nopo

Implementation of a matching-based decomposition technique with postestimation commands

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19th German Stata Conference, Berlin

16.6.2023

Decompositions in the Social Sciences

- Decomposition techniques are a common way to examine gaps in socio-economic outcomes between two groups (e.g., sex, race, nativity)
  - To what extent contribute observed differences in group characteristics to gaps?
    $\Rightarrow$ **Explained component**
  - Gaps not accounted for by observed differences in group characteristic might indicate differential returns or unobservables
    $\Rightarrow$ **Unexplained component**

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    ⇒ **Explained component**
  - Gaps not accounted for by observed differences in group characteristic might indicate differential returns or unobservables
    ⇒ **Unexplained component**

- Methods invoke different assumptions, can lead to different results, and provide different insights (Strittmatter and Wunsch 2021; Hamjediers and Sprengholz 2023)
  - Many applications rely on regression-based techniques (Blinder 1973; Oaxaca 1973), Nopo (2008) proposed a matching-based approach

Decomposition à la Ñopo

1. Each member of group B can be matched to all potential matches of group A along a set of characteristics $\mathbf{X}$ (one-to-many-matching), providing two pieces of information:

   - Who can be matched (subscript m) and who cannot be matched (subscript u)
   - Weights to calculate counterfactual outcome $\overline{Y}_{A^B,m}$ that reflects
     - outcome of group A if it had the same characteristics as group B
     - outcome of group B if it had the same returns to characteristics as group A

Decomposition à la Ñopo

2. If $D = \overline{Y}_B - \overline{Y}_A$,
   gap can be decomposed into four components after matching:

$$\begin{aligned}
D &= D_0 + D_X + D_A + D_B \\
&= \overbrace{\overline{Y}_{B,m} - \mathrlap{\underbrace{\phantom{\overline{Y}_{A^B,m} + \overline{Y}_{A^B,m}}}_{\substack{\text{splitting difference}\\ \text{among matched by}\\ \text{reweighted group A}}}}\overline{Y}_{A^B,m}} + \overbrace{\overline{Y}_{A^B,m} - \overline{Y}_{A,m}} + D_A + D_B \\
&= \underbrace{\substack{\text{unexplained}\\ \text{component}} + \substack{\text{explained}\\ \text{component}}}_{\text{pertains only to matched units}} + D_A + D_B
\end{aligned}$$

Decomposition à la Ñopo

2. If $D = \overline{Y}_B - \overline{Y}_A$,
   gap can be decomposed into four components after matching:

$$D = D_0 + D_X + \underbrace{D_A + D_B}_{\text{out of support}}$$

$$D_A = \overbrace{(\overline{Y}_{A,m} - \overline{Y}_{A,u})}^{\text{gap between matched and unmatched } A} \cdot \overbrace{(N_{A,u}/N_A)}^{\text{share of unmatched } A}$$

$$D_B = \underbrace{(\overline{Y}_{B,u} - \overline{Y}_{B,m})}_{\text{gap between unmatched and matched } B} \cdot \underbrace{(N_{B,u}/N_B)}_{\text{share of unmatched } B}$$

Links to other approaches

- Generally, similar to two-fold (Kitagawa-)Blinder-Oaxaca-Decomposition (Hamjediers and Sprengholz 2023)
    - Advantages of matching-based decomposition:
        - + Non-parametric estimation → no assumptions about functional form
        - + D_0 & D_X apply only to matched units → no model-based extrapolation
    - Disdvantages of matching-based decomposition:
        - – Suffers from curse of dimensionality → risk of attributing too much to D_A & D_B
        - – Does not allow to disentangle explained component across predictors

⇒ Similar arguments as for regression- vs. matching-based adjustment for confounders in estimating (local) treatment effects

Links to other approaches

- Component D_0 is equal to the average treatment effect on the treated ATT after matching

$$\begin{aligned} ATT &= Po^{T=1}_{t=1} - Po^{T=1}_{t=0} \\ &= \overline{Y}_{B,m} - \overline{Y}_{A^B,m} = D_0 \end{aligned}$$

⇒ All other components of Ñopo's approach are seldom assessed in estimations of treatment effects via matching

It's all about the matching

- Originally, exact matching on (coarsened) predictors

  (cf. ado-file `nopomatch` of Atal et al. (2013))

- We extend it to Propensity Score Matching (Rosenbaum and Rubin 1983) and Multivariate Distance Matching

- Trade-off between reaching balance on predictors between B, m and A^B, m vs. curse of dimensionality and lack of common support (Iacus et al. 2012)

New command: `nopo`

- More flexible, inference for all components

- Allows matching by different measures
  - `nopo` calls `kmatch` (Jann 2017) inherently or can be used as postestimation-command after matching via `kmatch` (Jann 2017)

- Provides postestimation commands for descriptives after matching, contribution of groups to D_A and D_B, and components across distribution of Y

- Illustration based on example from Hamjediers and Sprengholz (2023):
  - Data: GSOEP, 2014-2019, one observation per individual
  - Groups: Native men (A) and immgriant women (B)
  - Outcome: hourly gross wages
  - Predictors: age, married, educational attainment, labor market experience, 2-digit ISCO-08 occupations, part-time indicator *(all coarsened)*

Standalone usage

```
nopo decomp depvar varlist [if] [in] [weights], by(varname)
```

Standalone usage

```
. nopo decomp wage age_c married edu lmexp parttime isco2d, by(grp)

Nopo decomposition                        N                         =       8954
Exact matching:                           N strata                  =       1783
                                          N matched strata          =        488
                                          (unique combinations of matching set)

------------------------------------------------------------------------------
                             |             N / %                     Mean
                             | --------------------------------  ------------
Group                        |  Matched    Unmatched       Total          wage
-----------------------------+------------------------------------------------
A: Native Men                |     3110         2939        6049          20.5
   grp == 1                  |     51.4         48.6
B: Immigrant Women           |     1387         1518        2905          12.1
   grp == 4 (ref)            |     47.7         52.3
------------------------------------------------------------------------------

------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
           D |  -8.384027   .2067519   -40.55   0.000    -8.789253   -7.978801
          DO |  -2.406294   .5407104    -4.45   0.000    -3.466067   -1.346521
          DX |   -5.05875   .6061777    -8.35   0.000    -6.246837   -3.870664
          DA |   .7731673   .1257708     6.15   0.000     .5266609    1.019674
          DB |   -1.69215   .1285267   -13.17   0.000    -1.944057   -1.440242
------------------------------------------------------------------------------
```

Standalone usage

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Nopo decomposition                        N                          =       8954
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        wage | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
           D |  -8.384027   .2067519   -40.55   0.000    -8.789253   -7.978801
          D0 |  -2.406294   .5407104    -4.45   0.000    -3.466067   -1.346521
          DX |   -5.05875   .6061777    -8.35   0.000    -6.246837   -3.870664
          DA |   .7731673   .1257708     6.15   0.000     .5266609    1.019674
          DB |   -1.69215   .1285267   -13.17   0.000    -1.944057   -1.440242
------------------------------------------------------------------------------
```

Interpretations:

- D0 Among the matched, 2.41 Euro lower wages for group B are unexplained
- DX Compositional differences account for 5.06 Euro of the gap among matched units
- DA Unmatched units of group A earn lower wages than matched units, which accounts for 0.77 Euro of the gap
- DB Unmatched units of group B earn lower wages than matched units, which accounts for 1.69 Euro of the gap

Standalone usage

- General Options:
  - Swap groups: `swap`
  - Defining matching direction: `bref(`*varname*` == #)`
  - Normalize outcome to the reference group of the matching: `normalize`

Standalone usage

```
. nopo decomp wage ${pred}, by(grp) bref(grp == 1) swap normalize
Normalized outcome generated: _wage_norm

Nopo decomposition                       N                         =      8954
Exact matching:                          N strata                  =      1783
                                         N matched strata          =       488
                                         (unique combinations of matching set)

------------------------------------------------------------------------------
                             |              N / %                     Mean
                             | ---------------------------------  ------------
Group                        |  Matched    Unmatched       Total    _wage_norm
-----------------------------+------------------------------------------------
A: Immigrant Women           |     1387         1518        2905          .591
   grp == 4                  |     47.7         52.3
B: Native Men                |     3110         2939        6049             1
   grp == 1 (ref)            |     51.4         48.6
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  _wage_norm | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
           D |   .4090725   .0100878    40.55   0.000     .3893008    .4288443
          DO |   .1813969   .0299544     6.06   0.000     .1226873    .2401065
          DX |   .1828367   .0328027     5.57   0.000     .1185446    .2471288
          DA |   .0825632   .0062711    13.17   0.000     .0702721    .0948542
          DB |  -.0377243   .0061366    -6.15   0.000    -.0497518   -.0256968
------------------------------------------------------------------------------
```

Standalone usage

- General Options:
  - Swap groups: `swap`
  - Defining matching direction: `bref(`*varname*` == #)`
  - Normalize outcome to the reference group of the matching: `normalize`

- Options to adjust matching procedure correspondingly to `kmatch`:
  - `kmatch()` allows for exact matching `(em)` *(the default)*, propensity score matching `(ps)`, and multivariate distance matching `(md)`
  - Matching-specific options from `kmatch` can be implemented via `kmatchopt()`

```
. qui: nopo decomp wage ${pred}, by(grp)
. qui: est store em
. qui: nopo decomp wage ${pred}, by(grp) kmatch(ps)
. qui: est store ps
. qui: nopo decomp wage ${pred}, by(grp) kmatch(md)
. qui: est store md
. qui: nopo decomp wage ${pred}, by(grp) kmatch(ps) kmopt(pscmd(probit) bw(0.0001))
. qui: est store ps_probbw
```

Standalone usage

```
. esttab em ps md ps_probbw, se nonumbers nonotes ///
>          mtitles("exact" "prop. score" "multi. dist." "probit ps") ///
>         stats(nA mshareuwA nB mshareuwB bwidth, label("N(A)" "% matched A" "N(B)" "% matched B" "Bandwidth"))

----------------------------------------------------------------------------
                    exact     prop. score    multi. dist.        probit ps
----------------------------------------------------------------------------
D                  -8.384***       -8.384***       -8.384***       -8.384***
                  (0.207)         (0.207)         (0.207)         (0.207)

DO                 -2.406***       -2.652***       -3.957***       -3.680***
                  (0.541)         (0.504)         (0.402)         (0.788)

DX                 -5.059***       -5.650***       -4.427***       -3.673***
                  (0.606)         (0.546)         (0.452)         (0.829)

DA                  0.773***            0               0            0.260*
                  (0.126)             (.)             (.)         (0.101)

DB                 -1.692***      -0.0827***            0          -1.291***
                  (0.129)        (0.0232)             (.)         (0.111)
----------------------------------------------------------------------------
N(A)                 6049            6049            6049            6049
% matched A         51.41             100             100           62.08
N(B)                 2905            2905            2905            2905
% matched B         47.75           96.73             100           55.42
Bandwidth                         0.00265           2.325        0.000100
----------------------------------------------------------------------------
```

As postestimation after `kmatch`

- Can be used after `kmatch` by just prompting `nopo decomp`
- Needs that following options of `kmatch` are specified:
  - `tval(#)` to define reference group (if different from `tval(1)`)
  - `att` and/or `atc`; should be coherent to matching direction

As postestimation after kmatch

```
. qui: kmatch ps grp ${pred} (wage), ///
>          tval(4) atc att bw(0.001) pscmd(probit) generate wgenerate replace
```

```
. nopo decomp
```

```
Nopo decomposition                        N                          =       8954
Propensity-score matching:                Kernel bandwidth:          =    1.0e-03

-------------------------------------------------------------------------------
                             |              N / %                       Mean
                             | --------------------------------- -------------
Group                        |  Matched     Unmatched       Total          wage
-----------------------------+-------------------------------------------------
A: Native Men                |     5900           149        6049          20.5
   grp == 1                  |     97.5           2.5
B: Immigrant Women           |     2516           389        2905          12.1
   grp == 4 (ref)            |     86.6          13.4
-------------------------------------------------------------------------------

-------------------------------------------------------------------------------
        wage | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+-----------------------------------------------------------------
           D |  -8.384027   .2067519   -40.55   0.000    -8.789253   -7.978801
          D0 |   -3.02594   .6471339    -4.68   0.000    -4.294299   -1.757581
          DX |  -5.066576   .6814688    -7.43   0.000     -6.40223   -3.730921
          DA |  -.0101861   .0206239    -0.49   0.621    -.0506082    .0302361
          DB |  -.2813254    .049454    -5.69   0.000    -.3782534   -.1843973
-------------------------------------------------------------------------------
```

As postestimation after kmatch

- Invoked `kmatch`-command in standalone usage is returned and can be copied for case-specific adjustments

```
. qui: nopo decomp wage ${pred}, by(grp) kmatch(ps)

. display "`e(kmatch_cmdline)'"
kmatch ps grp age_c married edu lmexp parttime isco2d (wage) , tval(4) att generate wgenerate replace
```

Description by matching status

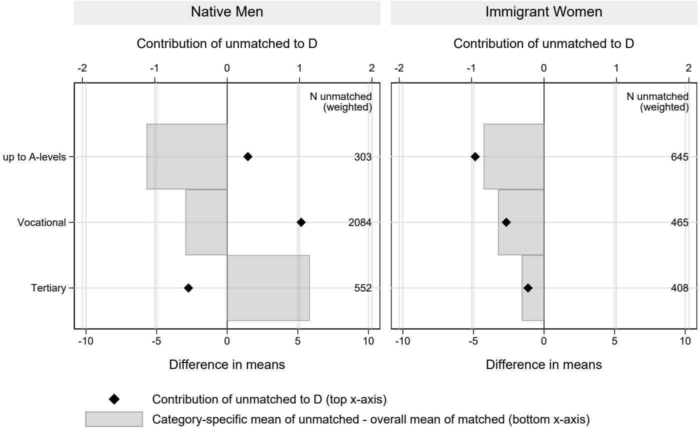
```
. qui: nopo decomp wage ${pred}, by(grp)

. nopo summarize wage age married edu_1 edu_2 edu_3, label

             |              Native Men              |      Immigrant Women
             |  unmatched |    matched | matched ~d |    matched |  unmatched
-------------+------------+------------+------------+------------+------------
Hourly wag~) |            |            |            |            |
        Mean |       19.7 |       21.3 |       16.2 |       13.8 |       10.6
          SD |       9.71 |       10.4 |       9.74 |       7.65 |       5.52
Age          |            |            |            |            |
        Mean |       44.8 |       43.9 |       38.9 |       38.9 |       41.6
          SD |       9.94 |         11 |       10.2 |         10 |       8.79
Married      |            |            |            |            |
        Mean |       .612 |       .669 |       .607 |       .607 |       .759
          SD |       .487 |       .471 |       .488 |       .489 |       .428
edu==up to~s |            |            |            |            |
        Mean |       .103 |       .055 |       .199 |       .199 |       .425
          SD |       .304 |       .228 |       .399 |       .399 |       .494
edu==Vocat~l |            |            |            |            |
        Mean |       .709 |       .504 |        .49 |        .49 |       .306
          SD |       .454 |         .5 |         .5 |         .5 |       .461
edu==Terti~y |            |            |            |            |
        Mean |       .188 |       .441 |       .311 |       .311 |       .269
          SD |       .391 |       .497 |       .463 |       .463 |       .443
```

Variable-specific contribution to D_A and D_B

```
. nopo dadb edu
```



See application in Sprengholz and Hamjediers (2022), Figure 2

Components along outcome-distribution

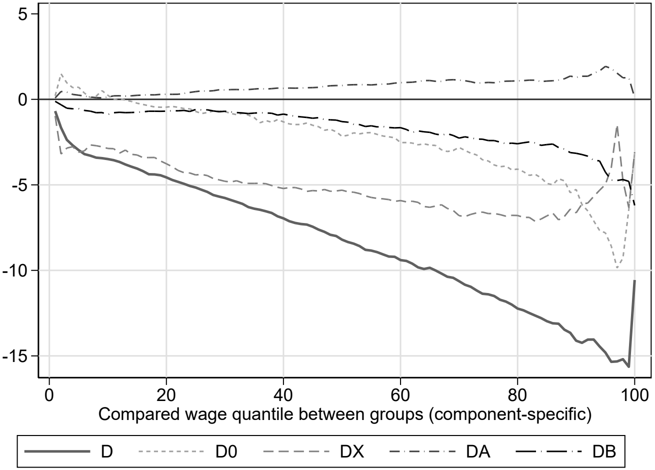
```
. nopo gapoverdist
Component distribution across 100 quantiles of wage requested

-------------------------------------------------------------------------------
    |                                       Minimum among compared groups
    |       Estimate      Sum over q     Unique q values                      N
----+--------------------------------------------------------------------------
  D |          -8.38           -8.39                 100                   2905
 DO |          -2.41           -2.36                  98                   1387
 DX |          -5.06            -5.1                  98                   3110
 DA |           .773            .773                 100                   2939
 DB |          -1.69           -1.69                 100                   1387
-------------------------------------------------------------------------------
Note:
- The component sum across quantiles should correspond to the estimates with
  well populated quantiles.
- There are less unique quantile values than quantiles requested which means
  that across some quantiles, the value of wage does not change for
  (one of) the groups compared to estimate the component.
- Use the nquantiles(#) option to set the number of quantiles.
```

Components along outcome-distribution

```
. nopo gapoverdist
Component distribution across 100 quant

---------------------------------------
    |
    |        Estimate      Sum over q
----+----------------------------------
  D |           -8.38           -8.39
 D0 |           -2.41           -2.36
 DX |           -5.06            -5.1
 DA |            .773            .773
 DB |           -1.69           -1.69
---------------------------------------
Note:
- The component sum across quantiles sh
  well populated quantiles.
- There are less unique quantile values
  that across some quantiles, the value
  (one of) the groups compared to estim
- Use the nquantiles(#) option to set t
```



See application in Nopo (2008), Figure 2

Outlook

- On our to-do-list:
  - Options for component-size relative to gap
  - Standard errors are still too large and need to be adjusted
    - `bootstrap`-prefix can be applied
  - Write a help-file
- Current version is available on git: github.com/mhamjediers/nopo_decomposition
- Any feedback is of course very welcome

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