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Fachbereich Wirtschafts- und
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`lgrgtest`: Lagrange-Multiplier Test after Constrained Maximum-Likelihood Estimation using Stata

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Outline

Motivation

Teach our students that after maximum-likelihood (ML) estimation **three** approaches to testing restrictions are available (if we teach ‘old-fashioned’ econometrics)

1. Wald test
2. Likelihood-ratio (LR) test
3. **Lagrange-multiplier** (LM) test (also known as **score** test)

Motivation (cont.)

- Official Stata includes commands implementing the
  1. Wald test (`test`)
  2. Likelihood-ratio test (`lrtest`)
- Yet no general command implementing the LM test
  - » LM tests available for specific settings (`xttest0` after `xtreg, re`; `estat scoretests` after `sem`)
- Implementing LM test ‘by hand’ somewhat cumbersome
- Having a (user written) Stata command (with straightforward syntax) available beneficial

User Written Commands to implement LM test?

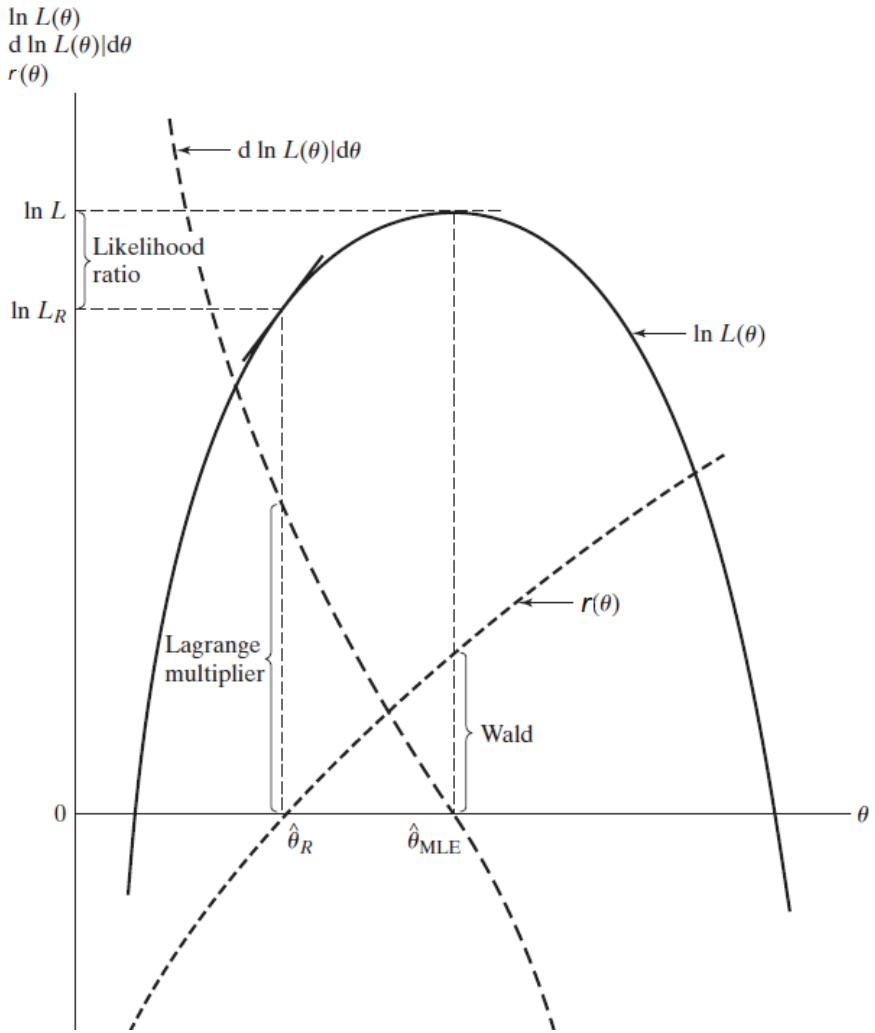
- Couldn’t find one
- Decided to write one myself
  - `lgrgtest` (earlier version `lmtest` available from SSC)
- Very recently, got a message from David Rodman via statalist
  - His `boottest` (`scoretest`) command not only implement bootstrap based inference (as I thought) but also the classical LM test
- Syntax of `lgrgtest` and `scoretest` different, yet they do the same after many (not all) Stata (ML) estimation commands

The Lagrange Multiplier Test

- Independently introduced by Rao (1948) and Silvey (1959)
- Tests constrained ($\boldsymbol{r}(\theta) = \mathbf{0}$) version ($H_0$) of a model against unconstrained model
- Exploits that gradient of log-likelihood function $\mathcal{L}(\theta)$ at constrained max. deviates from zero if restrictions bind
  - » Binding restrictions argue against constrained null model
- Test-statistic (score version): $LMS = \boldsymbol{g}(\widetilde{\theta})' \boldsymbol{I}(\widetilde{\theta})^{-1} \boldsymbol{g}(\widetilde{\theta})$
  - » $\widetilde{\theta}$: parameter estimates that maximize $\mathcal{L}(\theta)$ respecting the constraints
  - » $\boldsymbol{g}(\widetilde{\theta})$: gradient/score vector $d\mathcal{L}(\theta)/d\theta$, evaluated at $\widetilde{\theta}$
  - » $\boldsymbol{I}(\widetilde{\theta})$: information matrix, evaluated at $\widetilde{\theta}$
- LMS asymptotically χ^2-distributed with q dofs
  - » q: number of constraints

▸ multiplier version

The Lagrange Multiplier Test (cont.)



LM, LR, and Wald Test in ML estimation context, **Source**: Greene (2012)

Implementation in Stata trough `lgrgtest`

- (Jointly) tests restrictions imposed through option **`constraints()`** on most recently estimated model
  - » Confines `lgrgtest` to testing linear constraints
  - » Requires **`constraints()`** to be allowed in upstream estimation command
  - » After `sem`, and `gsem`, also restrictions specified using path notion (symbol `@`) tested
- Degrees of freedoms identified from **`e(Cns)`**
  - » Rank of `e(Cns)`
  - » Difference in ranks, if also 'unconstraint' model saves `e(Cns)` (e.g. to deal with collinearity)
- `lgrgtest` implements LM test in Stata based on score version ($\rightarrow$ estimates of $\boldsymbol{g}(\widetilde{\theta})$ and $\boldsymbol{I}(\widetilde{\theta})$ required)

Estimating the Score Vector

- Score vector needs to be saved in **`e(gradient)`**
  - » Confines `lgrgtest` to be used after ML estimators
  - » `lgrgtest` runs also after `cnsreg` ($\rightarrow$ interprets OLS as ML estimator [iid. normals errors]))
- After estimation with option `constraints()`, $\boldsymbol{g}(\widetilde{\theta})$ **not saved** in **`e(gradient)`**
- `lgrgtest` **re-executes** estimation command
  - (i) Without option `constraints()`
  - (ii) Using $\widetilde{\theta}$ as starting values
  - (iii) Disallowing maximization algorithm to iterate
- Requires options `from()` and `iterate()`
  - » For some command (`fmm`) more options need to be specified in re-execution to prevent re-estimation
- Makes $\boldsymbol{g}(\widetilde{\theta})$ be saved in **`e(gradient)`**

Estimating the Inverse Information Matrix

- **e(V)** from internal re-run used as estimate of $\boldsymbol{I}(\widetilde{\boldsymbol{\theta}})^{-1}$
- Only valid if **vcetype** either `oim` or `opg`
  - » If `vcetype` is neither `oim` nor `opg`, `lgrgtest` denies carrying out the test
  - » `lgrgtest` option `forcevce` makes command use `e(V_modelbased)` instead of `e(V)` ($\rightarrow$ one needs to know what one is doing)

Further Requirements for upstream Est. Command

- Re-executing estimation command – with adjusted options – requires interpreting command-line entry
  - `lgrgtest` only runs after commands with ‘standard’ Stata syntax
    - Also runs after some commands with more complex syntax (`sem`, `gsem`, `nlogit`, `fmm`)
  - Command-line entry needs to be saved in `e(cmdline)`
  - Command name needs to be saved in `e(cmd)` (or in `e(cmd2)`)

Stata Cmds allowing for `lgrgtest` in Postestimation

[CM] cmclogit, cmmixlogit, cmmprobit, cmroprobit, cmxtmixlogit, nlogit; [DSGE] dsge; [ERM] eintreg, xteinreg, eprobit, xteprobit, eregress, xteregress; [FMM] fmm; [ME] mecloglog, meglm, meintreg, melogit, menbreg, meologit, meoprobit, mepoisson, meprobit, mestreg, metobit; [R] betareg, binreg, clogit, cloglog, cnsreg, cpoisson, fracreg, frontier, glm, heckman, heckoprobit, heckpoisson, heckprobit, hetoprobit, hetprobit, hetregress, intreg, ivprobit, ivtobit, logistic, logit, mlexp, mlogit, mprobit, nbreg, gnbreg, ologit, oprobit, poisson, probit, scobit, slogit, tnbreg, tobit, tpoisson, truncreg, zinb, ziologit, zioprobit, zip; [SEM] sem, gsem; [ST] stcrreg, stintreg, streg; [TE] etpoisson, etregress; [TS] arch, arfima, arima, dfactor, mgarch ccc, mgarch dcc, mgarch dvech, mgarch vcc, sspace, ucm; [XT] xtcloglog, xtfrontier, xtheckman, xtintreg, xtlogit, xtmlogit, xtnbreg, xtologit, xtoprobit, xtpoisson, xtprobit, xttobit

Syntax of lgrgtest

```
lgrgtest [, notest nocnsreport noomitted df(#)
forcevce]
```

- Syntax does not specify restrictions to be tested
- Done in the preceding estimation syntax via specifying the option `constraints()`

Options for lgrgtest

- notest prevents lgrgtest from displaying any output on the screen
- nocnsreport prevents lgrgtest from displaying the imposed constraints
- noomitted makes lgrgtest not consider omitted variables as exclusion restrictions to be tested; specifying noomitted will not affect number of test degrees-of-freedom but only which restrictions are displayed
- df() makes lgrgtest use a user-specified number of degrees-of-freedom; specifying df() will rarely be required
- forcevce makes lgrgtest perform the LM test even if *vcetype* is neither oim nor opg (not ols after cnsreg); with option forcevce, lgrgtest issues a warning and uses e(V_modelbased) as estimate of the inverse information matrix

Stored Results for lgrgtest

lgrgtest stores in r():

- Scalars
  - r(p): *p*-value
  - r(chi2): LM test statistic (chi-squared)
  - r(df): test constraints degrees of freedom
  - r(rank): rank of e(Cns) adjusted for the number automatically imposed constraints (only saved if option df() is specified)
- Macros
  - r(modelbased): modelbased if e(V_modelbased) used as estimate of the inverse information matrix

lgrgtest and boottest

- boottest way more powerful than lgrgtest
- scoretest (boottest package) deals with special case in which no robustification through bootstrapping is required
- After many popular commands (logit, probit, ...) lgrgtest and scoretest yield identical results
- After some commands scoretest failed but not lgrgtest
  - David Roodman published new version of scoretest that have fixed some issues
  - See discussion on statalist

lgrgtest and boottest (cont.)

- After some commands scoretest and lgrgtest yield different results
  - » Needs still to be checked (from my side)
  - » See discussion on statalist
- Would probably not have written lgrgtest if I had been aware of scoretest
- lgrgtest might still be of some value since classical LM-test functionality of boottest apparently not well known among Stata users

Egger et al. (2011) [AEJ: EP] in a Nutshell

- Research question: Do preferential trade agreements (PTA) positively affect bilateral trade flows? ("The Trade Effects of Endogenous Preferential Trade Agreements")
- Country-level, cross-sectional data (year 2005, 15 750 country dyads)
- Existence of PTA possibly endogenous
- When considering extensive margin of trade $[\mathcal{I}_{ij} = 1\,(\text{exports from } i \text{ to } j > 0)]$, endogeneity addressed through estimating recursive bivariate probit model
- Exclusion restrictions (instruments for PTA_{ij}):
  1. Past colonizer-colony relationship ($COLONY_{ij}$)
  2. Common past colonizer ($COMCOL_{ij}$)
  3. Common history as one joint country ($SMCTRY_{ij}$)
- Non-linearity of probit model allows for conventionally testing validity of exclusion restrictions

Testing Exclusion Restrictions

- Egger et al. (2011) use `test` (Wald test)
  - » Based on estimating unconstrained model (no exclusion restrictions)
  - » Yet, discussion focusses on constrained model (exclusion restrictions imposed)
- Basing test on estimating constrained model has intuitive appeal
- LM test based on restricted model ($\rightarrow$ implemented by `lgrgtest`)

Testing Exclusion Restrictions (cont.)

- Wald test originally used in Egger et al. (2011):

```
.  quietly biprobit (pta = $z $instr const _x_*, nocons) (i =
> pta $z $instr const _x_*, nocons)
.  test [i]colony [i]curcol [i]smctry
( 1)  [i]colony = 0
( 2)  [i]curcol = 0
( 3)  [i]smctry = 0
          chi2( 3) = 3.78
        Prob > chi2 = 0.2864
```

- Exclusion restrictions not rejected

Testing Exclusion Restrictions (cont.)

- LM test using `lgrgtest`:

```
.  constraint 1 [i]colony = 0
.  constraint 2 [i]curcol = 0
.  constraint 3 [i]smctry = 0
.  constraint 4 [i]comcol = 0
.  quietly biprobit (pta = $z $instr const _x_*, nocons) (i =
> pta $z $instr const _x_*, nocons), constraints(1 2 3)
.  lgrgtest
LM test of constraints(1 2 3)
( 1)  [i]colony = 0
( 2)  [i]curcol = 0
( 3)  [i]smctry = 0
      LM chi2( 3) = 3.79
      Prob > chi2 = 0.2856
```

Testing Exclusion Restrictions (cont.)

- Virtually identical result from `test` and `lgrgtest`
  - » Asymptotic equivalence of LM test and Wald test
  - » Rather large sample
- Interesting secondary aspect:
  - » Test exclusion restrictions not those that are actually imposed (in preferred model)
  - » $COMCOL_{ij}$, not $CURCOL_{ij}$ (post 1945 colonizer-colony relationship), used as instrument for PTA_{ij}
  - » Actually imposed exclusion restrictions clear rejected (irrespective of whether LM test or Wald test is used)

Testing Exclusion Restrictions (cont.)

- LM test using `lgrgtest` of actually imposed restrictions:

```
.  quietly biprobit (pta = $z $instr const _x_*, nocons) (i =
> pta $z $instr const _x_*, nocons), constraints(1 3 4)
.  lgrgtest
LM test of constraints(1 3 4)
( 1)  [i]colony = 0
( 3)  [i]smctry = 0
( 4)  [i]comcol = 0
      LM chi2( 3) = 171.21
      Prob > chi2 = 0.0000
```

Conclusions

- `lgrgtest` provides convenient way to employ classic Lagrange-multiplier test in Stata postestimation
- Complements real Stata commands `test` and `lrtest`
- Alternative to `scoretest`
- Major limitations (`lgrgtest` shares with `scoretest`):
  - Confined to testing linear restrictions
  - Cannot be used after `ml maximize`

Multiplier Version of LM Test

Multiplier version of LM-test statistic:

$$LMS = \widetilde{\lambda}' \boldsymbol{R}(\widetilde{\theta})' \boldsymbol{I}(\widetilde{\theta})^{-1} \boldsymbol{R}(\widetilde{\theta}) \widetilde{\lambda}$$

with Jacobian $\boldsymbol{R}(\widetilde{\theta})$, and Lagrange multipliers $\widetilde{\lambda}$. From first order condition

$$\boldsymbol{g}(\widetilde{\theta}) - \boldsymbol{R}(\widetilde{\theta}) \widetilde{\lambda} = \boldsymbol{0}$$

of the constrained maximization problem

$$\max_{\theta} \mathcal{L}(\theta) - \lambda' \boldsymbol{r}(\theta)$$

equivalence of score and multiplier version becomes obvious. (cf. Arellano, 2004)