Performing and interpreting discrete choice analyses in Stata

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Discrete choice analysis with alternative-specific variables

```
. webuse transport
(Transportation choice data)

. list id t alt choice trcost trtime age income in 1/12, sepby(t) noobs
```

id	t	alt	choice	trcost	trtime	age	income
1	1	Car	1	4.14	0.13	3.0	3
1	1	Public	0	4.74	0.42	3.0	3
1	1	Bicycle	0	2.76	0.36	3.0	3
1	1	Walk	0	0.92	0.13	3.0	3
1	2	Car	1	8.00	0.14	3.2	5
1	2	Public	0	3.14	0.12	3.2	5
1	2	Bicycle	0	2.56	0.18	3.2	5
1	2	Walk	0	0.64	0.39	3.2	5
1	3	Car	1	1.76	0.18	3.4	5
1	3	Public	0	2.25	0.50	3.4	5
1	3	Bicycle	0	0.92	1.05	3.4	5
1	3	Walk	0	0.58	0.59	3.4	5

Examples of things we want to learn from discrete choice analyses

- How does the probability of choosing public transportation change if yearly income increases from $30,000 to $40,000?
- How does travel time and cost affect the probability of choosing each transportation mode?
- If travel cost related to car travel increases, how does that affect the probability of using a car?
- If travel time is increasing for public transportation, how does that affect the probability of choosing car travel?

Some estimation results from a discrete choice model

```
<snip>
------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
alt          |
      trcost |  -.8388216   .0438587   -19.13   0.000    -.9247829   -.7528602
      trtime |  -1.508756   .2641554    -5.71   0.000    -2.026492   -.9910212
------------------------------------------------------------------------------
<snip>
```

- We can conclude that people generally don't like to waste either time or money!
- In this talk, we will see how we can use **`margins`** to discover more interesting results

Theoretical motivation of discrete choice models

- Random utility models
- $U_{ijt} = V_{ijt} + \epsilon_{ijt}$
  - $U_{ijt} \rightarrow$ Utility of person *i* for the *j*th alternative at time *t*
  - $V_{ijt} \rightarrow$ Observed component of utility
  - $\epsilon_{ijt} \rightarrow$ Unobserved component of utility
- Decision makers choose alternative *j* if $U_{ijt} > U_{ikt} \quad \forall \quad k \neq j$
- Specification of V_{ijt} and assumptions about ϵ_{ijt} constitute different discrete choice estimators (e.g., logit or probit)
- New estimation command in Stata 16: **`cmxtmixlogit`** for fitting panel-data mixed logit models

The mixed logit model (1)

- The mixed multinomial logit model uses random coefficients to model the correlation of choices across alternatives, thereby relaxing IIA
- With mixed logit, for the random utility model $U_{ijt} = V_{ijt} + \epsilon_{ijt}$ we have:
  - $V_{ijt} \quad = \quad x_{ijt}\beta_i$
  - $\epsilon_{ijt} \quad \sim$ iid type I extreme value
- The random coefficients β_i induce correlation across the alternatives
- We estimate the parameters of a specified distribution for β_i

The mixed logit model (2)

- The probability of unit i choosing alternative j at time t is
  - $P_{ijt} = \int P_{ijt}(\beta) f(\beta) d\beta \quad (1)$
  - $P_{ijt}(\beta)$ is the probability of unit i choosing alternative j at time t, conditional on β_i
    - ★ $P_{ijt}(\beta) = e^{x_{ijt}\beta_i} / \sum_{j=1}^{J} e^{x_{ijt}\beta_i}$
    - ★ $f(\beta)$ is the mixing distribution of the random coefficients
  - The integral in (1) needs to be approximated because it has no closed form solution
  - Using Monte Carlo integration, we draw β_i from $f(\beta)$ and have simulated probabilities $\widehat{P}_{ijt} = 1/M \sum_{m=1}^{M} P_{ijt}(\beta^m)$
- The simulated likelihood for the i^{th} unit is $L_i = \prod_{t=1}^{T} \sum_{j=1}^{J} d_{ijt} \widehat{P}_{ijt}$

`cmxtmixlogit`

- Random coefficient distributions $f(\beta)$:
  - (multivariate) normal
  - lognormal
  - truncated normal
  - uniform
  - triangle
- Estimates the parameters of the mixed logit model by **maximum simulated likelihood**
- Halton, Hammersley, and pseudo-random draws with uni- and multidimensional **antithetics**
- Full support of **factor variables** and **time-series operators**
- Support of complex **survey** data
- Case-specific variables
- **`margins`**

cmset – declaring cm data

```
. cmset id t alt
panel data: panels id and time t
note: case identifier _caseid generated from id t
note: panel by alternatives identifier _panelaltid generated from id alt
                   caseid variable:  _caseid
             alternatives variable:  alt
      panel by alternatives variable:  _panelaltid (strongly balanced)
                     time variable:  t, 1 to 3
                             delta:  1 unit

note: data have been xtset
```

cmchoiceset – exploring choice sets

```
. cmchoiceset

Tabulation of choice-set possibilities

 Choice set |      Freq.     Percent        Cum.
------------+-----------------------------------
    1 2 3 4 |      1,053       70.20       70.20
    1 2 3 5 |        210       14.00       84.20
    1 2 5 6 |         90        6.00       90.20
    2 3 4 7 |        147        9.80      100.00
------------+-----------------------------------
      Total |      1,500      100.00

Total is number of cases.
```

`cmsample` – reasons for sample exclusion

```
. preserve

. webuse transport, clear
(Transportation choice data)

. replace trcost = . in 5
(1 real change made, 1 to missing)

. replace alt = . in 2
(1 real change made, 1 to missing)

. replace choice = 0 if t==3 & id==1
(1 real change made)

. replace income = 1 in 1
(1 real change made)
```

`cmsample` – reasons for sample exclusion

```
. cmset id t alt
panel data: panels id and time t
note: case identifier _caseid generated from id t
note: panel by alternatives identifier _panelaltid generated from id alt
note: alternatives are unbalanced across choice sets; choice sets of
      different sizes found

                 caseid variable:  _caseid
           alternatives variable:  alt
     panel by alternatives variable:  _panelaltid (unbalanced)
                   time variable:  t, 1 to 3
                           delta:  1 unit

note: data have been xtset
```

`cmsample` – reasons for sample exclusion

```
. cmsample trcost trtime, choice(choice) casevars(age income)
```

Reason for exclusion	Freq.	Percent	Cum.
observations included	5,988	99.80	99.80
caseid variable missing	1	0.02	99.82
varlist missing	4	0.07	99.88
choice variable all 0	4	0.07	99.95
casevars not constant within case*	3	0.05	100.00
Total	6,000	100.00	

```
* indicates an error

. restore
```

Panel-data mixed logit model using `cmxtmixlogit` (1)

```
. cmxtmixlogit choice trcost, random(trtime) casevars(age income) nolog
Mixed logit choice model                        Number of obs      =      6,000
                                                Number of cases    =      1,500
Panel variable: id                              Number of panels   =        500
Time variable: t                                Cases per panel: min =        3
                                                                 avg =      3.0
                                                                 max =        3
Alternatives variable: alt                      Alts per case:   min =        4
                                                                 avg =      4.0
                                                                 max =        4
Integration sequence:      Hammersley
Integration points:               594               Wald chi2(8)     =     432.68
Log simulated likelihood = -1005.9899               Prob > chi2      =     0.0000

------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
<snip>
```

Panel-data mixed logit model using `cmxtmixlogit` (2)

```
<snip>
------------------------------------------------------------------------------
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
alt          |
      trcost |  -.8388216    .0438587   -19.13   0.000    -.9247829   -.7528602
      trtime |  -1.508756    .2641554    -5.71   0.000    -2.026492   -.9910212
-------------+----------------------------------------------------------------
/Normal      |
   sd(trtime)|   1.945596    .2594145                      1.498161    2.526661
-------------+----------------------------------------------------------------
Car          |  (base alternative)
------------------------------------------------------------------------------
<snip>
```

Panel-data mixed logit model using `cmxtmixlogit` (3)

```
<snip>
------------------------------------------------------------------------------
Car          |  (base alternative)
-------------+----------------------------------------------------------------
Public       |
         age |   .1538915   .0672638     2.29   0.022     .0220569    .2857261
      income |  -.3815444   .0347459   -10.98   0.000    -.4496451   -.3134437
       _cons |  -.5756547   .3515763    -1.64   0.102    -1.264732    .1134222
-------------+----------------------------------------------------------------
Bicycle      |
         age |     .20638   .0847655     2.43   0.015     .0402426    .3725174
      income |  -.5225054   .0463235   -11.28   0.000    -.6132978   -.4317131
       _cons |  -1.137393   .4461318    -2.55   0.011    -2.011795   -.2629909
-------------+----------------------------------------------------------------
Walk         |
         age |   .3097417   .1069941     2.89   0.004     .1000372    .5194463
      income |  -.9016697   .0686042   -13.14   0.000    -1.036132   -.7672078
       _cons |  -.4183279   .5607111    -0.75   0.456    -1.517302    .6806458
------------------------------------------------------------------------------
```

What would be the expected choice probabilities if every person in the population had a yearly income of $30,000?

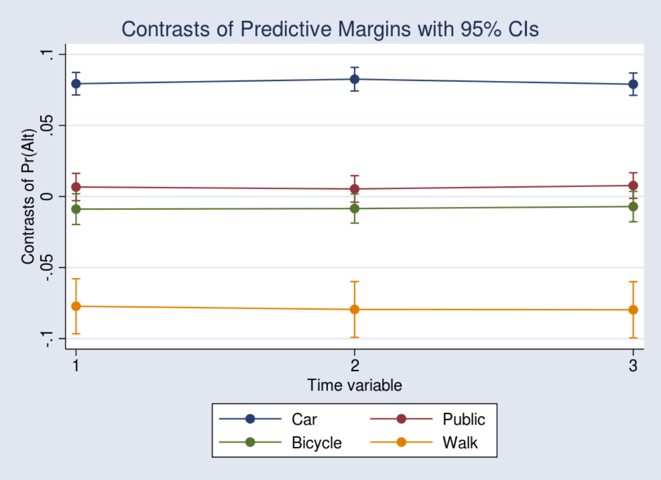
```
. margins, at(income=3)

Predictive margins                              Number of obs     =      6,000
Model VCE    : OIM

Expression   : Pr(alt), predict()
at           : income          =           3

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    _outcome |
         Car |   .3331611   .0196734    16.93   0.000      .294602    .3717203
      Public |   .2210964   .0184285    12.00   0.000     .1849772    .2572156
     Bicycle |   .1676081   .0181511     9.23   0.000     .1320325    .2031837
        Walk |   .2781343   .0243791    11.41   0.000     .2303521    .3259166
------------------------------------------------------------------------------
```

What would be the differences between an income of $40,000 and $30,000 over time?

```
. margins, at(income=(3 4)) contrast(at(r) nowald) over(t)
Contrasts of predictive margins                 Number of obs     =      6,000
Model VCE      : OIM

Expression     : Pr(alt), predict()
over           : t

1._at          : 1.t
                     income          =           3
1._at          : 2.t
                     income          =           3
1._at          : 3.t
                     income          =           3

2._at          : 1.t
                     income          =           4
2._at          : 2.t
                     income          =           4
2._at          : 3.t
                     income          =           4

------------------------------------------------------------------------
                     |            Delta-method
                     |   Contrast   Std. Err.     [95% Conf. Interval]
---------------------+--------------------------------------------------
     _at@_outcome#t  |
    (2 vs 1) Car#1   |   .0793997   .0040536      .0714548    .0873446
    (2 vs 1) Car#2   |   .0825786   .0042477      .0742532     .090904
    (2 vs 1) Car#3   |   .0790618   .0040101      .0712022    .0869214
 (2 vs 1) Public#1   |   .0066981   .0049098      -.002925    .0163212
 (2 vs 1) Public#2   |   .0053644     .00474     -.0039258    .0146547
 (2 vs 1) Public#3   |   .0077187   .0046076     -.0013121    .0167495
(2 vs 1) Bicycle#1   |  -.0088805   .0055205     -.0197005    .0019396
(2 vs 1) Bicycle#2   |  -.0084672   .0052449      -.018747    .0018126
(2 vs 1) Bicycle#3   |  -.0070729   .0054537      -.017762    .0036161
   (2 vs 1) Walk#1   |  -.0772173   .0098791       -.09658   -.0578546
   (2 vs 1) Walk#2   |  -.0794758   .0100246     -.0991236    -.059828
   (2 vs 1) Walk#3   |  -.0797076   .0100757     -.0994556   -.0599596
------------------------------------------------------------------------
```

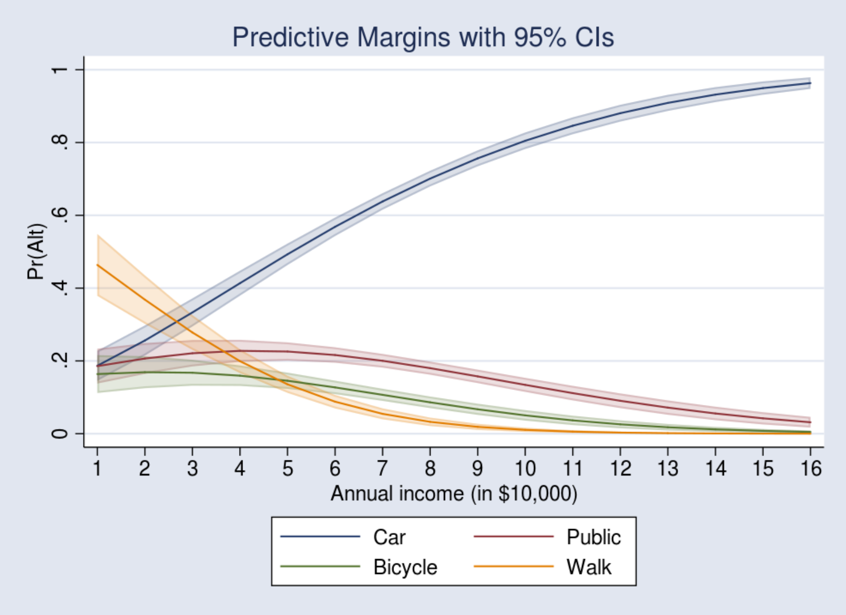
We better plot these:

```
. marginsplot
  Variables that uniquely identify margins: t _outcome
```



What are the averaged choice probabilities over the entire income range?

```
. margins, at(income=(1(1)16))
<output omitted>

. marginsplot, recast(line) ciopts(recast(rarea) color(%20))
  Variables that uniquely identify margins: income _outcome
```


Predictive Margins with 95% CIs
Pr(Alt)
0
.2
.4
.6
.8
1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Annual income (in $10,000)
Car
Public
Bicycle
Walk

Marginal predictions with alternative-specific variables

- Direct and indirect effects
- If travel costs related to cars increased by 25%, how would that affect the probability of choosing a car?
- How would that increase affect the probability of choosing any of the other transportation modes?

margins specification

```
. margins, alternative(Car)                              ///
>          at(trcost = generate(trcost))                 ///
>          at(trcost = generate(1.25*trcost))            ///
>          subpop(if t==1)
```

Applying the counterfactual

```
. webuse transport
(Transportation choice data)

. generate trcost_cf = trcost

. qui replace trcost_cf = 1.25*trcost if alt == 1

. format trcost_cf %3.2f

. list id t alt choice trcost trcost_cf in 1/12, sepby(t) noobs
```

id	t	alt	choice	trcost	trcost~f
1	1	Car	1	4.14	5.17
1	1	Public	0	4.74	4.74
1	1	Bicycle	0	2.76	2.76
1	1	Walk	0	0.92	0.92
1	2	Car	1	8.00	10.00
1	2	Public	0	3.14	3.14
1	2	Bicycle	0	2.56	2.56
1	2	Walk	0	0.64	0.64
1	3	Car	1	1.76	2.20
1	3	Public	0	2.25	2.25
1	3	Bicycle	0	0.92	0.92
1	3	Walk	0	0.58	0.58

`margins` output

```
Predictive margins                              Number of obs     =      6,000
Model VCE    : OIM                              Subpop. no. obs   =      2,000

Expression   : Pr(alt), predict()
Alternative  : Car

1._at        : trcost          = trcost

2._at        : trcost          = 1.25*trcost
```

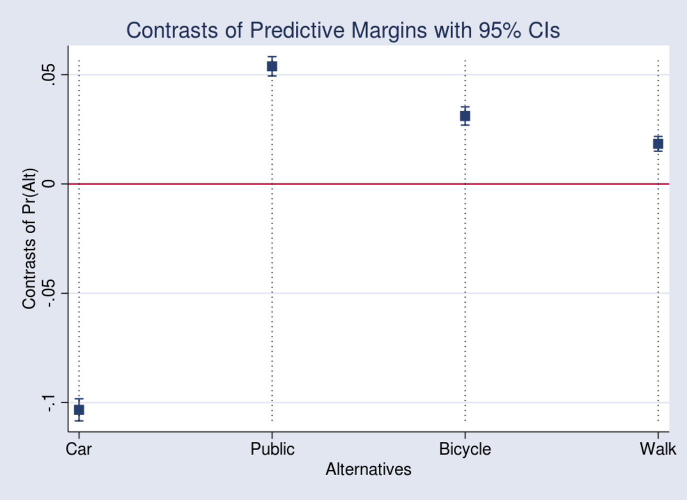
	Delta-method					
	Margin	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_outcome#_at						
Car#1	.5439062	.0113994	47.71	0.000	.5215638	.5662486
Car#2	.4405694	.0101017	43.61	0.000	.4207704	.4603683
Public#1	.2010082	.0104382	19.26	0.000	.1805497	.2214668
Public#2	.2548516	.0117988	21.60	0.000	.2317264	.2779769
Bicycle#1	.1255662	.0095539	13.14	0.000	.1068409	.1442914
Bicycle#2	.1566796	.0110237	14.21	0.000	.1350736	.1782856
Walk#1	.1295194	.0101536	12.76	0.000	.1096187	.1494201
Walk#2	.1478994	.0110109	13.43	0.000	.1263185	.1694803

Contrasts with alternative-specific variables

```
. margins, alternative(Car)                    ///
>          at(trcost = generate(trcost))       ///
>          at(trcost = generate(1.25*trcost)) ///
>          contrast(at(r) nowald)              ///
>          subpop(if t==1)

Contrasts of predictive margins                 Number of obs     =      6,000
Model VCE    : OIM                              Subpop. no. obs   =      2,000

Expression   : Pr(alt), predict()
Alternative  : Car

1._at        : trcost          = trcost

2._at        : trcost          = 1.25*trcost

------------------------------------------------------------------
                   |            Delta-method
                   |   Contrast   Std. Err.     [95% Conf. Interval]
-------------------+----------------------------------------------
     _at@_outcome  |
     (2 vs 1) Car  |  -.1033369   .0025876     -.1084084   -.0982653
  (2 vs 1) Public  |   .0538434   .0022563      .0494212    .0582656
 (2 vs 1) Bicycle  |   .0311134   .0021237      .0269511    .0352757
    (2 vs 1) Walk  |     .01838   .0017167      .0150153    .0217448
------------------------------------------------------------------
```

Plotting contrasts

```
. marginsplot, recast(dot) yline(0) plotopts(msymbol(square))
<output omitted>
```



Average marginal effects: how does the probability of choosing a car change with car travel time?

```
. margins, dydx(trtime) outcome(Car) alternative(Car)

Average marginal effects                        Number of obs     =      6,000
Model VCE    : OIM

Expression   : Pr(alt), predict()
Alternative  : Car
Outcome      : Car
dy/dx w.r.t. : trtime

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
trtime       |
       _cons |  -.1581844   .0269102    -5.88   0.000    -.2109275   -.1054414
------------------------------------------------------------------------------
```

Average marginal effects: how does the probability of choosing public transportation change with travel time related to car use?

```
. margins, dydx(trtime) outcome(Public) alternative(Car)

Average marginal effects                        Number of obs     =      6,000
Model VCE      : OIM

Expression     : Pr(alt), predict()
Alternative    : Car
Outcome        : Public
dy/dx w.r.t.   : trtime

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
trtime       |
       _cons |   .1055447   .0171745     6.15   0.000     .0718834     .139206
------------------------------------------------------------------------------
```

Average direct & indirect marginal effects

```
. margins, dydx(trtime) outcome(Car)

Average marginal effects                        Number of obs     =      6,000
Model VCE    : OIM

Expression   : Pr(alt), predict()
Outcome      : Car
dy/dx w.r.t. : trtime

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
trtime       |
         alt |
        Car  |  -.1581844   .0269102    -5.88   0.000    -.2109275   -.1054414
     Public  |   .1055447   .0171745     6.15   0.000     .0718834     .139206
    Bicycle  |   .0374872   .0073318     5.11   0.000     .0231171    .0518573
       Walk  |   .0151526   .0043034     3.52   0.000      .006718    .0235871
------------------------------------------------------------------------------
```

Discrete choice estimators in Stata 16

Stata's new **cm** commands:

- **cmclogit** (formerly asclogit)
- **cmmprobit** (formerly asmprobit)
- **cmroprobit** (formerly asroprobit)
- **cmrologit** (formerly rologit)
- **cmmixlogit** (formerly asmixlogit)
- **cmxtmixlogit** (new in Stata 16)

All **cm** commands now support **margins**

New **[CM]** manual

Other discrete choice estimators:

- nlogit, mlogit, mprobit, logit, probit, ...

Thank you!