# Intervention time-series model using transfer functions


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# Introduction

- I will present a Stata command **tstf** to estimate the intervention time series with transfer functions.
- The method has been described by Box and Tiao (1975, JASA).
- Estimation, inference, and graphs will be given for both the original data and the log-transformed data.
- The method will be illustrated using the Swedish National Tobacco Quitline (SRL)

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- Time series analysis is a quasi-experimental design useful to evaluate the longitudinal effects of interventions on a population level.
- Intervention time series analysis is widely used in areas like finance, economics, labor markets, transportation, public health and so on .

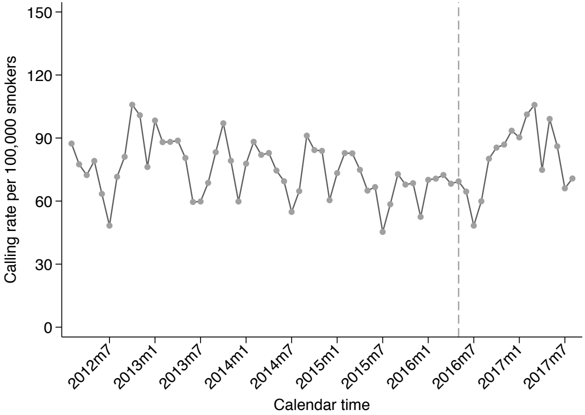
# SRL


The Swedish National Tobacco Quitline (SRL) established in 1998 is a nationwide, free service, providing telephone counseling for tobacco users who want to quit the habit.

# SRL continue


According to Swedish proposition 2015/16:82, started from May 2016, new cigarette packages sold in Sweden will have to display pictorial warnings together with text warnings and the SRL telephone number.

# SRL, Calling per 100,000 smokers



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- To which extent this measure has been effective in inducing a behavioral change among Swedish tobacco users is not known.
- A change in the inflow of calls received at the quitline may be used as an estimator of the population impact of policy measures.

# What already have in Stata?

Stata package **itsa** analyses interrupted time series using segmented regression.

$$Y_t = \beta_0 + \beta_1 T + \beta_2 X_t + \beta_3 XT_t \tag{1}$$

$\beta_0$ represents the baseline level at $T = 0$,
$\beta_1$ is interpreted as the change in outcome associated with a time unit increase
$\beta_2$ is the level change following the intervention
$\beta_3$ indicates the slope change following the intervention.

shortcomings: hard to reduce the auto-correlation among the residuals.

# Intervention time series

The intervention time series model (Box and Tiao, 1975, JASA) can be expressed as:

$$Y_t = M_t + X_t, \tag{2}$$

where $Y_t$ represents the monthly (log) calling rate per 100,000 smokers;

# Intervention time series

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where $Y_t$ represents the monthly (log) calling rate per 100,000 smokers;

$X_t$ is a seasonal Box and Jenkin's ARIMA $(p, d, q)(P, D, Q)$ model which represents the baseline or background monthly (log) calling rate per 100,000 smokers throughout the selected interval of time;

# Time series background

A Box and Jenkin's ARIMA $(p, d, q)$ model can be written as

$$\phi_p(B)(1-B)^d X_t = \theta_q(B)\epsilon_t, \tag{3}$$

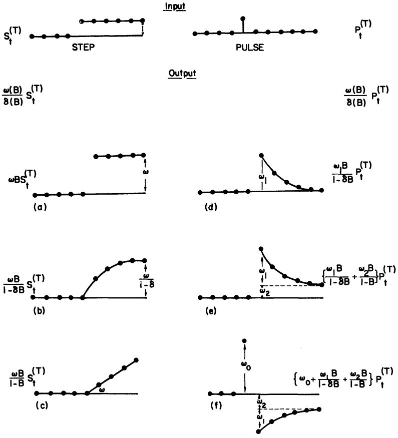
where $B$ is the back-shift operator such that $BX_t = X_{t-1}$, $d$ is the number of trend differences, $\phi_p(B)$ and $\theta_q(B)$ are the polynomials in $B$ of order $p$ and $q$ separately, that is $\phi_p(B) = 1 + \sum_{i=1}^{p} \phi_i B^i$, $\theta_q(B) = 1 - \sum_{j=1}^{q} \theta_j B^j$. If we consider seasonality, Model (3) can be modified as

$$\phi_p(B)\Phi_P(B^s)(1-B)^d(1-B^s)^D X_t = \theta_q(B)\Theta_Q(B^s)\epsilon_t, \tag{4}$$

where $D$ is the number of seasonal differences, $s$ is the seasonal period, and $\Phi_P(B^s)$ and $\Theta_Q(B^s)$ are polynomials in $B^s$ of order $P$ and $Q$, respectively.

# Transfer function

Two types of interventions: step and pulse interventions (Box and Tiao, 1975)




[a] (a), (b), (c) show the response to a step input for various simple transfer function models; (d), (e), (f) show the response to a pulse for some models of interest.

# Transfer function: continue

$M_t$ represents the additive change in the log calling rate due to the intervention. In other words $M_t$ is the log rate ratio at certain time $t$.

$M_t$ is a transfer function of intervention (or dummy) variable $I_t$,

$$I_t = \begin{cases} 1 & \text{if } t >= T_0, \\ 0 & \text{otherwise .} \end{cases}$$

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$$I_t = \begin{cases} 1 & \text{if } t >= T_0, \\ 0 & \text{otherwise }. \end{cases}$$

A flexible yet parsimonious form of $M_t$ could be a "first order" dynamic process of $I_t$, e.g., $M_t = \delta M_{t-1} + \omega I_t$. The value of the transfer function $M_t$ is as following:

$$M_t = \begin{cases} \frac{\omega(1-\delta^{k+1})}{1-\delta} & \text{if } k >= 0, \\ 0 & \text{otherwise }, \end{cases}$$

# Transfer function: continue

Compared to the calling rates that would have been observed in the absence of intervention, the relative change in the calling rate $k$ months after intervention is a non-linear function of two parameters $(\omega, \delta)$

$$\mathrm{RR}_k = \begin{cases} \exp(\omega(\frac{1-\delta^{k+1}}{1-\delta})) & \text{if } k \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

# Transfer function: continue

The limits of the function are
$RR(\omega, \delta, k \to 0) = exp(\omega)$ "immediate" intervention effect occurring exactly the intervention month;

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The parameter $\delta$ provides information about how quickly the rate ratio converges toward its long-term value. The cloest $\delta$ to 0, the quickest is the convergence.

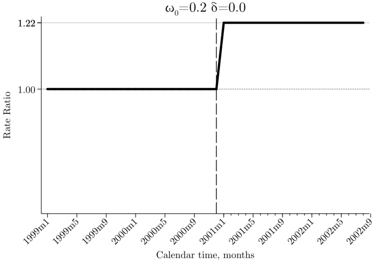
# Transfer function: continue

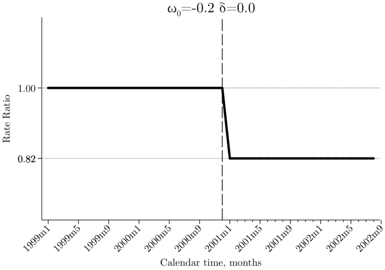
(1) No intervention effect:

If $\omega = 0$ the $RR_k = 1$ regardless of the magnitude and sign of $\delta$

(2) Immediate (either positive or negative) and no further effect over time:

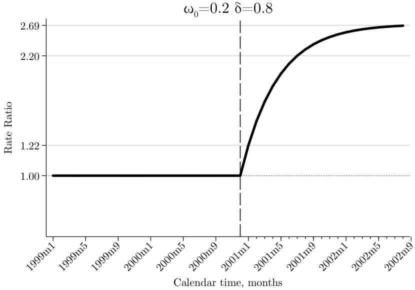
If $\delta = 0$ the $RR_k = exp(\omega)$

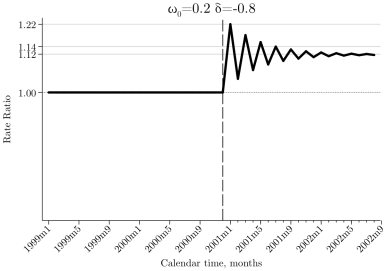


# Transfer function: continue

(3) Immediate positive effect plus a smooth and gradual increasing effect over time or an oscillating effect over time



# Algorithm

Let $\theta = (\theta_1', \theta_2')'$, $\theta_1$ includes the parameters from the transfer function such that $\theta_1 = (\delta, \omega)'$; and $\theta_2$ includes the parameters coming from the time series models.

(1) Create the likelihood function in Mata

(a) Use $\theta_1$ to calculate $m_t$, where $m_t = \delta * m_{t-1} + \omega * I_t$;

(b) Update $y_t^* = y_t - m_t$;

(c) Use $\theta_2$ and $y_t^*$ to calculate the likelihood, by calling Stata **arima** command

(d) Return the likelihood

(2) Maximize the likelihood function using Mata **optimize()**

(i) Use the BFGS algorithm

(3) Inference on the intervention effects

(i) Delta method using **nlcom**

(4) Tabulate the estimated results, plot the graphs and so on

# Syntax

```
// Simple step function
tstf lograte after
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// Controlling the ARIMA and transfer parameters
tstf lograte after, arima(1,0,1) sarima(0,1,0,12) t(1,0)
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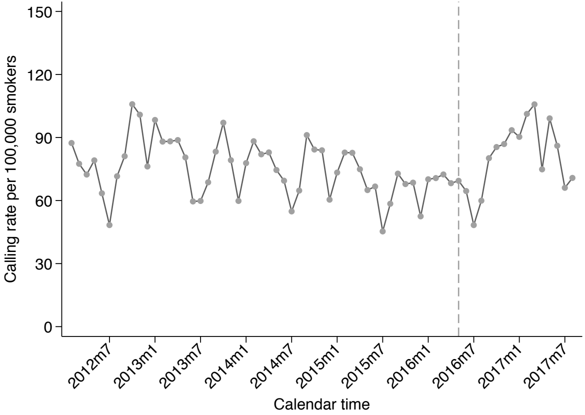
# Steps when using tstf command

- Choose the orders of the (seasonal) ARIMA background through the pre-intervention time points;
- Choose an intervention model;
- Estimate the model through ML and get the common dynamic effect of the intervention;
- Inference on the effect $k$-months after each intervention.

# Data: Calling rate per 100,000 smokers




Larger pictorial warnings on cigarette packs (May 2016)

# Output

```
// Simple step function
   tstf lograte after

ARIMA regression with a transfer function            No. of obs     =          67
Optimization = CSS-ML
Log likelihood = 26.707402
Sample = 2012m2 - 2017m8   Intervention starts = 2016m5
------------------------------------------------------------------------------
     lograte |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ARIMA        |
         ar1 |    .499028   .1063989     4.69   0.000     .2904899    .7075661
       _cons |   4.307029   .0438631    98.19   0.000     4.221059    4.392999
-------------+----------------------------------------------------------------
TRANSFER     |
       omega |   .0429717   .0866812     0.50   0.620    -.1269204    .2128638
------------------------------------------------------------------------------
```

# Output

```
//Controlling the ARIMA and transfer parameters
tstf lograte after, arima(1,0,1)  sarima(0,1,0,12)   t(1,0) ///
                    method(ML)  gre grd eform tabulate

ARIMA regression with a transfer function           No. of obs     =          67
Optimization = ML
Log likelihood = 42.170879
Sample = 2012m2 - 2017m8   Intervention starts = 2016m5
------------------------------------------------------------------------------
     lograte |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ARIMA        |
         ar1 |   .8558569   .1183034     7.23   0.000     .6239865    1.087727
         ma1 |  -.5901846   .1807572    -3.27   0.001    -.9444622   -.2359069
-------------+----------------------------------------------------------------
TRANSFER     |
       delta |   .9054777   .0727619    12.44   0.000      .762867    1.048088
       omega |   .0472376   .0212599     2.22   0.026      .005569    .0889062
------------------------------------------------------------------------------
```

# Output, continue

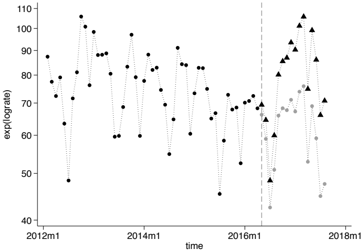
```
// tabulate the confidence intervals
tstf lograte after, arima(3,1,2)  sarima(1,0,0,12)   t(1,0) ///
                    method(ML)    eform tabulate
```

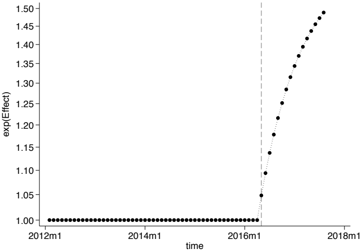
Table of effects k units of time after intervention

| time | k | exp(Eff) | LB | UB | P-value |
|---|---|---|---|---|---|
| 2016m5 | 0 | 1.05 | 1.01 | 1.09 | 0.026 |
| 2016m6 | 1 | 1.09 | 1.02 | 1.18 | 0.017 |
| 2016m7 | 2 | 1.14 | 1.03 | 1.26 | 0.011 |
| 2016m8 | 3 | 1.18 | 1.05 | 1.32 | 0.006 |
| 2016m9 | 4 | 1.22 | 1.07 | 1.39 | 0.003 |
| 2016m10 | 5 | 1.25 | 1.09 | 1.44 | 0.002 |
| 2016m11 | 6 | 1.28 | 1.11 | 1.49 | 0.001 |
| 2016m12 | 7 | 1.32 | 1.13 | 1.54 | 0.001 |
| 2017m1 | 8 | 1.34 | 1.14 | 1.58 | 0.000 |
| 2017m2 | 9 | 1.37 | 1.16 | 1.62 | 0.000 |
| 2017m3 | 10 | 1.39 | 1.17 | 1.65 | 0.000 |
| 2017m4 | 11 | 1.42 | 1.19 | 1.69 | 0.000 |
| 2017m5 | 12 | 1.44 | 1.19 | 1.73 | 0.000 |
| 2017m6 | 13 | 1.46 | 1.20 | 1.76 | 0.000 |
| 2017m7 | 14 | 1.47 | 1.20 | 1.80 | 0.000 |

# Output, continue

```
// graphs
tstf lograte after, arima(3,1,2)  sarima(1,0,0,12)   t(1,0) ///
                        method(ML)  gre grd eform tabulate
```




The calling rate gradually and significantly ($p$-value $< 0.001$) increased after the introduction of the larger pictorial warnings on the cigarette packs.

# Summary

- We are working on a Stata package **tstf** to estimate intervention time series model with transfer functions
- We focus on the transfer functions with two parameters (shape and scale parameter), the background is a seasonal time series model
- Estimation, inference, and graphs are given for both the original data and the log-transformed data

# Future work

- Keep on working the **tstf** package for multiple interventions
- Power calculations
- Time-vary confounders

# Team members

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Nicola Orsini, Associate Professor Medical Statistics, Karolinska Institutet

# References

(1) Box, G.E. P., Tiao G. C. Intervention analysis with applications to economic and environmental problems. Journal of the American Statistical Association. 1975. 70(349), 70-79.

(2) Shadish, W. R., Cook, T. D., & Campbell, D. T. Experimental and quasi-experimental designs for generalized causal inference. 2002. Belmont, CA: Wadsworth Cengage Learning.