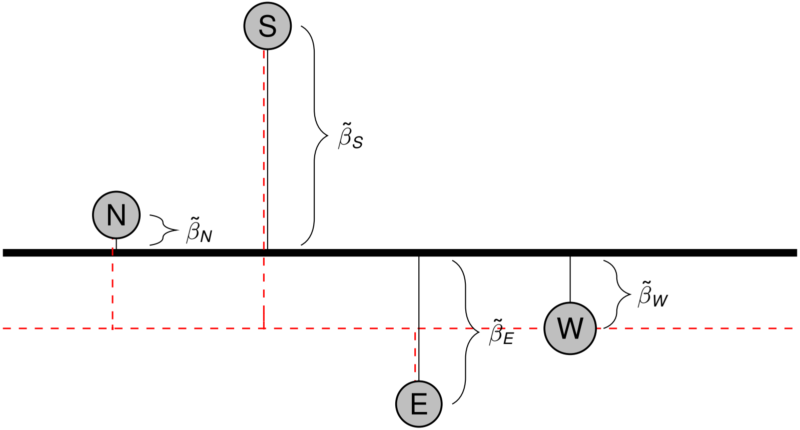


- $Y_i = \alpha + \beta_{Male} Male_i + \gamma X_i + \epsilon_i$
- We have defined Females to be the "reference"
- Effect of Females is zero (0) by definition: $\beta_{Female} = 0$
- Effect of Males is given by the bar (negative): $\hat{\beta}_{Male}$
- $\hat{\beta}_{Male} - \hat{\beta}_{Female} = \hat{\beta}_{Male} - 0 = \hat{\beta}_{Male}$

Remove Arbitrary Choice of Reference

- What if you have 300 dummies in a set? What is a natural reference?
- We have seen very different coefficient estimates depending on the choice of reference category; coefficients can change signs and significance
- You must always leave out one category to be the reference
- **So let us select a reference category such that there are all categories included**
- Let us create a **weighted average** of all the effects and calculate deviations from that weighted average; we can talk about **all** coefficients
- The zero line or weighted average will depend on the **sample weights** of each dummy variable
- K Weights: $0 < w_k < 1; \sum_{k=1}^{K} w_k = 1$
- $\sum_{k=1}^{K} \beta_k \cdot w_k = 0$

Deviations from a Weighted Average



General Solution for Centering all Explanatory Vars

- $T_{[k+1,k+1]} =$

$$\begin{bmatrix} 1_{Continuous} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\bar{\mu}_N & -\bar{\mu}_S & -\bar{\mu}_E & -\bar{\mu}_W & 0 & 0 & 0 \\ 0 & -\bar{\mu}_N & 1-\bar{\mu}_S & -\bar{\mu}_E & -\bar{\mu}_W & 0 & 0 & 0 \\ 0 & -\bar{\mu}_N & -\bar{\mu}_S & 1-\bar{\mu}_E & -\bar{\mu}_W & 0 & 0 & 0 \\ 0 & -\bar{\mu}_N & -\bar{\mu}_S & -\bar{\mu}_E & 1-\bar{\mu}_W & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\bar{\mu}_M & -\bar{\mu}_F & 0 \\ 0 & 0 & 0 & 0 & 0 & -\bar{\mu}_M & 1-\bar{\mu}_F & 0 \\ \bar{\mu}_{Continuous} & \bar{\mu}_N & \bar{\mu}_S & \bar{\mu}_E & \bar{\mu}_W & \bar{\mu}_M & \bar{\mu}_F & 1_{Constant} \end{bmatrix}$$

- For continuous and dummy variables after centering, the constant must be adjusted by $+\bar{\mu}_X\beta_X$ for **each** variable X. All explanatory variables are functionally "demeaned". Constant is now the unconditional mean of Y.
- $\hat{\hat{\beta}} = \hat{\beta} \cdot T$, where $\hat{\beta}_{[1,k+1]} = [\ \hat{\beta}_{Continuous}, \hat{\beta}_N, \hat{\beta}_S, \hat{\beta}_E, \hat{\beta}_W, \hat{\beta}_M, \hat{\beta}_F, \hat{\beta}_{Constant}\ ]$
- $V(\hat{\hat{\beta}}) = T' \cdot V(\hat{\beta}) \cdot T$

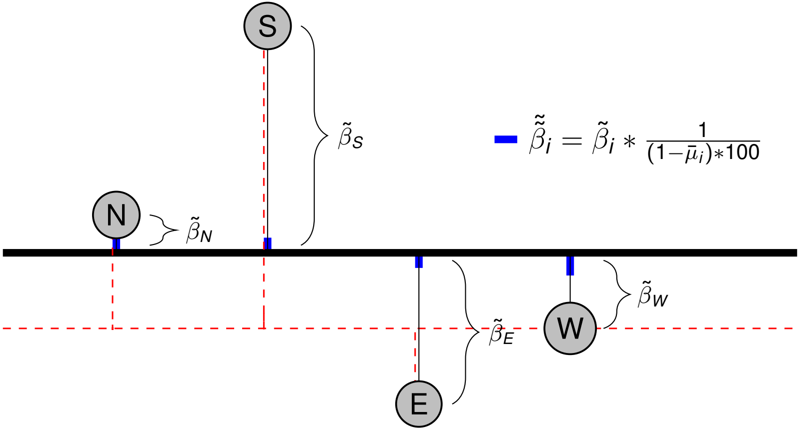
Overall Measure of Dispersion

- How much variation is there in a dummy set?
- Is the set of dummies jointly significant?
- $\hat{\sigma}_{\beta}^{weighted} = ?$
- $B2 = \bar{\mu}_i \cdot diag(\tilde{\beta}) \cdot \tilde{\beta}'$, i.e. weighted sum over i of $\tilde{\beta}_i^2$
- $V2 = \bar{\mu}_i \cdot vecdiag(V(\tilde{\beta}))'$, i.e. weighted sum over i of variance
- $\hat{\sigma}_{\beta}^{weighted} = sqrt(trace(B2 - V2))$
- The standard deviation ($\hat{\sigma}_{\beta}^{weighted}$) of the β coefficients is **reduced** by the imprecision in se(β)'s
- F-test for joint significance, H0: $\tilde{\beta}_k = 0$ for all k, HA: $\tilde{\beta}_k \neq 0$
- F-test with degrees of freedom $(k - 1, N - k - 1)$

Infinitesimally Small Changes

- If $\bar{\mu}_{d1}$ is the mean/share of dummy $d1$, using Haisken-DeNew/Schmidt (1997), then the coefficients are interpreted as going from "average" $\bar{\mu}_{d1}$ to 100% of the dummy in question ($d1$)
- Thus there is only $(1 - \bar{\mu}_{d1})$ "left" until 100%
- To add **only** 1 percentage-point to the mean of $d1$ means:
  $Factor = \frac{1}{(1-\bar{\mu}_{d1})*100}$
  $\tilde{\tilde{\beta}}_{d1} = \tilde{\beta}_{d1} * Factor$
- This gives the unconditional partial effect of 1 additional percentage point for a dummy variable coefficient
- Note, for any continuous variable, *Factor* is simply 1
- What does that look like?

Deviations from a Weighted Average



$$\tilde{\tilde{\beta}}_i = \tilde{\beta}_i * \frac{1}{(1-\bar{\mu}_i)*100}$$

Comparisons over time, place, outcome

- The exposed constant is the **unconditional mean** of the outcome variable
- All unconditional partial effects (UPEs) can be expressed in relation to the magnitude of the unconditional mean of the outcome variable by simply dividing through each coefficient by the (estimated coefficient of the) constant
- This is a **non-linear** transformation as we are **dividing** by the **estimated** coefficient of the constant (with its own standard error), not just a scalar.
- We use Stata's **nlcom** based on the Delta Method to divide each coefficient by the constant (except the constant itself)

$$\tilde{\tilde{\tilde{\beta}}}_k = \frac{\tilde{\tilde{\beta}}_k}{\tilde{\tilde{\beta}}_{\_cons}}$$

- We can now compare the **relative magnitudes** of all unconditional partial effects **within** the regression, to other time periods or other datasets with same specification, to other outcome variables

Conclusions

- Through **centering**, we can cleanly **identify the constant** as the **unconditional mean** or **functional** (percentile, statistic of interest, etc)
- All dummy coefficients are in the **appropriate interpretation and magnitude** for RIF regression as **infinitesimally small changes**: percent point changes
- This corresponds exactly to $RIF = v(F) + IF$, where $v(F) = E(Y)^{Unconditional}$
- Can also obtain magnitude of Unconditional Treatment Effects, **relative** to the magnitude of the outcome variable: divide through by constant using non-linear combinations of coefficients (but maintain correct standard errors)
- We provide a post-estimation tool “creg.ado” for Stata that achieves this Run after **reg**, **xtreg**, **rifhdreg**, etc
- This is straightforward to deal with standard dummy variables. But, how to deal with complex interactions with the dummy variables? Marginals!

Conclusions

- We cannot use Stata's margins command (based on *Delta Method*), as we can get **either** marginals, **or** the unconditional constant, but **not both** at the same time. We need everything all at once. What to do?
- Using linear combinations of coefficients, we allow for complicated forms of Stata's **##** interactions:
  1. dummy interactions: *i.dummy***##***i.dummy* ...**##***i.dummy*
  2. dummy/continuous interactions: *i.dummy***##***c.continuous*
  3. continuous polynomials: *c.continuous***##***c.continuous* ...**##***c.continuous*
- First we calculate marginal effects by hand using linear combinations, then to run RIF simulation for dummy and continuous variables; divide by _cons
- This is the first method that we are aware of, of estimating and presenting the unconditional partial effects (UPEs) appropriate to RIF-based estimators. Also, these UPEs can be displayed relative to E(Y) allowing comparison.

Implementation in Stata

- We have several options open to estimate these RIF models. We can use `rifhdreg` or `egen RIFwage=rifvar(wage), ...` and then `regress RIFwage`.
- `rifhdreg wages i.educ##i.occ##i.sex c.exper##c.exper ten, rif(q(25) kernel(gaussian))`
- **`creg`**`, eval radn divbycons pp(1) eststub(wages1)`
- `egen RIFwages = rifvar(wages), q(75) kernel(gaussian)`
- `regress RIFwages i.educ exper c.tenure##i.ftpt [aw=wgt]`
- **`creg`**`, eval radn divbycons pp(1) eststub(wages2)`

Implementation in Stata: Unconditional Mean

```
. sysuse nlsw88, clear
. numlabel, add mask("[#] ")
. regress wage

      Source |       SS           df       MS      Number of obs   =     2,246
-------------+----------------------------------   F(0, 2245)      =      0.00
       Model |           0         0           .   Prob > F        =         .
    Residual |  74367.9674     2,245  33.1260434   R-squared       =    0.0000
-------------+----------------------------------   Adj R-squared   =    0.0000
       Total |  74367.9674     2,245  33.1260434   Root MSE        =    5.7555

------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   7.766949   .1214451    63.95   0.000     7.528793    8.005105
------------------------------------------------------------------------------
```

Implementation in Stata: Regression Dummy Base 1

```
. regress wage b1.race

      Source |       SS           df       MS      Number of obs   =     2,246
-------------+----------------------------------   F(2, 2243)      =     10.28
       Model |  675.510282         2  337.755141   Prob > F        =    0.0000
    Residual |  73692.4571     2,243  32.8544169   R-squared       =    0.0091
-------------+----------------------------------   Adj R-squared   =    0.0082
       Total |  74367.9674     2,245  33.1260434   Root MSE        =    5.7319
------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        race |
   [2] Black |  -1.238442   .2764488    -4.48   0.000    -1.780564   -.6963193
   [3] Other |   .4677818   1.133005     0.41   0.680    -1.754067    2.689631
             |
       _cons |   8.082999   .1416683    57.06   0.000     7.805185    8.360814
------------------------------------------------------------------------------
```

Implementation in Stata: Regression Dummy Base 2

```
. regress wage b2.race

      Source |       SS           df       MS      Number of obs   =     2,246
-------------+----------------------------------   F(2, 2243)      =     10.28
       Model |  675.510282         2  337.755141   Prob > F        =    0.0000
    Residual |  73692.4571     2,243  32.8544169   R-squared       =    0.0091
-------------+----------------------------------   Adj R-squared   =    0.0082
       Total |  74367.9674     2,245  33.1260434   Root MSE        =    5.7319

------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        race |
   [1] White |   1.238442   .2764488     4.48   0.000     .6963193    1.780564
   [3] Other |   1.706223   1.148906     1.49   0.138    -.5468071    3.959254
             |
       _cons |   6.844558   .2373901    28.83   0.000     6.379031    7.310085
------------------------------------------------------------------------------
```

Implementation in Stata: Deviations from Weighted Average

```
.  creg, eval

Restricted Least Squares for Dummy Variable Sets (Stata Factor Variables)

 Authors      : Prof Dr John P. de New and Prof Dr Christoph M. Schmidt
                Version: 22 Dec 2021

 Citation     : Haisken-DeNew, J.P. and Schmidt C.M. (1997):
                "Interindustry and Interregion Wage Differentials:
                Mechanics and Interpretation," Review of Economics
                and Statistics, 79(3), 516-521. REStat Reprint

------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        race |
   [1] White |   .3160504   .0737694     4.28   0.000     .1713869    .4607138
   [2] Black |  -.9223912   .2042697    -4.52   0.000    -1.322969   -.5218139
   [3] Other |   .7838322   1.117588     0.70   0.483    -1.407783    2.975448
             |
       _cons |   7.766949   .1209461    64.22   0.000     7.529771    8.004127
------------------------------------------------------------------------------

 Sampling-Error-Corrected Standard Deviation of Differentials
 Joint test of all coefficients in dummy variable set = 0, Prob > F = p
------------------------------------------------------------------------------
                race |  0.521062     F(2,2243)  = 10.28        p=0.0000
------------------------------------------------------------------------------
```

Implementation in Stata: 1-% point Dummy Share Increase

```
. creg, eval radn
 Idea          : Interpreting RIFreg regressions: RIF = v(F) + IF

 Authors       : Dr Fernando Rios-Avila and Prof Dr John P. de New
                 Version: 22 Dec 2021

 Citation      : Fernando Rios-Avila, and de New, J.P. (2021):
                 "Interpreting RIFreg regressions", mimeo.

               : Haisken-DeNew, J.P. and Schmidt C.M. (1997):
                 "Interindustry and Interregion Wage Differentials:
                 Mechanics and Interpretation," Review of Economics
                 and Statistics, 79(3), 516-521. REStat Reprint

 Interpret     : Dummy:      a  percentage-point increase in dummy share
                 Continuous: a 1 unit increase in centered variable
                 Constant:   functional statistic, unconditional mean of LHS

------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        race |
  [1] White |    .011656   .0027206     4.28   0.000     .0063208    .0169912
  [2] Black |   -.0124576   .0027588    -4.52   0.000    -.0178676   -.0070475
  [3] Other |    .0079301   .0113068     0.70   0.483    -.0142427     .030103
             |
       _cons |    7.766949   .1209461    64.22   0.000     7.529771    8.004127
------------------------------------------------------------------------------
```

Implementation in Stata: Now Relative to Outcome Variable

```
.  creg, eval radn divbycons
 Idea          : Interpreting RIFreg regressions: RIF = v(F) + IF

 Authors       : Dr Fernando Rios-Avila and Prof Dr John P. de New
                 Version: 22 Dec 2021

 Citation      : Fernando Rios-Avila, and de New, J.P. (2021):
                 "Interpreting RIFreg regressions", mimeo.

               : Haisken-DeNew, J.P. and Schmidt C.M. (1997):
                 "Interindustry and Interregion Wage Differentials:
                 Mechanics and Interpretation," Review of Economics
                 and Statistics, 79(3), 516-521. REStat Reprint

 Interpret     : Dummy:       a 1 percentage-point increase in dummy share
                 Continuous: a 1 unit increase in centered variable
                 Constant:   functional statistic, unconditional mean of LHS
                 RHS coeffs: all are divided by _b[_cons] via nlcom.

------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        race |
  [1] White  |   .0000299   6.94e-06     4.31   0.000     .0000163    .0000435
  [2] Black  |   -.000031   6.96e-06    -4.45   0.000    -.0000446   -.0000173
  [3] Other  |   3.36e-06   .0000152     0.22   0.825    -.0000264    .0000331
             |
       _cons |   7.763807   .1209483    64.19   0.000     7.526625    8.000989
------------------------------------------------------------------------------
```

Implementation in Stata

Thank you!

Follow us on twitter: @FRiosAvila and @DeNewJohn

For Further Reading I

- John P. Haisken-DeNew and Christoph M. Schmidt (1997)
  "Inter-Industry and Inter-Region Differentials: Mechanics and Interpretation"
  Review of Economics and Statistics, 79(3), 516-521.
- Fernando Rios-Avila (2019)
  Recentered Influence Functions (RIF) in Stata RIF-regression and RIF-decomposition (Presentation Slides)
  Stata Conference-Chicago
- Fernando Rios-Avila (2020)
  Recentered influence functions (RIFs) in Stata: RIF regression and RIF decomposition
  The Stata Journal, 2(1), 51-94

For Further Reading II

- Sergio Firpo, Nicole Fortin, Thomas Lemieux (2009)
  ***Unconditional** Quantile Regressions*
  Econometrica, 77(3), 953-973.
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For Further Reading III

James Foster, Joel Greer, Erik Thorbecke (2010)
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