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Efficient Programming in Stata and Mata II: Obtaining Non-Standard Distributions for a Cointegration Test via Simulation

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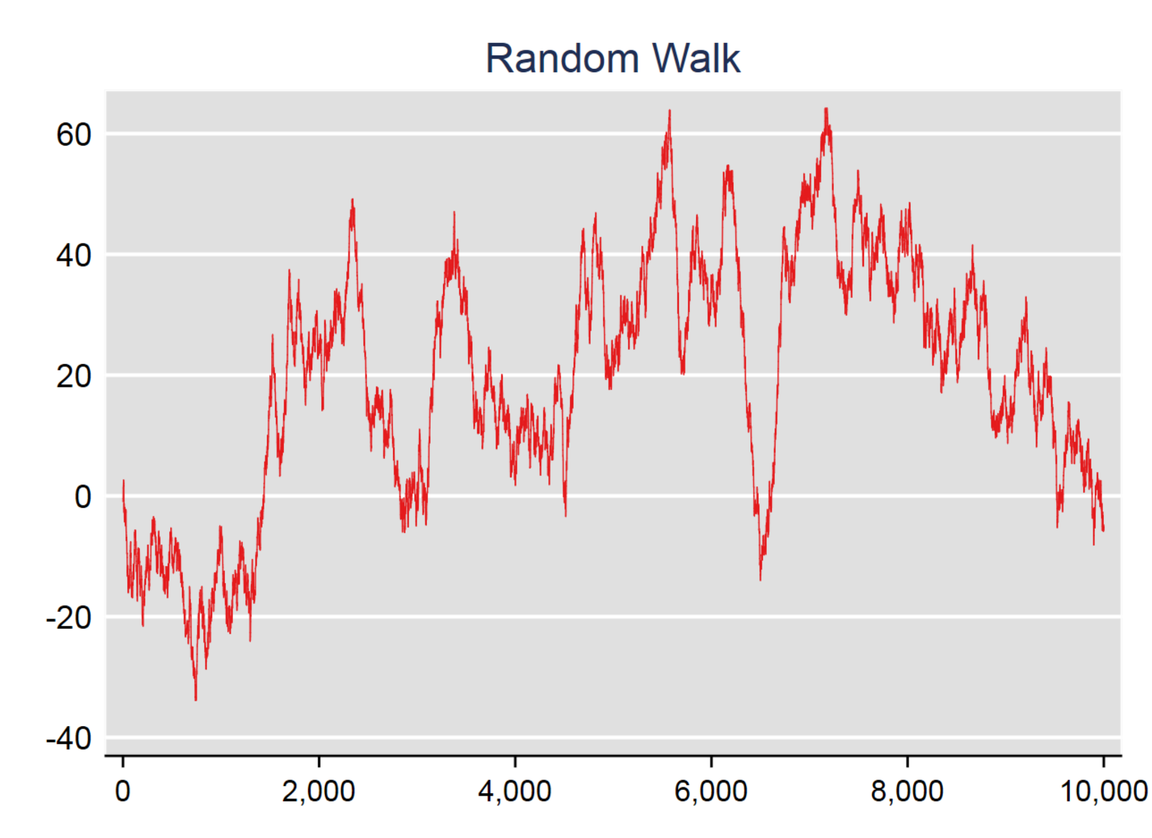
Last Year's Talk

- efficient coding strategies:
  - use common sense
  - use your knowledge of your software (Stata, of course!)
  - use your knowledge of matrix algebra
- case study: the -ardl- estimation command
  - last year: optimal lag selection
  - this talk: simulation of finite sample distributions

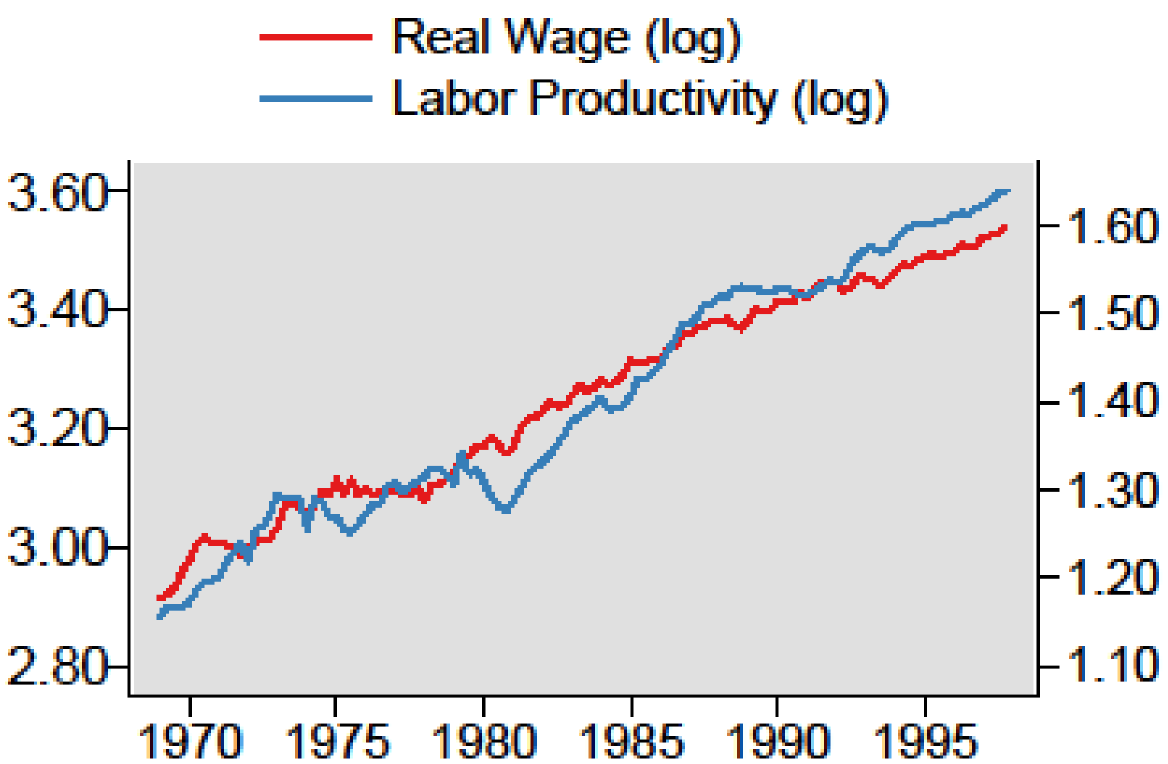
Stationarity vs. Non-Stationarity

- fundamental distinction in time series analysis (TSA)
- mostly about time series with a unit root: I(0) vs. I(1)
- non-stationary TS behave fundamentally different



Multiple Time Series Analysis

Long-run relationship: Some time series are bound together due to equilibrium forces even though the individual time series might move considerably.



Data source: Pesaran, Shin, and Smith (2001).

The ARDL Model and the Bounds Test

- ARDL$(p, q, \ldots, q)$ model:

$$y_t = c_0 + c_1 t + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=0}^{q} \boldsymbol{\beta}_i' \mathbf{x}_{t-i} + \varepsilon_t,$$

  with $\mathbf{x}_t$ a $K \times 1$ vector.

- Reparameterization in error-correction (EC) form:

$$\begin{aligned} \Delta y_t = c_0 + c_1 t - \alpha(y_{t-1} - \boldsymbol{\theta}\mathbf{x}_{t-1}) \\ + \sum_{i=1}^{p-1} \psi_{yi} \Delta y_{t-i} + \boldsymbol{\omega}' \Delta \mathbf{x}_t + \sum_{i=1}^{q-1} \boldsymbol{\psi}_{xi}' \Delta \mathbf{x}_{t-i} + \varepsilon_t, \end{aligned}$$

- Pesaran / Shin / Smith (2001) (PSS) derive the asymptotic coefficient distributions under the opposing assumptions of stationary vs. non-stationary regressors, the basis for their bounds test for a levels relationship.
- They provide critical values (CV) tables obtained via simulation.

ARDL Toy Model Estimation

```
. ardl w prod union ur , ec maxlag(6) dots trend(qtime) restricted vsquish
```

```
Optimal lag selection, % complete:
----+--20%--+--40%--+--60%--+--80%--+--100%
.............................................
BIC optimized over 2058 lag combinations

ARDL(2,0,2,0) regression

Sample: 1971q3 - 1997q4                      Number of obs     =        106
                                             R-squared         =     0.2637
                                             Adj R-squared     =     0.2029
Log likelihood =   330.70424                  Root MSE          =     0.0112
```

D.w	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ADJ						
w						
L1.	-0.240	0.063	-3.827	0.000	-0.365	-0.116
LR						
prod	0.416	0.208	1.998	0.049	0.003	0.829
union	-0.210	0.235	-0.893	0.374	-0.676	0.256
ur	0.039	0.017	2.382	0.019	0.007	0.072
qtime	0.003	0.001	2.962	0.004	0.001	0.005
SR						
w						
LD.	-0.203	0.094	-2.161	0.033	-0.389	-0.017
union						
D1.	0.058	0.597	0.097	0.923	-1.128	1.243
LD.	-1.535	0.596	-2.574	0.012	-2.719	-0.351
_cons	0.527	0.153	3.454	0.001	0.224	0.830

ARDL Toy Model Estimation

```
. estat btest

note:  estat btest has been superseded by  estat ectest
       as the prime procedure to test for a levels relationship.
       (click to run)

Pesaran/Shin/Smith (2001) ARDL Bounds Test
H0: no levels relationship                F =      3.863
                                          t =     -3.827

Critical Values (0.1-0.01),  F-statistic, Case 4
```

	[I_0] L_1	[I_1] L_1	[I_0] L_05	[I_1] L_05	[I_0] L_025	[I_1] L_025	[I_0] L_01	[I_1] L_01
k_3	2.97	3.74	3.38	4.23	3.80	4.68	4.30	5.23

```
accept if F < critical value for I(0) regressors
reject if F > critical value for I(1) regressors

Critical Values (0.1-0.01),  t-statistic, Case 4
```

	[I_0] L_1	[I_1] L_1	[I_0] L_05	[I_1] L_05	[I_0] L_025	[I_1] L_025	[I_0] L_01	[I_1] L_01
k_3	-3.13	-3.84	-3.41	-4.16	-3.65	-4.42	-3.96	-4.73

```
accept if t > critical value for I(0) regressors
reject if t < critical value for I(1) regressors

k: # of non-deterministic regressors in long-run relationship
Critical values from Pesaran/Shin/Smith (2001)
```

Simulation Project Outline

- PSS bounds test very popular, but CV tables only cover a limited number of cases

$\Rightarrow$computational / simulation project:

1. simulate distributions for all combinations of c, l, k, q, T
2. store calculated statistics / distributions
3. run response surface regressions (RSR), where the depvars are distributional quantiles
4. implement and distribute an ARDL postestimation feature that displays RSR-based CVs / p-values

Response Surface Regressions (RSR)

- idea:
  for each c, l, k: regress quantile of distr ~ g(T,q)
  We implement variations thereof.
- use predicted values for a particular T, q as CVs in applied work
- introduced by MacKinnon (1991, 1994, 1996)
- Other Stata commands, e.g.
  - `ersur` (Baum/Otero 2017)
  - `kssur`, `ksur` (Otero/Smith 2017)

The Computational Task

Similar to PSS, the DGP is

$$y_t = y_{t-1} + \epsilon_{yt}$$
$$\boldsymbol{x}_t = \boldsymbol{P}\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_{xt}$$

for $t = 1, 2, \ldots, T + 50$ (including 50 burn-in periods), and where

$$(y_0, \boldsymbol{x}'_0)' = \mathbf{0}, \epsilon_t \sim N(0, I_{k+1})$$

and

$$\boldsymbol{P} = 0 \;\; (I(0) \text{ regressors})$$
$$\boldsymbol{P} = \boldsymbol{I_k} \;\; (I(1) \text{ regressors})$$

The Computational Task

project size:

Symbol	Meaning	Values	# values
c	deterministics cases	1, 2, …, 5 (F); 1, 3, 5 (t)	8
I	integration order	0, 1	2
k	# of regressors	0, 1, …, 10	11
q	# of lags	0, 1, …, 4, 6, 8, 12	8
T	sample size	20, 22, …, 400, 500, 1000	18
r	# replications		100,000
m	# meta replications		100

Results in ~160,000,000,000 stats

Implies several months of computation (“Oh my!”)

Implies ~600GB disk space (“Oh dear!”)

Reducing Data Size

Idea, omitting details: i) round to 3 decimal places, ii) store tabulation

cIkr_group	stat	stat3	tmpdif	statdif	mult
		(...)			
2310	2.345145	2345	2345	-28655	1
2310	2.761234	2761	416	-30584	2
2310	2.761411	2761	0	-31000	2
2310	2.761932	2762	1	-30999	4
2310	2.761944	2762	0	-31000	4
2310	2.761948	2762	0	-31000	4
2310	2.762331	2762	0	-31000	4
		(...)			
2310	10.85794	10858	100	-20142	1
2310	10.99043	10990	132	-20010	1
		(...)			
2311	2.118192	2118	2118	-28882	1
2311	2.239101	2239	121	-30879	1
2311	2.241233	2241	2	-30998	1
2311	2.241708	2242	1	-30999	2
2311	2.241744	2242	0	-31000	2

statdif	mult	first
	(...)	
-28655	1	1
-30584	2	0
-30999	4	0
	(...)	
-20142	1	0
-20010	1	0
	(...)	
-28882	1	1
-30879	1	0
-30998	1	0
-30999	2	0
	(...)	

Reducing Data Size

- Achieved size reduction: over 90%
- After -zipfile-, data occupy 10GB
- Solving this was crucial as now computational steps can be separated.
- But: Takes up 20% computation time
- . help data types, . help compress
- Data transformations and data types
  - Years, age in years
- Wish list item: if Mata supported all numeric types of Stata
  - Could implement more complex storage ideas in Mata and its mmat files
  - Could write (de-)compression in terms of a class

Simulation & Multiple Stata Instances

```
// --------------- beg dosim.do -----------
args inputarg
if "`inputarg'"!="" {
     confirm integer number `inputarg'
     // (...) potentially some setup statements here
     //   like startup scripts that set matsize, maxvar, etc.
}

set rng mt64s
local laglist 1 2 3 4 6 8 12
if "`inputarg'"!="" local laglist `inputarg'

foreach lag of local laglist {
     set seed 123456
     set rngstream `lag'
     mata : dosim(`lag')
}
// --------------- end dosim.do -----------
```

Simulation & Multiple Stata Instances

Windows / DOS batch file to fire up Stata instances

```
rem -------- beg multiinstance.bat ---------
for %%c in (1 2 3 4 6 8 12) do (
    copy dosim.do dosim_multiinst_ %%c.do /Y
    start "sim%%c" /D "PROJECTPATH" "STATAPATH\StataMP-64.exe" ^
          /e do dosim_multiinst_ %%c.do   %%c
)
rem -------- end multiinstance.bat ---------
```

Simulation & Multiple Stata Instances

- Multiple instances
  - help entry: [GSW] B.5 Stata batch mode
  - careful with any kind of file saving operations, e.g. logs
  - batch file to kill processes?
- RNG streams
  - new in Stata 15
  - . help set rngstream

Mata Code Optimization

- necessary to examine each expression for speed improvements
- examples of smaller improvements
  - row extraction instead of column extraction
  - inner vector product: sum of squares vs. cross() vs. multiplication
- most important code features
  - pre-calculation of cross-products, accessing through indexing
  - use pointer variables to facilitate storing numbers
  - experiment with inverters / solvers
- not pursued: C/C++
  - Stata/Mata has a MUCH better convenience-speed trade-off
  - Stata/Mata great in other respects too: version control

Mata Code Optimization

Usage of pointer variables

```
/*
Structure of returned results:

   pstatlidx                      pFkI , ptkI                     unnamed but referenced matrices
(returned matrix)

       | lag-idx                    | I                                     |  c
       | 0 1 ...                    | 0  1                                  |  1 (2) 3 (4) 5
---------------- p point to:   -----------   p point to:   -----------------------------
stat=F | p p ...                k 0 | p  p                 statdata 1    |  #  #  #  #  #
stat=t | p p ...                  1 | p  p                          2    |  #  #  #  #  #
                                 ...| ...                          ...   |  ...
                                kmax| p  p                         reps  |  #  #  #  #  #
*/
```

Mata Code Optimization

Loop structure

```
for [T] {
  for [lags] {
    // - calculate deterministics for all cases (X1)
    //   cross products thereof (XX11)
    for [reps] {
      // - random draws
      // - calculation of levels variables (X2)
      //   cross products thereof (XX22)
      // - calculation of first-difference variables (X3))
      //   cross products thereof (XX33)
      // - also calculate cross products among y, X1, X2, X3 variables (XX12, ...)
      for [cases] {
        for [k] {
          // - check degree-of-freedom requirement
          for [I-order] {
            // - select / assemble matrices from parts for (un-)restricted models (F-test)
            //   calculate (un-)restricted SSR (solver: lusolve())
} } } } } }
```

Project Results: ARDL Toy Example

```
. quietly ardl w prod union ur , ec maxlag(6) dots trend(qtime) restricted  vsquish

. estat ectest

Pesaran, Shin, and Smith (2001) bounds test

H0: no level relationship                                      F =       3.863
Case 4                                                             t =      -3.827

Finite sample ( 3 variables,  106 observations,  3 short-run coefficients)

Kripfganz and Schneider (2018) critical values and approximate p-values

   | 10%               | 5%                | 1%                | p-value
   |    I(0)     I(1)  |    I(0)     I(1)  |    I(0)     I(1)  |    I(0)     I(1)
---+-------------------+-------------------+-------------------+------------------
 F |   3.011    3.829  |   3.486    4.373  |   4.530    5.548  |   0.028    0.096
 t |  -3.116   -3.829  |  -3.419   -4.162  |  -4.016   -4.803  |   0.017    0.100

do not reject H0 if
    both F and t are closer to zero than critical values for I(0) variables
      (if p-values > desired level for I(0) variables)
reject H0 if
    both F and t are more extreme than critical values for I(1) variables
      (if p-values < desired level for I(1) variables)
```

Project Results: ARDL Toy Example

PSS values

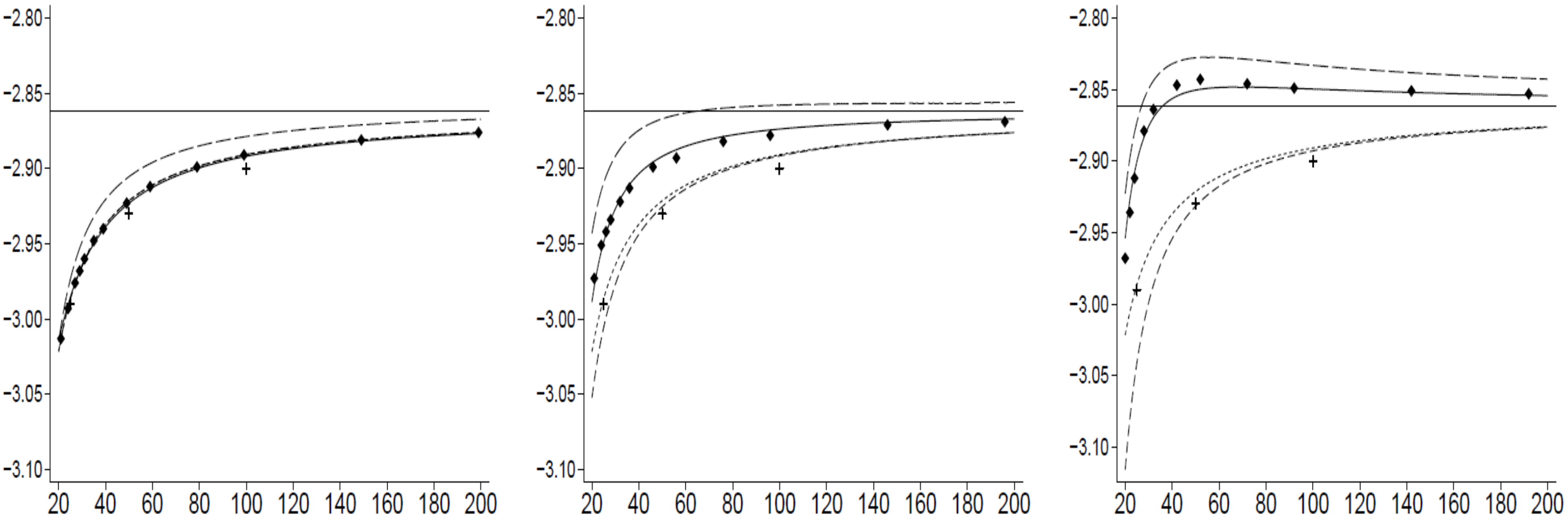
	[I_0] L_1	[I_1] L_1	[I_0] L_05	[I_1] L_05	[I_0] L_025	[I_1] L_025	[I_0] L_01	[I_1] L_01
k_3	2.97	3.74	3.38	4.23	[redacted]	[redacted]	4.30	5.23
k_3	-3.13	-3.84	-3.41	-4.16	[redacted]	[redacted]	-3.96	-4.73

Response surface regression based values

	10% I(0)	10% I(1)	5% I(0)	5% I(1)	1% I(0)	1% I(1)	p-value I(0)	p-value I(1)
F	3.011	3.829	3.486	4.373	4.530	5.548	0.028	0.096
t	-3.116	-3.829	-3.419	-4.162	-4.016	-4.803	0.017	0.100

Project Results: E.g. Dickey-Fuller

Besides Cheung and Lai (1995), the existing literature largely neglects the lag-order dependence of the finite-sample critical values (t-statistic, k=0, case (iii), α=5%)



(a) $q = 1$ (b) $q = 4$ (c) $q = 8$

Recap

- Non-stationary time series and cointegration, ardl and the PSS bounds test
- Simulation project: Improve CV tables for bounds test
  - Storing large quantity of numbers
  - Computation time
    - Multiple Stata instances
    - Code improvements within Mata

Thank you!

Questions? Comments?

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See also: the ardl discussion thread on the Stata Forum

```
. net install ardl, from(http://www.kripfganz.de/stata/)
```

Paper available at http://www.kripfganz.de/research/index.html

References

Cheung, Y.-W. and K. S. Lai (1995a). Lag order and critical values of the augmented Dickey-Fuller test. Journal of Business & Economic Statistics 13 (3), 277-280.

Kripfganz, S. and D. C. Schneider (2018). Response Surface Regressions for Critical Value Bounds and Approximate p-values in Equilibrium Correction Models. Manuscript, University of Exeter and Max Planck Institute for Demographic Research. Available at www.kripfganz.de/research/Kripfganz_Schneider_ec.html.

MacKinnon, J. G. (1991). Critical values for cointegration tests. In R. F. Engle and C. W. J. Granger (Eds.), Long-Run Economic Relationships: Readings in Cointegration, Chapter 13, pp. 267-276. Oxford: Oxford University Press.

MacKinnon, J. G. (1994). Approximate asymptotic distribution functions for unit-root and cointegration tests. Journal of Business & Economic Statistics 12 (2), 167-176.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. Journal of Applied Econometrics 11 (6), 601-618.

Otero, J. and C. F. Baum (2017). Response surface models for the Elliott, Rothenberg, and Stock unit-root test. Stata Journal 17 (4), 985-1002.

Otero, J. and J. Smith (2017). Response surface models for OLS and GLS detrending-based unit-root tests in nonlinear ESTAR models. Stata Journal 17 (3), 704-722.

Pesaran, M. H., Y. Shin, and R. J. Smith (2001). Bounds testing approaches to the analysis of level relationships. Journal of Applied Econometrics 16 (3), 289-326.