动态随机一般均衡分析 (Dynamic Stochastic General Equilibrium Model)

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内容

DSGE

DSGE 是一种多元时间序列分析方法，用于宏观经济政策分析和预测。

在经济运行系统中，家庭、厂商、央行、政府各自决策。家庭最大化效用，决定消费、投资和劳动力的供给。 企业最大化利润，在已有技术水平下利用资本和劳动力决定产量。

均衡模型的基本构成：价格系统（工资、利率和价格）、商品和要素（产量、消费、投资、劳动力、资本）和均衡条件（供给=需求）。

当代宏观经济学的两大流派：实际经济周期理论（RBC）、新凯恩斯理论（NK）。

RBC 理论认为经济扩张和衰退都是经济对技术状态的自然而有效的响应，不需要财政或货币政策来改变宏观经济条件。

新凯恩斯理论则认为经济波动源于市场失灵，宏观政策有必要干预市场运行。

线性 DSGE

在 DSGE 中，个体行为由非线性动态系统方程来描述，决策规则由动态随机优化方法来解决。DSGE 模型假定理性预期，即个体对未来数值的期望是正确的。

DSGE 模型包含三类变量：控制变量（control），状态变量（state），冲击（shock）。

DSGE 包含两类方程：控制方程描述控制变量与状态变量的关系，状态方程描述状态变量的动态变化方程。

$$\begin{aligned} A_0 y_t &= A_1 \mathbb{E}_t y_{t+1} + A_2 y_t + A_3 x_t \\ B_0 x_{t+1} &= B_1 \mathbb{E}_t y_{t+1} + B_2 y_t + B_3 x_t + C_3 \epsilon_{t+1} \end{aligned}$$

其中，x_t为状态变量，y_t为控制变量（包括可观测变量y_{1t}和不可观测变量y_{2t}）。A_0, B_0是对角矩阵，A_2的对角线元素为 0。

控制变量：控制变量是内生的，可观测或不可观测，当期数值由系统方程决定。

状态变量：状态变量总是不可观测的，在给定时期是固定或外生的。

线性 DSGE

将上述模型转化为状态空间的形式，其中控制变量为状态变量的函数，状态变量为 AR(1)过程：

$$\begin{aligned} y_t &= G(\theta) x_t \\ x_{t+1} &= H(\theta) x_t + M(\theta) \epsilon_{t+1} \end{aligned}$$

其中，G叫做政策矩阵，H叫做状态转移矩阵。

y_{1t}的个数必须与带有冲击项的状态方程的个数相同。

线性 DSGE 和非线性 DSGE

非线性 DSGE（nonlinear）: 关于变量和参数都是非线性的。

线性 DSGE（Linear, linearized）: 方程关于变量是线性的，但关于参数仍有可能是否非线性的。

对于非线性 DSGE，可以首先线性化，然后采用 `dsge` 进行估计；或者采用 `dsgenl`，Stata 做线性近似进行估计。

解 DSGE 模型是指将内生变量写为外生变量的函数，即将控制变量写为状态变量的函数。DSGE 的解是将模型表达为状态空间的形式，对于估计和分析是至关重要的一个环节。

利用 Kalman 滤子计算线性 DSGE 模型的似然函数。

例：非线性 DSGE

$$
\begin{aligned}
R_t &= \alpha \frac{Y_t}{K_t} \\
1 &= \beta \left[\frac{C_t}{C_{t+1}} (1 + R_{t+1} - \delta) \right] \\
Y_t &= Z_t K_t^{\alpha} \\
K_{t+1} &= Y_t - C_t + (1 - \delta) K_t \\
\ln(z_{t+1}) &= \rho \ln(z_t) + e_{t+1}
\end{aligned}
$$

```
. dsgenl (r = {alpha}*y/k) ///
(1 = {beta}*(c/F.c)*(1 + F.r - {delta})) ///
(y = z*k^({alpha})) ///
(F.k = y - c + (1-{delta})*k) ///
(ln(F.z) = {rho}*ln(z)) , ///
observed(y) unobserved(c r) exostate(z) endostate(k)
```

例：线性 DSGE

$$
\begin{aligned}
& \text{(output-gap Euler equation)} \\
x_t &= \mathbb{E}_t(x_{t+1}) - (r_t - \mathbb{E}_t \Pi_{t+1} - g_t), \\
& \text{(New Keynesian Phillips Curve)} \\
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t, \\
r_t &= \frac{1}{\beta} \pi_t + u_t, \text{(Taylor rule)} \\
u_{t+1} &= \rho_u u_t + \epsilon_{t+1} \\
g_{t+1} &= \rho_g g_t + \xi_{t+1}
\end{aligned}
$$

```
dsge (x = F.x - (r - F.p - g), unobserved) ///
(p = {beta}*F.p + {kappa}*x) ///
(r = (1/{beta})*p + u) ///
(F.u = {rhou}*u, state) ///
(F.g = {rhog}*g, state), nolog
```

Stata

时间序列符号：F, L。参数用{}括起来。每个方程用()括起来。

默认为控制方程。如果是状态方程，则加入 state 选项，状态方程的因变量必须为 F.的形式。

控制方程默认为控制变量是可观测的。如果不可观测，加入 unobserved 选项。

状态方程默认为含有冲击项。如果没有冲击项，加入 noshock 选项。

或者用选项 observed() unobserved() exostate() endostate()指明各个变量的属性。

- observed(): 可观测的控制变量；unobserved(): 不可观测的控制变量。
- exostate(): 外生状态变量（即带有冲击项的状态变量）；endostate(): 内生状态变量（即不带有冲击项的状态变量）

外生状态变量的个数必须与可观测的控制变量的个数相同。

Stata

允许方程表示为其它等价的形式。比如，(r = {alpha}*y/k)也可以写为(r*k = {alpha}*y)或(r*k/y = {alpha})。

变量必须是零均值的弱平稳过程。

结构模型的形式约束

控制方程只能包括三类变量：$(E(y_{t+1}), y_t, x_t)$， 状态方程只能包含四类变量$(E(y_{t+1}), y_t, x_t, \epsilon_t)$。具体包括的情形：

- 控制变量的方程不能含有冲击项。
- 所有方程不能包含控制变量的滞后项$(y_{t-1}, y_{t-2}, \ldots)$，也不能包含状态变量的滞后项$(x_{t-1}, x_{t-2}, \ldots)$。
- 所有方程不能包含控制变量的高阶超前项的预期$E(y_{t+2}, E(y_{t+3}), \ldots)$。

所有的形式约束都可以通过增加状态变量或者新的控制变量来解决。比如，

- 控制变量的方程含有冲击项，可以将冲击项作为新的状态变量来引入。
- 状态变量的方程含有状态变量的滞后项，可以将滞后项作为新的状态变量来引入。
- 控制变量方程含有控制变量的高阶超前项，将其高阶超前项作为新的控制变量。

DSGE 理论将所有观测变量都视作内生的。如果模型中存在外生控制变量，那么引入外生状态变量，令其等于外生控制变量。

对数线性化

均衡状态：

$$\mathbb{E}_t x_{t+1} = x_t = x_{t-1} = x_{ss}$$

Uhlig(1999)： log-linearization

对数线性化的常用公式：定义$\tilde{X}_t = \log X_t - \log X_{ss}$,

$$\begin{aligned} X_t &\approx X_{ss} e^{\tilde{X}_t} \\ e^{(\tilde{X}_t + a\tilde{Y}_t)} &\approx 1 + \tilde{X}_t + a\tilde{Y}_t \\ \tilde{X}_t \tilde{Y}_t &\approx 0 \\ \mathbb{E}_t\left[a e^{\tilde{X}_{t+1}}\right] &\approx a + a\mathbb{E}_t[\tilde{X}_{t+1}]. \end{aligned}$$

例：新凯恩斯模型

Ref: Celso Jose (2016, P42, Table 2.1)

定义	方程	均衡
labor supply	$C_t^\sigma L_t^\psi = \frac{W_t}{P_t}$	$C_{ss}^\sigma L_{ss}^\psi = \frac{W_{ss}}{P_{ss}}$
Euler equation of consumption	$\left(\frac{\mathbb{E}_t C_{t+1}}{C_t}\right)^\sigma = \beta\left[(1-\delta) + \mathbb{E}_t\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$	$1 = \beta\left(1 - \delta + \frac{R_{ss}}{P_{ss}}\right)$
law of motion of capital	$K_{t+1} = (1-\delta)K_t + I_t$	$I_{ss} = \delta K_{ss}$
production function	$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$	$Y_{ss} = K_{ss}^\alpha L_{ss}^{1-\alpha}$
demand for capital	$\frac{K_t}{P_t} = \alpha\frac{Y_t}{R_t}$	$\frac{K_{ss}}{P_{ss}} = \alpha\frac{Y_{ss}}{R_{ss}}$
demand for labor	$\frac{L_t}{P_t} = (1-\alpha)\frac{Y_t}{W_t}$	$\frac{L_{ss}}{P_{ss}} = (1-\alpha)\frac{Y_{ss}}{W_{ss}}$

定义	方程	均衡
price level	$P_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$	$P_{ss} = \left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_{ss}}{\alpha}\right)^{\alpha}$
equilibrium	$Y_t = C_t + I_t$	$Y_{ss} = C_{ss} + I_{ss}$
productivity shock	$\ln(A_t) = (1-\rho_A)\ln(A_{ss}) + \rho_A \ln(A_{t-1}) + \epsilon_t$	$A_{ss} = 1$

对数线性化

例：劳动力供给: $C_t^{\sigma} L_t^{\psi} = \frac{W_t}{P_t}$。

根据$X_t \approx X_{ss} e^{\tilde{X}_t}$, 上述方程近似为

$$C_{ss}^{\sigma} L_{ss}^{\psi} e^{\sigma \tilde{C}_t + \psi \tilde{L}_t} = \frac{W_{ss}}{P_{ss}} e^{\tilde{W}_t - \tilde{P}_t}.$$

利用$e^{(\tilde{X}_t + a\tilde{Y}_t)} \approx 1 + \tilde{X}_t + a\tilde{Y}_t$, 继续化简为

$$C_{ss}^{\sigma} L_{ss}^{\psi}\left(1 + \sigma \tilde{C}_t + \psi \tilde{L}_t\right) = \frac{W_{ss}}{P_{ss}}\left(1 + \tilde{W}_t - \tilde{P}_t\right)$$

在均衡状态下，$C_{ss}^{\sigma} L_{ss}^{\psi} = \frac{W_{ss}}{P_{ss}}$，可得：

$$\sigma \tilde{C}_t + \psi \tilde{L}_t = \tilde{W}_t - \tilde{P}_t.$$

对数线性化

例：消费的欧拉方程: $\frac{1}{\beta}\mathbb{E}_t\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = (1-\delta) + \mathbb{E}_t\left(\frac{R_{t+1}}{P_{t+1}}\right)$

根据$X_t \approx X_{ss} e^{\tilde{X}_t}$, 上述方程近似为

$$\frac{1}{\beta} e^{\sigma \mathbb{E}_t \tilde{C}_{t+1} - \sigma \tilde{C}_t} = (1-\delta) + \frac{R_{ss}}{P_{ss}} e^{\mathbb{E}_t(\tilde{R}_{t+1} - \tilde{P}_{t+1})}$$

利用$e^{(\tilde{X}_t + a\tilde{Y}_t)} \approx 1 + \tilde{X}_t + a\tilde{Y}_t$, 继续化简为

$$\frac{1}{\beta}\left[1+\sigma\mathbb{E}_t\tilde{C}_{t+1}-\sigma\tilde{C}_t\right]=(1-\delta)+\frac{R_{ss}}{P_{ss}}\left[1+\mathbb{E}_t\left(\tilde{R}_{t+1}-\tilde{P}_{t+1}\right)\right]$$

在均衡状态下，$\frac{1}{\beta}=\frac{R_{ss}}{P_{ss}}+(1-\delta)$，可得：

$$\frac{1}{\beta}\left[\sigma\mathbb{E}_t\tilde{C}_{t+1}-\sigma\tilde{C}_t\right]=\frac{R_{ss}}{P_{ss}}\left[\mathbb{E}_t\left(\tilde{R}_{t+1}-\tilde{P}_{t+1}\right)\right]$$

对数线性化

练习：将下面的方程做对数线性化。

1. 资本需求方程：

$$\frac{R_t}{P_t}=\alpha\frac{Y_t}{K_t}$$

2. 生产函数方程: $Y_t=A_tK_t^{\alpha}L_t^{1-\alpha}$
3. 资本积累方程: $K_{t+1}=(1-\delta)K_t+I_t$
4. 均衡条件：$Y_t=C_t+I_t$
5. 技术冲击：

$$\ln(A_t)=(1-\rho_A)\ln(A_{ss})+\rho_A\ln(A_{t-1})+\epsilon_t$$

Key:

1. $\tilde{R}_t-\tilde{P}_t=\tilde{Y}_t-\tilde{K}_t$
2. $\tilde{Y}_t=\tilde{A}_t+\alpha\tilde{K}_t+(1-\alpha)\tilde{L}_t$
3. $\tilde{K}_{t+1}=(1-\delta)\tilde{K}_t+\delta\tilde{I}_t$
4. $Y_{ss}\tilde{Y}_t=C_{ss}\tilde{C}_t+I_{ss}\tilde{I}_t$
5. $\tilde{A}_t=\rho_A\tilde{A}_{t-1}+\epsilon_t$

对数线性化：新古典模型

King and Rebelo (1999); Stata DSGE Intro 3b.

C_t: 消费; R_t： 利率； H_t: 劳动力供给； W_t: 工资； X_t: 投资; G_t: 政府支出。K_t: 资本投入； Z_t: 生产率。

$$\begin{aligned}
\frac{1}{C_t} &= \beta \mathbb{E}_t\left[\frac{1}{C_{t+1}(1+R_{t+1}-\delta)}\right], \text{consumption} \\
H_t^{\delta} &= \frac{W_t}{C_t}, \text{labor supply} \\
Y_t &= C_t + X_t + G_t, \text{national accounting identity} \\
Y_t &= K_t^{\alpha}(Z_t H_t)^{1-\alpha} \\
W_t &= (1-\alpha)\frac{Y_t}{H_t}, \text{labor demand} \\
R_t &= \alpha\frac{Y_t}{K_t}, \text{capital demand} \\
K_{t+1} &= (1-\delta)K_t + X_t, \text{capital accumulation}
\end{aligned}$$

线性化模型：

$$\begin{aligned}
c_t &= \mathbb{E}_t c_{t+1} - (1-\beta+\beta\delta)\mathbb{E}_t r_{t+1} \\
\eta h_t &= w_t - c_t \\
\phi_1 x_t &= y_t - \phi_2 c_t - g_t \\
y_t &= (1-\alpha)(z_t + h_t) + \alpha k_t \\
w_t &= y_t - h_t \\
r_t &= y_t - k_t \\
k_{t+1} &= \delta x_t + (1-\delta)k_t \\
z_{t+1} &= \rho_z z_t + \epsilon_t \\
g_{t+1} &= \rho_g g_t + \xi_{t+1}
\end{aligned}$$

6 个控制变量，3 个状态变量($k_{t+1}, z_{t+1}, g_{t+1}$)。

$(1-\alpha)$为劳动力份额，δ为资本折旧率，η为劳动力供给曲线的斜率，ϕ_1和ϕ_2为国民收入中投资份额和消费份额，ρ_z和ρ_g为状态变量的自回归系数。

对数线性化：新古典模型

```
. use usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. dsge (c = F.c - (1-{beta}+{beta}*{delta})*F.r, unobserved) ///
>  ({eta}*h = w - c, unobserved) ///
>  ({phi1}*x = y - {phi2}*c - g, unobserved) ///
>  (y = (1-{alpha})*(z+h) + {alpha}*k) ///
>  (w = y - h, unobserved) ///
```

```
> (r = y - k, unobserved) ///
> (F.k = {delta}*x+ (1-{delta})*k, state noshock) ///
> (F.z = {rhoz}*z, state) ///
> (F.g = {rhog}*g, state), ///
> from(beta=0.96 eta=1 alpha=0.3 delta=0.025 phi1=0.2 phi2=0.6 rhoz=0.8 rhog=0.3)
  ///
> noidencheck solve nolog

DSGE model

Sample: 1955q1 thru 2015q4                               Number of obs = 244
Log likelihood = -1957.0261
------------------------------------------------------------------------------
           y | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
/structural  |
        beta |        .96          .        .       .            .           .
       delta |       .025          .        .       .            .           .
         eta |          1          .        .       .            .           .
        phi1 |         .2          .        .       .            .           .
        phi2 |         .6          .        .       .            .           .
       alpha |         .3          .        .       .            .           .
        rhoz |         .8          .        .       .            .           .
        rhog |         .3          .        .       .            .           .
-------------+----------------------------------------------------------------
     sd(e.z) |          1          .                             .           .
     sd(e.g) |          1          .                             .           .
------------------------------------------------------------------------------
Note: Skipped identification check.
Note: Model solved at specified parameters; maximization options ignored.
```

例

Woolford (2004, chap 4):

$$
\begin{aligned}
\frac{1}{Y_t} &= \beta \mathbb{E}_t\left(\frac{1}{Y_{t+1}}\frac{R_t}{\Pi_{t+1}}\right) &\rightarrow\quad & 1=\beta \mathbb{E}_t\left(\frac{X_t}{X_{t+1}}\frac{1}{G_t}\frac{R_t}{\Pi_{t+1}}\right)\\
(\Pi_t-\Pi)+\frac{1}{\phi} &= \phi\frac{Y_t}{Z_t}+\beta \mathbb{E}_t(\Pi_{t+1}-\Pi) &\rightarrow\quad & (\Pi_t-\Pi)+\frac{1}{\phi}=\phi X_t+\beta \mathbb{E}_t(\Pi_{t+1}-\Pi)\\
\frac{R_t}{R} &= & & \left(\frac{\Pi_t}{\Pi}\right)^{1/\beta} U_t\\
\ln(G_{t+1}) &= & & \rho_g \ln(G_t)+\xi_{t+1},\\
\ln(U_{t+1}) &= & & \rho_u \ln(U_t)+e_{t+1}
\end{aligned}
$$

定义$X_t=Y_t/Z_t$， $G_t=Z_{t+1}/Z_t$.

Y_t: 产出(output), Π_t: 通胀率(inflation), R_t: 名义利率(nominal interest rate); β: 折现因子(discount factor, willingness to delay consumption); Z_t: 产出的自然水平 (natural level of output). Π: 通胀率均衡水平 (steady state of inflation); U_t为除了通胀以外其它影响利率变化的因素。

例

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left(\frac{X_t}{X_{t+1}} \frac{1}{G_t} \frac{R_t}{\Pi_{t+1}} \right) \\
(\Pi_t - \Pi) + \frac{1}{\phi} &= \phi X_t + \beta \mathbb{E}_t (\Pi_{t+1} - \Pi) \\
\frac{R_t}{R} &= \left(\frac{\Pi_t}{\Pi} \right)^{1/\beta} U_t \\
\ln(G_{t+1}) &= \rho_g \ln(G_t) + \xi_{t+1}, \\
\ln(U_{t+1}) &= \rho_u \ln(U_t) + e_{t+1}
\end{aligned}
$$

均衡状态下，$G_t = U_t = 1, R = \Pi/\beta$，定义$x = X_t/\Pi$，$r_t = R_t/\Pi$，$p_t = \Pi_t/\Pi$。

```
. use rates2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-02-10)

. dsgenl (1 = {beta}*(x/F.x)*(1/g)*(r/F.p) ) ///
>   (1/{phi} + (p-1) = {phi}*x + {beta}*(F.p-1)) ///
>   ({beta}*r = p^(1/{beta})*u) ///
>   (ln(F.u) = {rhou}*ln(u)) ///
>   (ln(F.g) = {rhog}*ln(g)), ///
>   exostate(u g) observed(p r) unobserved(x) nolog
Solving at initial parameter vector ...
Checking identification ...

First-order DSGE model

Sample: 1954q3 thru 2016q4                          Number of obs = 250
Log likelihood = -768.09383
```

	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
/structural						
beta	.5112889	.0757723	6.75	0.000	.3627778	.6597999
phi	5.895129	1.652261	3.57	0.000	2.656756	9.133502
rhou	.6989168	.0449139	15.56	0.000	.6108871	.7869465
rhog	.9556408	.0181345	52.70	0.000	.9200978	.9911838

```
             sd(e.u)   2.317583   .2987338                      1.732076    2.903091
             sd(e.g)   .6147329    .097312                      .4240048    .8054609
```

例

上述模型的线性化形式(Woolford, 2004, chap 4):

$$
\begin{aligned}
& && \text{(output-gap Euler equation)} \\
x_t &=& & \beta\mathbb{E}_t(x_{t+1}) - (r_t - \mathbb{E}_t\Pi_{t+1} - g_t), \\
& && \text{(New Keynesian Phillips Curve)} \\
\pi_t &=& & \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t, \\
r_t &=& & \frac{1}{\beta}\pi_t + u_t, \text{(Taylor rule)} \\
u_{t+1} &=& & \rho_u u_t + \epsilon_{t+1} \\
g_{t+1} &=& & \rho_g g_t + \xi_{t+1}
\end{aligned}
$$

```
dsge (x = F.x - (r - F.p - g), unobserved) ///
(p = {beta}*F.p + {kappa}*x) ///
(r = (1/{beta})*p + u) ///
(F.u = {rhou}*u, state) ///
(F.g = {rhog}*g, state), nolog
```

例

```
. use rates2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-02-10)

. dsge (x = F.x - (r - F.p - g), unobserved) (p = {beta}*F.p + {kappa}*x) ///
> (r = (1/{beta})*p + u) (F.u = {rhou}*u, state) (F.g = {rhog}*g, state), nolog

DSGE model

Sample: 1954q3 thru 2016q4                                 Number of obs = 250
Log likelihood = -768.09383
------------------------------------------------------------------------------
             | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
```

/structural						
beta	.5112882	.0757912	6.75	0.000	.3627402	.6598363
kappa	.1696295	.0475493	3.57	0.000	.0764346	.2628243
rhou	.698919	.0449193	15.56	0.000	.6108789	.7869591
rhog	.9556407	.0181342	52.70	0.000	.9200983	.9911831
sd(e.u)	2.317588	.2988029			1.731945	2.903231
sd(e.g)	.6147348	.097328			.4239754	.8054942

政策矩阵

状态变量对控制变量的影响矩阵。

```
. estat policy
```

Policy matrix

	Coefficient	Delta-method std. err.	z	P>|z|	[95% conf.	interval]
x						
u	-1.580154	.392635	-4.02	0.000	-2.349704	-.8106033
g	2.658669	.9045318	2.94	0.003	.885819	4.431518
p						
u	-.4170859	.0389324	-10.71	0.000	-.4933921	-.3407798
g	.8818842	.2330577	3.78	0.000	.4250994	1.338669
r						
u	.184245	.0567981	3.24	0.001	.0729228	.2955672
g	1.724828	.2210262	7.80	0.000	1.291624	2.158031

转移矩阵

状态变量的动态转移矩阵。

```
. estat transition
```

Transition matrix of state variables

	Coefficient	Delta-method std. err.	z	P>|z|	[95% conf. interval]	
F.u						
u	.698919	.0449193	15.56	0.000	.6108789	.7869591
g	4.44e-16	.	.	.	.	.
F.g						
u	0	(omitted)				
g	.9556407	.0181342	52.70	0.000	.9200983	.9911831

Note: Standard errors reported as missing for constrained transition matrix value
s.

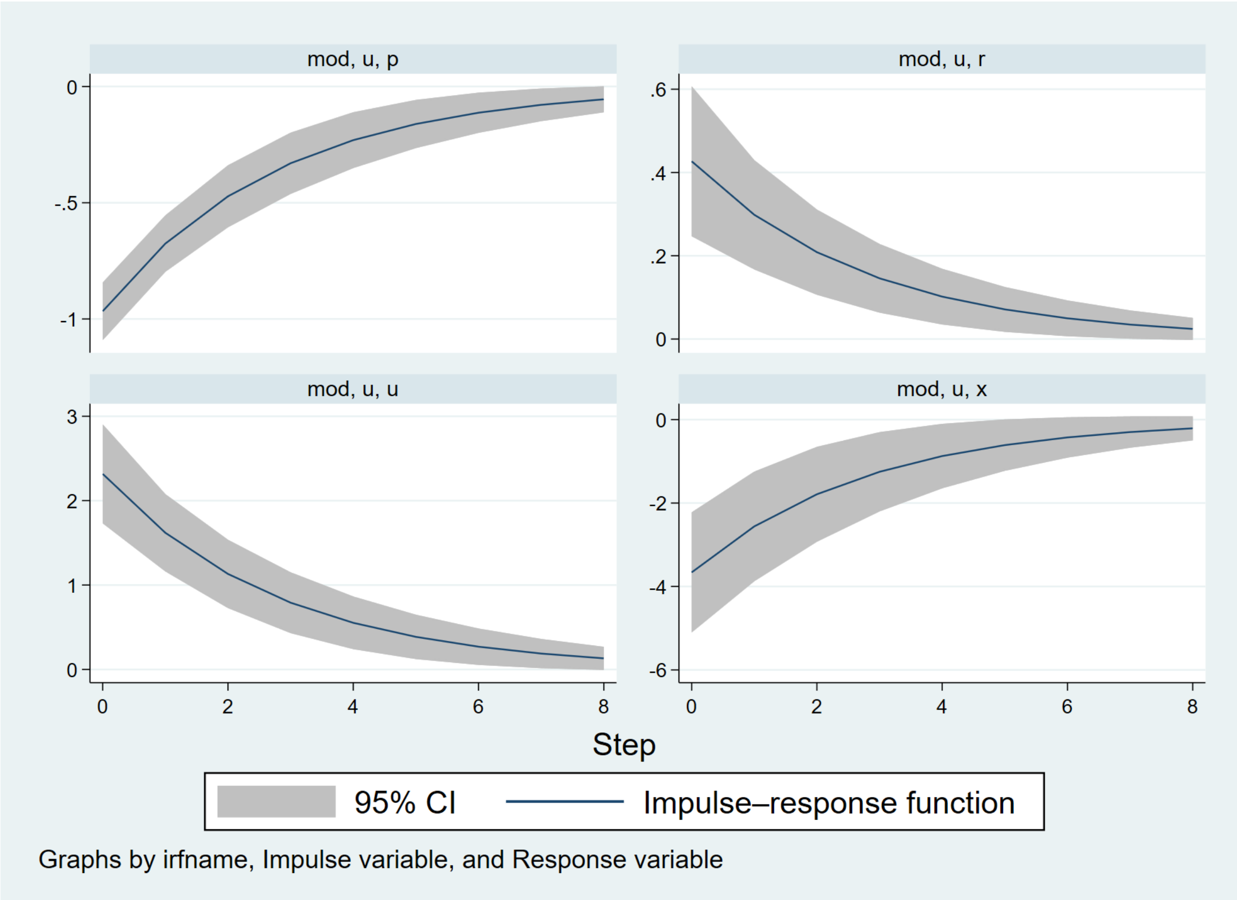
脉冲响应

控制变量或状态变量对状态变量的冲击的响应。

```
. irf set nkirf.irf
(file nkirf.irf now active)

. irf create mod, replace
(file nkirf.irf updated)

. irf graph irf, impulse(u) response(x p r u) byopts(yrescale)
```



例: 新凯恩斯模型

$$
\begin{aligned}
& && \text{(phillips curve)} \\
p_t &= && \beta \mathbb{E}_t p_{t+1} + \kappa x_t \\
x_t &= && \mathbb{E}_t x_{t+1} - [r_t - \mathbb{E}_t p_{t+1}] + g_t \\
r_t &= && \psi p_t + u_t \\
u_{t+1} &= && \rho_u u_t + \epsilon_{t+1} \\
g_{t+1} &= && \rho_g g_t + \xi_{t+1}
\end{aligned}
$$

控制变量：p_t: 通胀率（可观测）；r_t: 利率（可观测）, x_t： 产出缺口(不可观测)

状态变量: u_t, g_t

β: 折现因子

```
. constraint 1 _b[beta]=0.96
. dsge (p = {beta}*F.p + {kappa}*x)  ///
 (x = F.x - (r - F.p) + g, unobserved) ///
 (r = {psi}*p + u) ///
 (F.g = {rhog}*g, state) ///
 (F.u = {rhou}*u, state)
```

例: 新凯恩斯模型

```
. use usmacro2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. constraint 1 _b[beta]=0.96

. dsge (p = {beta}*F.p + {kappa}*x)  (x = F.x - (r - F.p) + g, unobserved) ///
>  (r = {psi}*p + u)  (F.g = {rhog}*g, state) (F.u = {rhou}*u, state), nolog cons
traint(1) from(psi=1.5)

DSGE model

Sample: 1955q1 thru 2015q4                              Number of obs = 244
Log likelihood = -753.57131
 ( 1)  [/structural]beta = .96
```

	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
/structural						
beta	.96	(constrained)				
kappa	.0849631	.0287692	2.95	0.003	.0285765	.1413497
psi	1.943004	.2957868	6.57	0.000	1.363272	2.522735
rhog	.9545256	.0186424	51.20	0.000	.9179872	.991064
rhou	.7005484	.0452602	15.48	0.000	.61184	.7892568
sd(e.g)	.5689891	.0982974			.3763298	.7616485
sd(e.u)	2.318204	.3047433			1.720918	2.91549

结构模型的形式约束: 控制方程含有冲击项

形式约束 1：控制方程不含有冲击项。

$$c_t = \mathbb{E}_t c_{t+1} - r_t + \epsilon_t$$

由于引入新的状态变量u_t，将方程等价地写为，

$$\begin{aligned} c_t &= \mathbb{E}_t c_{t+1} - r_t + u_t \\ u_{t+1} &= \epsilon_{t+1} \end{aligned}$$

Stata:

```
. dsge ... (c = F.c - r + u) (F.u = , state)
```

结构模型的形式约束: 控制方程含有冲击项

$$\begin{aligned} c_t &= (1-h)w_t + h\mathbb{E}_t c_{t+1} + \epsilon_t \\ n_t &= w_t - \gamma c_t \\ w_{t+1} &= \rho w_t + \xi_{t+1} \end{aligned}$$

(1) consumption growth c_t is a linear combination of wage growth w_t, expected future consumption growth $\mathbb{E}_t c_{t+1}$, and a consumption shock ϵ_t.

(2) the growth rate of hours worked n_t depends on wage growth and consumption growth.

(3) the wage growth follows an autoregressive process.

$$\begin{aligned} c_t &= (1-h)w_t + h\mathbb{E}_t c_{t+1} + z_t \\ n_t &= w_t - \gamma c_t \\ w_{t+1} &= \rho w_t + \xi_{t+1} \\ z_t &= \epsilon_t \end{aligned}$$

结构模型的形式约束: 控制方程含有冲击项

```
. use usmacro2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. dsge (c = (1-{h})*(w) + {h}*F.c + z)  (n = w - {gamma}*c) ///
>  (F.w = {rho}*w, state)  (F.z = , state), nolog

DSGE model

Sample: 1955q1 thru 2015q4                          Number of obs = 244
```

```
Log likelihood = -1131.2826
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
h	.7642505	.0360272	21.21	0.000	.6936386	.8348625
gamma	.2726181	.1059468	2.57	0.010	.0649663	.4802699
rho	.6545212	.0485627	13.48	0.000	.55934	.7497023
sd(e.w)	2.904762	.1958651			2.520873	3.28865
sd(e.z)	2.077219	.1400659			1.802695	2.351743

结构模型的形式约束: 方程含有控制变量的滞后项

形式约束：所有方程不能含有控制变量或状态变量的$(t-j)(j>0)$滞后项。

$$y_t = \alpha y_{t-1} + (1-\alpha)\mathbb{E}_t y_{t+1} - r_t$$

引入新的状态变量$u_t = y_{t-1}$，该方程可以等价地写为，

$$\begin{aligned} y_t &= \alpha u_t + (1-\alpha)\mathbb{E}_t y_{t+1} - r_t \\ u_{t+1} &= y_t \end{aligned}$$

Stata:

```
. dsge ... (y = {alpha}*u + (1-{alpha})*F.y - r) (F.u = y, state noshock)
```

结构模型的形式约束: 方程含有控制变量的滞后项

$$\begin{aligned} p_t &= \beta \mathbb{E}_t p_{t+1} + \kappa y_t \\ y_t &= \mathbb{E}_t y_{t+1} - (r_t - \mathbb{E}_t p_{t+1} - \rho_z z_t) \\ r_t &= \rho_r r_{t-1} + (1-\rho_r)\left(\frac{1}{\beta} p_t + u_t\right) \\ z_{t+1} &= \rho_z z_t + \epsilon_{t+1} \\ u_{t+1} &= \rho_u u_t + \xi_{t+1} \end{aligned}$$

(1) inflation p_t is a linear combination of expected future inflation $\mathbb{E}_t p_{t+1}$ and output growth y_t.

(2) Output growth is a linear combination of expected future output growth $\mathbb{E}_t y_{t+1}$, the interest rate r_t, expected future inflation, and the state z_t. The state z_t is the AR(1) process.

(3) The interest rate depends on its own lagged value, the inflation rate, and the state u_t. The state u_t is the AR(1) process that drives the interest rate.

(4) The control variables are p_t, y_t, and r_t. The state variables are u_t and z_t.

$$\begin{aligned}
p_t &= \beta \mathbb{E}_t p_{t+1} + \kappa y_t \\
y_t &= \mathbb{E}_t y_{t+1} - (r_t - \mathbb{E}_t p_{t+1} - \rho_z z_t) \\
r_t &= \rho_r r1_t + (1-\rho_r)\left(\frac{1}{\beta} p_t + u_t\right) \\
r1_{t+1} &= r_t \\
z_{t+1} &= \rho_z z_t + \epsilon_{t+1} \\
u_{t+1} &= \rho_u u_t + \xi_{t+1}
\end{aligned}$$

结构模型的形式约束: 方程含有控制变量的滞后项

```
. use usmacro2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. dsge (p = {beta}*F.p + {kappa}*y) (y = F.y - (r - f.p - {rhoz}*z), unobserved)
///
>  (r = {rhor}*lr + (1-{rhor})*((1/{beta})*p + u))  (F.lr = r, state noshock) ///
>  (F.u = {rhou}*u, state) (F.z = {rhoz}*z, state), nolog

DSGE model

Sample: 1955q1 thru 2015q4                          Number of obs = 244
Log likelihood = -753.07026
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
beta	.5026373	.0791864	6.35	0.000	.3474349	.6578398
kappa	.1760191	.0511049	3.44	0.001	.0758553	.2761829
rhoz	.9591408	.0181341	52.89	0.000	.9235986	.994683
rhor	-.0939028	.093966	-1.00	0.318	-.2780728	.0902673
rhou	.7094626	.0447808	15.84	0.000	.6216939	.7972312
sd(e.u)	2.324309	.3236225			1.690021	2.958598
sd(e.z)	.6111535	.108498			.3985014	.8238056

结构模型的形式约束: 方程含有状态变量的高阶滞后项

形式约束：所有方程不能含有控制变量或状态变量的$(t-j)(j>0)$滞后项。

$$z_{t+1} = \rho_1 z_t + \rho_2 z_{t-1} + \epsilon_{t+1}$$

引入新的状态变量$u_t = y_{t-1}$，该方程可以等价地写为，

$$\begin{aligned} z_{t+1} &= \rho_1 z_t + \rho_2 z2_t + \epsilon_{t+1} \\ z2_{t+1} &= z_t \end{aligned}$$

Stata:

```
. dsge ... (F.z = {rho1}*z + {rho2}*Lz, state) (F.Lz = z, state noshock)
```

结构模型的形式约束: 方程含有状态变量的高阶滞后项

$$\begin{aligned} p_t &= \beta \mathbb{E}_t p_{t+1} + \kappa y_t \\ y_t &= \mathbb{E}_t y_{t+1} - (r_t - \mathbb{E}_t p_{t+1} - \rho_z z_t) \\ r_t &= \frac{1}{\beta} p_t + u_t \\ z_{t+1} &= \rho_z z_t + \rho_2 z_{t-1} + \epsilon_{t+1} \\ u_{t+1} &= \rho_u u_t + \xi_{t+1} \end{aligned}$$

(1) inflation p_t is a linear combination of expected future inflation $\mathbb{E}_t p_{t+1}$ and output growth y_t.

(2) Output growth is a linear combination of expected future output growth $\mathbb{E}_t y_{t+1}$, the interest rate r_t, expected future inflation, and the state z_t. The state z_t is the AR(1) process that drives output growth.

(3) The interest rate depends on the inflation rate, and the state u_t. The state u_t is the AR(1) process.

(4) The control variables are p_t, y_t, and r_t. The state variables are u_t and z_t.

$$
\begin{aligned}
p_t &= \beta \mathbb{E}_t p_{t+1} + \kappa y_t \\
y_t &= \mathbb{E}_t y_{t+1} - (r_t - \mathbb{E}_t p_{t+1} - \rho_z z_t) \\
r_t &= \frac{1}{\beta} p_t + u_t \\
z_{t+1} &= \rho_z z_t + \rho_2 z2_t + \epsilon_{t+1} \\
z2_{t+1} &= z_t \\
u_{t+1} &= \rho_u u_t + \xi_{t+1}
\end{aligned}
$$

结构模型的形式约束: 方程含有状态变量的高阶滞后项

```
. use usmacro2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. dsge (p = {beta}*F.p + {kappa}*y) (y = F.y - (r - F.p - z), unobserved) ///
>   (r = (1/{beta})*p + u) (F.u = {rhou}*u, state)  ///
>   (F.z = {rhoz1}*z + {rhoz2}*Lz, state) (F.Lz = z, state noshock), nolog

DSGE model

Sample: 1955q1 thru 2015q4                          Number of obs = 244
Log likelihood = -753.07788
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
beta	.5154875	.0770493	6.69	0.000	.3644737	.6665013
kappa	.1662425	.0468472	3.55	0.000	.0744237	.2580614
rhou	.6979877	.0452071	15.44	0.000	.6093834	.786592
rhoz1	.6735487	.2665988	2.53	0.012	.1510247	1.196073
rhoz2	.2709998	.2564557	1.06	0.291	-.2316442	.7736438
sd(e.u)	2.315262	.299217			1.728807	2.901717
sd(e.z)	.7720663	.1716206			.4356962	1.108436

结构模型的形式约束: 方程含有控制变量的高阶超前项

形式约束：方程不能含有控制变量的高阶超前项$y_{(t+j)}(j > 1)$。

$$
c_t = (1-h) w_t + h \mathbb{E}_t c_{t+2} + r_t
$$

引入新的控制变量$z_t = c_{t+1}$，该方程可以等价地写为，

$$\begin{aligned} c_t &= (1-h)w_t + h\mathbb{E}_t z_{t+1} + r_t \\ z_t &= c_{t+1} \end{aligned}$$

Stata:

```
. dsge ... (c = (1-{h})*w + {h}*F.Fc + r) (Fc = F.c, unobserved)
```

结构模型的形式约束: 方程含有控制变量的高阶超前项

$$\begin{aligned} c_t &= (1-h)w_t + h\mathbb{E}_t c_{t+2} + r_t \\ n_t &= w_t - \gamma c_t \\ w_{t+1} &= \rho_w w_t + \epsilon_{t+1} \\ r_{t+1} &= \rho_r r_t + \xi_{t+1} \end{aligned}$$

(1) consumption growth c_t is a linear combination of wage growth w_t, the expected value of consumption growth two periods ahead $\mathbb{E}_t c_{t+2}$, and the interest rate r_t.

(2) the growth rate of hours worked n_t depends on wage growth and consumption growth.

(3) AR(1) for wage growth and for the interest rate.

(4) The control variables are c_t and n_t, and the state variables are w_t and r_t.

$$\begin{aligned} c_t &= (1-h)w_t + h\mathbb{E}_t fc_{t+1} + r_t \\ fc_t &= \mathbb{E}_t c_{t+1} \\ n_t &= w_t - \gamma c_t \\ w_{t+1} &= \rho_w w_t + \epsilon_{t+1} \\ r_{t+1} &= \rho_r r_t + \xi_{t+1} \end{aligned}$$

结构模型的形式约束: 方程含有控制变量的高阶超前项

```
. use usmacro2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)

. dsge (c = (1-{h})*(w) + {h}*F.fc + r) (n = w - {gamma}*c)  (fc = F.c, unobserve
d) ///
>  (F.w = {rho_w}*w, state) (F.r = {rho_r}*r, state) , nolog
```

```
DSGE model

Sample: 1955q1 thru 2015q4                        Number of obs = 244
Log likelihood = -1129.9357
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
h	.6617911	.0427916	15.47	0.000	.577921	.7456612
gamma	.2733813	.1120734	2.44	0.015	.0537214	.4930411
rho_w	.6545231	.0485628	13.48	0.000	.5593417	.7497044
rho_r	.1050388	.0638171	1.65	0.100	-.0200405	.230118
sd(e.w)	2.905808	.2023205			2.509267	3.302349
sd(e.r)	2.049915	.1437862			1.768099	2.331731

结构模型的形式约束: 方程含有状态变量的高阶滞后项

形式约束：所有方程不能含有控制变量或状态变量的$(t-j)(j>0)$滞后项。

$$k_t = (1-\delta)k_{t-1} + x_{t-2}$$

引入新的控制变量$k1_t = k_{t-1}$，$x1_t = x_{t-1}$，$x2_t = x1_{t-1}$, 该方程可以等价地写为,

$$\begin{aligned} k_t &= (1-\delta)k1_{t-1} + x2_t \\ k1_{t+1} &= k_t \\ x1_{t+1} &= x_t \\ x2_{t+1} &= x1_t \end{aligned}$$

Stata:

```
. dsge ... (k = (1-{delta})*k1 + L2x) (F.k1 = k, state noshock) (F.L2x = Lx, stat
e noshock) (F.Lx = x, state noshock)
```

结构模型的形式约束: 可观测的外生变量

形式约束：所有观测变量都是内生的。

比如，实际汇率$\mathbb{E}_t$为外生控制变量，那么引入新的状态变量s_t，假定s_t为 AR(1)过程：

$$\begin{aligned} e_t &= s_t \\ s_{t+1} &= \rho s_t + \epsilon_t \end{aligned}$$

Stata:

```
. dsge ... (e = s) (F.s = {\rho}*s, state)...
```

结构模型的形式约束: 状态变量之间存在相关

比如，国内产出y_t取决于国内因素g_t和国际因素z_t，g_t受到z_t的影响:

$$
\begin{aligned}
y_t &= E_t y_{t+1} + \alpha p_t + g_t \\
p_t &= z_t \\
g_{t+1} &= \rho_g g_t + \rho_{gz} z_t + \epsilon_t \\
z_{t+1} &= \rho_z z_t + \xi_t
\end{aligned}
$$

Stata:

```
dsge (y = F.y + {alpha}*p + g) ///
 (p = z) ///
 (F.g = {rho_g}*g + {rho_gz}*z, state) ///
 (F.z = {rho_z}*z, state)
```

例: 金融摩擦

$$
\begin{aligned}
R_t &= \alpha \frac{Y_t}{K_t} \\
1 &= \beta \left[\frac{C_t}{C_{t+1}} (1 + R_{t+1} - \delta) \right] \\
Y_t &= Z_t K_t^{\alpha} \\
K_{t+1} &= Y_t - C_t + (1-\delta) K_t \\
\ln(z_{t+1}) &= \rho \ln(Z_t) + e_{t+1}
\end{aligned}
$$

```
. dsgenl (r = {alpha}*y/k) ///
(1 = {beta}*(c/F.c)*(1 + F.r - {delta})) ///
(y = z*k^({alpha})) ///
(F.k = y - c + (1-{delta})*k) ///
(ln(F.z) = {rho}*ln(z)) , ///
observed(y) unobserved(c r) exostate(z) endostate(k)
```

例：非线性新凯恩斯模型

$$\begin{aligned} R_t &= \alpha \frac{Y_t}{K_t} \\ 1 &= \beta \left[\frac{C_t}{C_{t+1}} (1 + R_{t+1} - \delta) \right] \\ Y_t &= Z_t K_t^{\alpha} \\ K_{t+1} &= Y_t - C_t + (1-\delta) K_t \\ \ln(z_{t+1}) &= \rho \ln(Z_t) + e_{t+1} \end{aligned}$$

```
. dsgenl (r = {alpha}*y/k) ///
(1 = {beta}*(c/F.c)*(1 + F.r - {delta})) ///
(y = z*k^({alpha})) ///
(F.k = y - c + (1-{delta})*k) ///
(ln(F.z) = {rho}*ln(z)) , ///
observed(y) unobserved(c r) exostate(z) endostate(k)
```

例：非线性新凯恩斯模型

$$\begin{aligned} R_t &= \alpha \frac{Y_t}{K_t} \\ 1 &= \beta \left[\frac{C_t}{C_{t+1}} (1 + R_{t+1} - \delta) \right] \\ Y_t &= Z_t K_t^{\alpha} \\ K_{t+1} &= Y_t - C_t + (1-\delta) K_t \\ \ln(z_{t+1}) &= \rho \ln(Z_t) + e_{t+1} \end{aligned}$$

```
. dsgenl (r = {alpha}*y/k) ///
(1 = {beta}*(c/F.c)*(1 + F.r - {delta})) ///
(y = z*k^({alpha})) ///
(F.k = y - c + (1-{delta})*k) ///
(ln(F.z) = {rho}*ln(z)) , ///
observed(y) unobserved(c r) exostate(z) endostate(k)
```

平稳性

只有有解的模型是可估计的，只有满足平衡条件的模型是可求解的。

有A, B构成的广义特征值，如果特征值绝对值小于 1，则该特征值是平稳的。如果平稳特征值的个数等于状态的个数，则该模型是鞍路径平稳（saddle-path stable），意味着既不会发散（失去控制），也不会收敛到一个点（退化）。

平稳性

$$
\begin{aligned}
p_t &= (1/\gamma)\mathbb{E}_t p_{t+1} + \kappa y_t \\
y_t &= \mathbb{E}_t - [r_t - \mathbb{E}_t p_{t+1} - z_t] \\
r_t &= \gamma p_t + u_t \\
u_{t+1} &= \rho_u u_t + \xi_{t+1} \\
z_{t+1} &= \rho_z z_t + \rho_2 L z_t + \epsilon_t \\
L z_{t+1} &= z_t
\end{aligned}
$$

平稳性

```
. use usmacro2, clear
. dsge (p = (1/{gamma})*F.p + {kappa}*y) ///
 (y = F.y - (r - F.p - z), unobserved) ///
 (r = {gamma}*p + u) ///
 (F.u = {rho_u}*u, state) ///
 (F.z = {rho_z1}*z + {rho_z2}*Lz, state) ///
 (F.Lz = z, state noshock)

 model is not saddle-path stable at current parameter values
    The number of stable eigenvalues is greater than the number of state variable
s.
r(498);
```

平稳性

solve 将模型表示为状态空间的形式，但不进行估计。用户可以设定参数数值进行求解，分析模型的含义，给出的结果与数据没有关系。

```
. dsge (p = (1/{gamma})*F.p + {kappa}*y)  (y = F.y - (r - F.p - z), unobserved) /
//
>  (r = {gamma}*p + u)  (F.u = {rho_u}*u, state)  (F.z = {rho_z1}*z + {rho_z2}*Lz,
 state) ///
>  (F.Lz = z, state noshock), solve noidencheck

DSGE model

Sample: 1955q1 thru 2015q4                              Number of obs = 244
Log likelihood = .
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
gamma	.21	.	.	.	.	.
kappa	.22	.	.	.	.	.
rho_u	.23	.	.	.	.	.
rho_z1	.24	.	.	.	.	.
rho_z2	.25	.	.	.	.	.
sd(e.u)	1	.			.	.
sd(e.z)	1	.			.	.

```
Note: Skipped identification check.
Warning: Model cannot be solved at current parameter values. Current values imply
 a model that is not saddle-path stable.
```

平稳性

```
. estat stable

Stability results
```

	Eigenvalues
stable	-.3942
stable	.6342
stable	.21
stable	.23

```
    unstable |      2.605e+16
    unstable |          1.046
```

```
The process is not saddle-path stable.
The process is saddle-path stable when there are 3 stable eigenvalues and 3 unsta
ble eigenvalues.
```

4 个平稳特征值，3 个状态变量，因此模型不是鞍路径平稳的。

平稳性

```
. dsge (p = (1/{gamma})*F.p + {kappa}*y)  (y = F.y - (r - F.p - z), unobserved) /
//
>  (r = {gamma}*p + u)  (F.u = {rho_u}*u, state)  (F.z = {rho_z1}*z + {rho_z2}*Lz,
 state) ///
>  (F.Lz = z, state noshock), solve noidencheck from(gamma = 1.2)

DSGE model

Sample: 1955q1 thru 2015q4                            Number of obs = 244
Log likelihood = -1504.7564
```

	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
/structural						
gamma	1.2	.	.	.	.	.
kappa	.22	.	.	.	.	.
rho_u	.23	.	.	.	.	.
rho_z1	.24	.	.	.	.	.
rho_z2	.25	.	.	.	.	.
sd(e.u)	1	.			.	.
sd(e.z)	1	.			.	.

```
Note: Skipped identification check.
Note: Model solved at specified parameters.
```

平稳性

```
estat stable
```

3 个平稳特征值，3 个状态变量，因此模型是鞍路径平稳的。

识别

Iskrev (2010)通过校验观测到的控制变量的自协方差与参数是否存在一一对应关系来判断模型是否可识别。

$$
\begin{aligned}
p_t &= \beta \mathbb{E}_t p_{t+1} + \kappa y_t \\
y_t &= \mathbb{E}_t y_{t+1} - \gamma[r_t - \mathbb{E}_t p_{t+1} - \rho z_t] \\
\beta r_t &= p_t + u_t \\
u_{t+1} &= \rho_u u_t + \xi_{t+1} \\
z_{t+1} &= \rho_z z_t + \epsilon_t
\end{aligned}
$$

识别

```
dsge (p = {beta}*F.p + {kappa}*y) (y = F.y -{gamma}*(r - F.p - {rhoz}*z), unobser
ved) ///
 (r = (1/{beta})*p + u) (F.u = {rhou}*u, state) (F.z = {rhoz}*z, state)
identification failure at starting values
    Constrain some parameters or specify option noidencheck.  Likely source of id
entification failure: {kappa} {gamma}
r(498);

. constraint 2 _b[gamma]=1

. dsge (p = {beta}*F.p + {kappa}*y) (y = F.y -{gamma}*(r - F.p - {rhoz}*z), unobs
erved) ///
>  (r = (1/{beta})*p + u) (F.u = {rhou}*u, state) (F.z = {rhoz}*z, state), constr
aint(2) nolog

DSGE model

Sample: 1955q1 thru 2015q4                              Number of obs = 244
Log likelihood = -753.57131
 ( 1)  [/structural]gamma = 1
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
/structural						
beta	.5146676	.078349	6.57	0.000	.3611064	.6682289
kappa	.1659054	.0474074	3.50	0.000	.0729887	.2588222
gamma	1	(constrained)				
rhoz	.9545256	.0186424	51.20	0.000	.9179872	.991064
rhou	.7005484	.0452604	15.48	0.000	.6118396	.7892572
sd(e.u)	2.318202	.3047439			1.720915	2.915489

sd(e.z)	.6507116	.1123847	.4304417	.8709815

内容

DSGE 模型

DSGE 模型的贝叶斯估计

贝叶斯推断

对于计量模型，数据为y，参数为θ。贝叶斯定理表示为

$$\pi(\theta|y)=\frac{f(y|\theta)\pi(\theta)}{f(y)}\propto f(y|\theta)\pi(\theta).$$

其中，$f(y|\theta)$为似然函数(概率密度函数)，$\pi(\theta)$为先验分布，$\pi(\theta|y)$为后验分布。

$f(y|\theta)\pi(\theta)$叫做贝叶斯核(kernel)。

$$\text{后验分布}\propto\text{似然函数}\times\text{先验分布}$$

MCMC (Markov Chain Monte Carlo) simulation

Metropolis-Hastings(MH)抽样: 设后验分布为$f(\theta)$（θ代表模型的参数）

(1) 给定θ_t, 定义工具分布$q(y|\theta_t)$ (proposed distribution), 从$q(y|\theta_t)$抽取随机数θ_{new}。其中，工具分布：(a)容易抽样; (b)定义域覆盖后验分布的定义域。

(2) 定义接受概率(acceptance probability): $\rho(\theta_t,\theta_{new})=\min\left[\frac{f(\theta_{new})}{f(\theta_t)}\frac{q(\theta_t|\theta_{new})}{q(\theta_{new}|\theta_t)},1\right]$,

$$\theta_{t+1}=\begin{cases}\theta_{new} & \rho(\theta_t,\theta_{new})\\ \theta_t & 1-\rho(\theta_t,\theta_{new})\end{cases}$$

对称情形：$q(\theta_t|\theta_{new})=q(\theta_{new}|\theta_t),\rho(\theta_t,\theta_{new})=\min\left[\frac{f(\theta_{new})}{f(\theta_t)},1\right]$,

独立情形：$q(\theta_t|\theta_{new})=q(\theta_t),q(\theta_{new}|\theta_t)=q(\theta_{new}),\rho(\theta_t,\theta_{new})=\min\left[\frac{f(\theta_{new})}{f(\theta_t)}\frac{q(\theta_t)}{q(\theta_{new})},1\right]$。

1. 如果$q(\theta_t)=q(\theta_{new})$，那么$\rho=\min[f(\theta_{new})/f(\theta_t),1]$.

2. 计算接受概率时，分布中只有核起作用，其它常数都被略掉。

适应性随机游走 MH 抽样

适应性 MH 抽样是指每隔一定的区间调整ρ_t和Σ_t，以实现最优的接受率(TAR)。

$$\theta_{new} = \theta_{t-1} + e_t, e_t \sim N(0, \rho_t^2 \Sigma_t)$$

根据 Gelman, Gilks, and Roberts (1997)，单个参数的最优接受率为 0.44，多个参数的最优接受率为 0.234。 (ρ_k^2, Σ_k)的更新方程为

$$\begin{aligned} \rho_k &= \rho_{k-1}\exp\left[\beta_k\left(\Phi^{-1}(AR_k/2) - \Phi^{-1}(TAR/2)\right)\right]. \\ \Sigma_k &= (1-\beta_k)\Sigma_{k-1} + \beta_k \hat{\Sigma}_k. \end{aligned}$$

其中，$AR_k = (1-\alpha)AR_{k-1} + \alpha\widehat{AR}_k$.

根据AR_k的公式，如果当前的接受率超过最优水平，ρ_k会提高，即工具密度的方差会增加，这会降低接受率。根据 Gelman, Gilks, and Roberts (1997), Roberts and Rosenthal (2001), and Andrieu and Thoms (2008)，初始值$_0 设定为$2.38/\sqrt{d}$，其中，$d$表示参数的个数。$\widehat{AR}_k$表示第 k 段的接受率，$\hat{\Sigma}_k$表示第 k 段中模拟数值的协方差。$\alpha \in (0,1)$决定了 AR 的平滑水平，$\beta_k = \beta_0/k^\gamma$，$\beta_0 \in (0,1)$，$\gamma \in [0,1]$表示降低适应性调整的水平。可以参考的经验设定值，$\alpha = 0.75, \beta_0 = 0.8, \gamma = 0$。

可以设定适应性调整的区间长度、适应性调整的最小或最大次数、停止规则。比如，alen=100；最小调整次数为 5，最大调整次数为 max(25, burnin/alen)；当接受率 AR 与 TAR 接近时则停止调整，等等。

为了防止方差矩阵的退化，Roberts and Rosenthal (2009)建议用固定的方差矩阵，

$$\Sigma_k = (1-\beta_k)\Sigma_{k-1} + \beta_k \Sigma_{fixed}.$$

MCMC

贝叶斯估计

```
. use usmacro2, clear
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
```

```
. bayes, prior({beta}, beta(95, 5)) prior({kappa}, beta(30, 70))  prior({psi}, be
ta(67, 33)) ///
>  prior({rhou}, beta(75, 20)) prior({rhog}, beta(75, 20)) rseed(17) : ///
>  dsge (p = {beta}*F.p + {kappa}*x) (x = F.x - (r - F.p - g) , unobserved) ///
>  (r = 1/{psi}*p + u)  (F.u = {rhou}*u, state) (F.g = {rhog}*g, state)
note: initial parameter vector set to means of priors.

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  p r ~ dsgell({beta},{kappa},{psi},{rhou},{rhog},{sd(e.u)},{sd(e.g)})

Priors:
             {beta} ~ beta(95,5)
            {kappa} ~ beta(30,70)
              {psi} ~ beta(67,33)
        {rhou rhog} ~ beta(75,20)
  {sd(e.u) sd(e.g)} ~ igamma(.01,.01)
------------------------------------------------------------------------------

Bayesian linear DSGE model                       MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
Sample: 1955q1 thru 2015q4                       Number of obs    =        244
                                                 Acceptance rate  =      .2209
                                                 Efficiency:  min =     .01093
                                                              avg =     .02799
Log marginal-likelihood = -796.75515                          max =     .05246

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. dev.     MCSE     Median  [95% cred. interval]
-------------+----------------------------------------------------------------
        beta |  .9330586   .0273878   .002619   .9354095   .8728009    .975816
       kappa |   .150112   .0247365   .001796   .1485866   .1038851    .203525
         psi |  .5905049   .0405852   .002398   .5895752   .5146669   .6752993
        rhou |  .6275518   .0256116   .001511   .6283089   .5764364   .6770674
        rhog |  .9061189   .0131007   .001059   .9069891   .8800901   .9294626
     sd(e.u) |  2.114667   .1380783   .006028   2.104597   1.869307   2.400718
     sd(e.g) |  .5579649   .0578754   .002862   .5574834   .4491618   .6778793
------------------------------------------------------------------------------
```

MCMC 诊断

接受率(Acceptance Rate, AR): 试验值被接受的概率。Roberts and rosenthal (2001)认为，有效抽样的接受率在 0.15-0.5 之间。

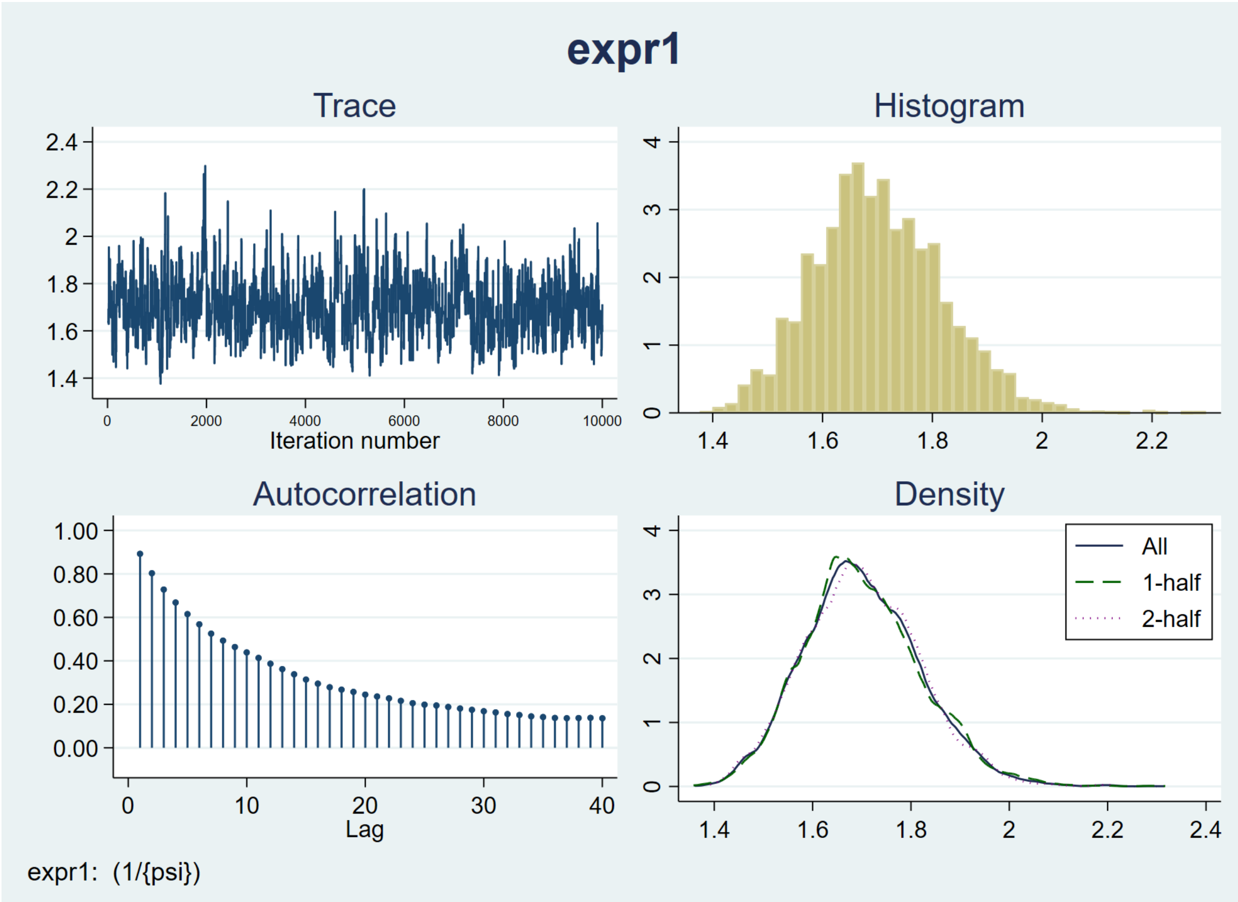
收敛性(是否达到平稳分布)

- 踪迹图（trace plot）： 围绕常数均值波动
- 核密度图（kernel density）：模拟样本分为前后两段，比较其分布。
- 自相关
- Gelman-Rubin (shrink factor)统计量：多条马尔科夫链进行抽样，那么链之间的平均差异与链内的平均差异应该进行相等。二者的比例称作收缩因子（shrink factor），作为经验规则， 收缩因子应低于 1.1。

有效性（Efficiency）是衡量自相关的一个指标。大于 0.1，视作良好。低于 0.01，严重关注。

MCMC 诊断

```
. bayesgraph diag (1/{psi})
```



MCMC 诊断

```
. bayesgraph kdensity {kappa}, lp(solid) ///
 addplot(function Prior = betaden(30,70, x), legend(on label(1 "Posterior")) lp(d
ash))
. bayesgraph kdensity _all
```

MCSE

由于模拟序列存在自相关，后验均值的标准差（也叫做蒙特卡洛标准误差，MCSE）不能通过$s/\sqrt{T}$来估计。

$$ESS = \frac{T}{1 + 2\sum_{k=1}^{K} \rho_k}; Efficiency = ESS/T; \text{Correlation time} = T/ESS.$$

$$\rho_k = \frac{\gamma_k}{\gamma_0};\ \ \gamma_k = \frac{1}{T}\sum_{t=1}^{T-k}(\theta_t - \bar{\theta})\,(\theta_{t+k} - \bar{\theta}); \gamma_0 = \frac{1}{T}\sum_{t=1}^{T-k}(\theta_t - \bar{\theta})^2.$$

其中，ρ_k为k阶自相关系数；γ_k为自协方差系数。

最高阶数K可以自行设定（`corrlag()`，Stata 默认值为 min(500,T/2)。）或者根据自相关系数高于某个阈值（`corrtol()`，默认值为 0.01）来判断。

$$MCSE(\bar{\theta}) = \frac{s}{\sqrt{ESS}};\ \text{or}\ MCSE(\bar{\theta}) = \frac{s}{\sqrt{\sum_{j=1}^{M} ESS_j}}.$$

MCSE

```
. bayesstats ess

Efficiency summaries    MCMC sample size =     10,000
                        Efficiency:  min =     .01093
                                     avg =     .02799
                                     max =     .05246
```

	ESS	Corr. time	Efficiency
beta	109.35	91.45	0.0109
kappa	189.60	52.74	0.0190
psi	286.40	34.92	0.0286
rhou	287.47	34.79	0.0287
rhog	153.01	65.36	0.0153
sd(e.u)	524.63	19.06	0.0525
sd(e.g)	408.84	24.46	0.0409

```
. bayesstats summary (1/{psi})

Posterior summary statistics                      MCMC sample size =     10,000

       expr1 : 1/{psi}
```

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]	
expr1	1.701543	.1181073	.006993	1.696136	1.480825	1.943004

先验分布

$$
\begin{aligned}
p_t &= \beta \mathbb{E}_t p_{t+1} + \kappa x_t \\
x_t &= \mathbb{E}_t x_{t+1} - [r_t - \mathbb{E}_t p_{t+1}] + g_t \\
r_t &= \psi p_t + u_t \\
u_{t+1} &= \rho_u u_t + \epsilon_{t+1} \\
g_{t+1} &= \rho_g g_t + \xi_{t+1}
\end{aligned}
$$

折现因子β，理论范围(0, 1)，常用范围(0.90, 0.99)

自回归因子ρ, 理论范围(-1, 1), 常用范围(0, 1)

Phill 曲线的价格调整因子κ，理论范围(0, .), 常用范围(0, 1)

泰勒规则的通胀率系数ψ，理论范围(0, .)，常用范围(1.5, 2)

先验分布

贝叶斯估计可以通过先验分布约束参数的取值范围。比如，资本份额、折旧率、折现率等参数落在(0, 1)，可以采用 Beta 先验分布。

如果参数严格在某个区间(a, b)取值，那么可以定义$(b-a)\theta+a$，其中θ服从 Beta 分布。比如，取值范围在(2, 4)的参数，可以将其写为 `2*{theta}+2`, 其中 theta 比如假定服从 Beta(5, 5)或 Beta(10, 10)。

比如，自回归系数落在(-1, 1)，可以将其写为 `2*{theta}-1`。

如果参数适宜的取值范围为(a, b)，但可以落在该区间之外，那么可以取其倒数采用 Beta 分布。比如，适宜的取值范围在(2, 4)，其倒数取值范围为(0.25, 0.5)，那么定义θ服从 Beta(37, 63)。

先验分布: Beta 分布

$f(x)=\frac{1}{Beta(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$

$$
\begin{aligned}
\text{Mean} &= \frac{\alpha}{\alpha+\beta} \\
\text{Variance} &= \frac{\alpha\beta}{(\alpha+\beta)^2(1+\alpha+\beta)} \\
\text{Skewness} &= \frac{2(\beta-\alpha)\sqrt{1+\alpha+\beta}}{\sqrt{\alpha\beta}(2+\alpha+\beta)}
\end{aligned}
$$

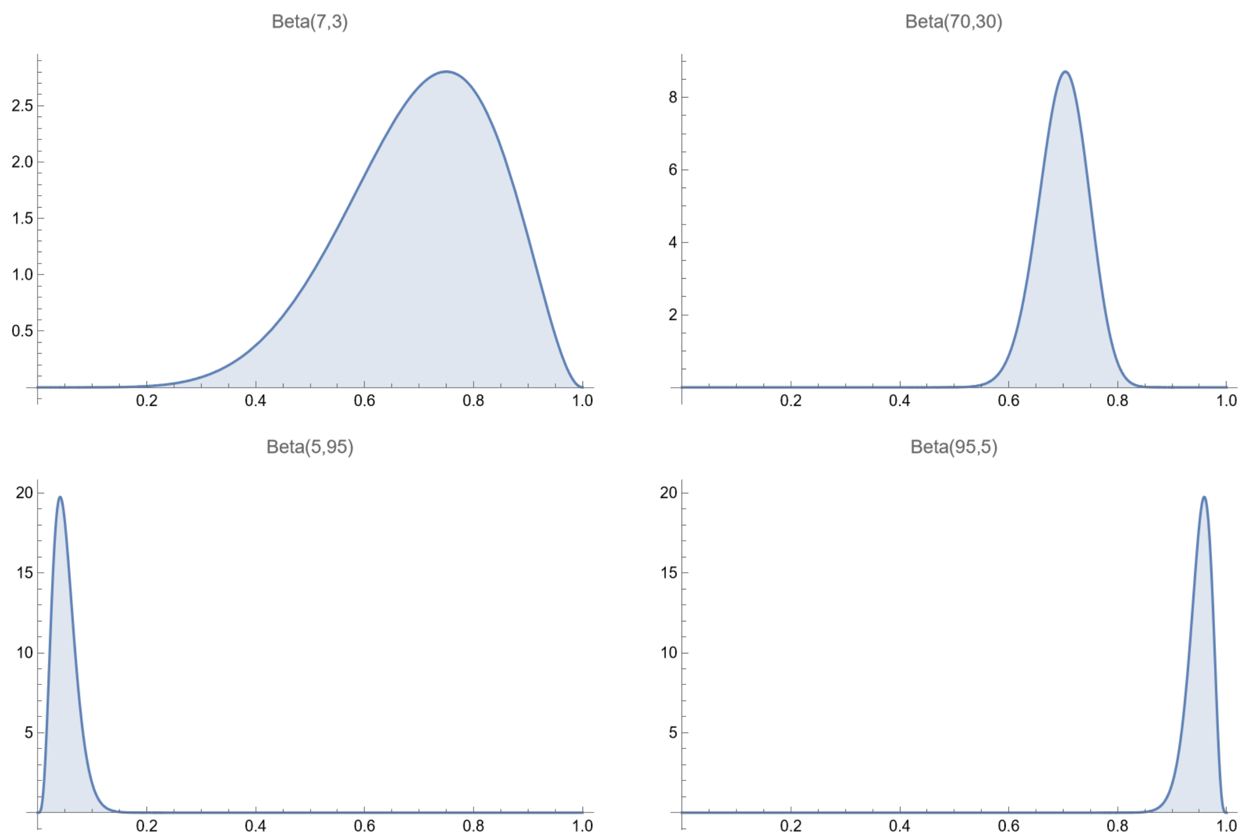
如果$\alpha < 1, \beta > 1$，pdf 单调递减。如果$\alpha > 1, \beta < 1$，pdf 单调递增。

如果$\alpha < 1, \beta < 1$，pdf 呈 U 型特征。如果$\alpha > 1, \beta > 1$，pdf 呈倒 U 型特征。

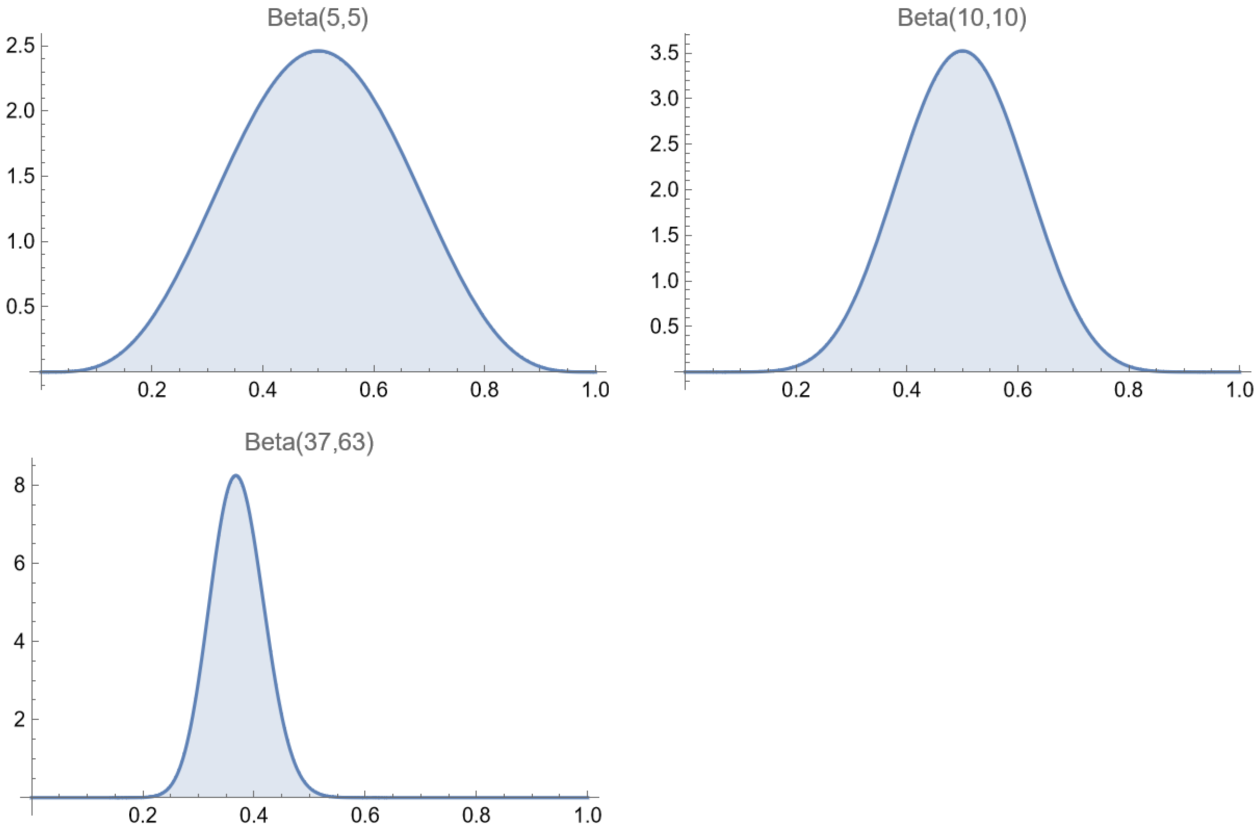
Beta(1,1) = Uniform(0, 1)

(α, β)越大，方差越小。$\beta < \alpha$，左偏；$\beta > \alpha$，右偏。

先验分布: Beta 分布



先验分布: Beta 分布



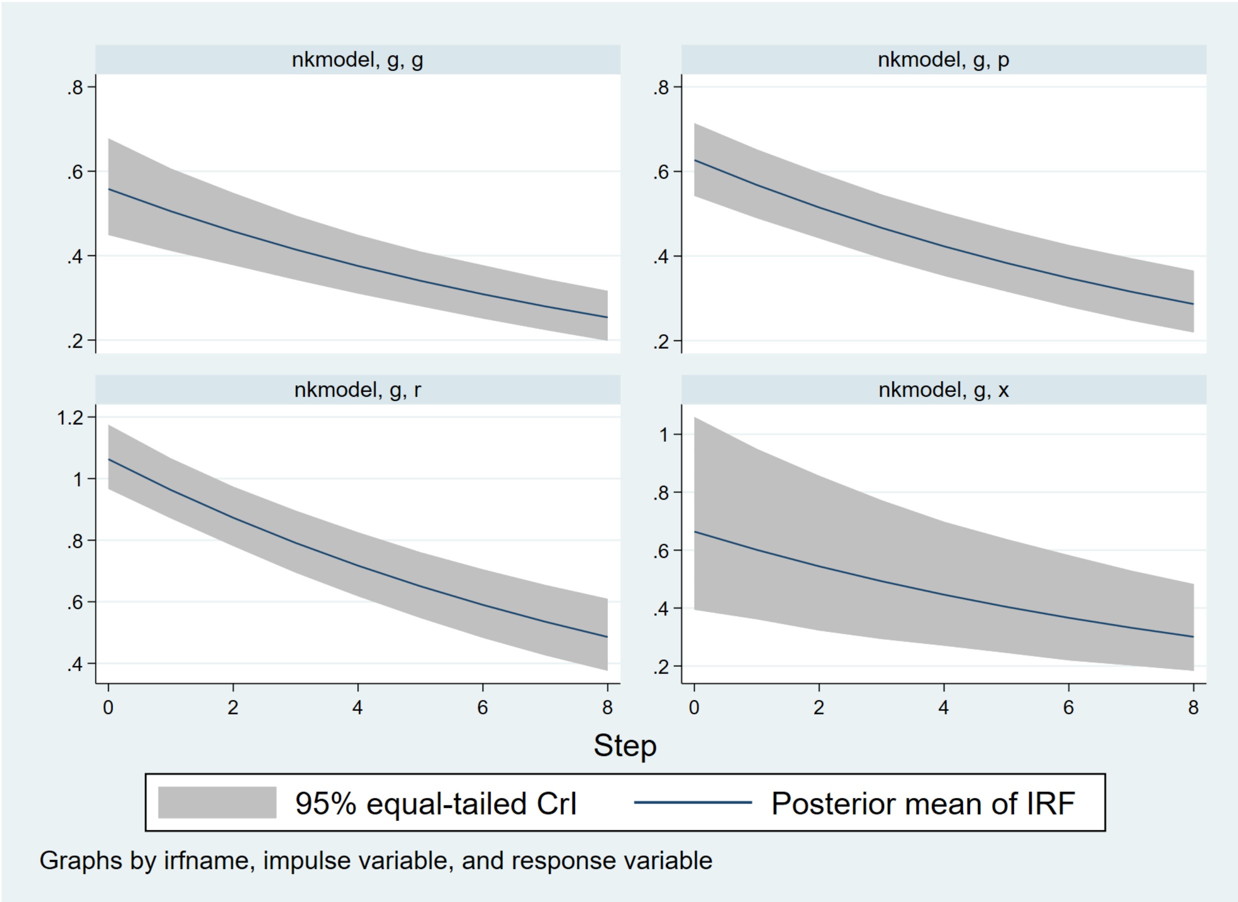
贝叶斯分析: 脉冲响应

```
. bayes, saving(bayesdsge, replace)
note: file bayesdsge.dta saved.

. bayesirf set bayesirf.irf
(file bayesirf.irf now active)

. bayesirf create nkmodel, replace
(file bayesirf.irf updated)

. bayesirf graph irf, impulse(g) response(x p r g) byopts(yrescale)
```



贝叶斯 DSGE: 例(随机增长模型)

产出Y_t, 利率R_t，消费C_t，资本K_t，生产率Z_t (Schmitt-Grohe and Uribe, 2004)。

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} (1 + R_{t+1} - \delta) \right] \\
Y_t &= Z_t K_t^{\alpha} \\
R_t &= \alpha Z_t K_t^{\alpha - 1} \\
K_{t+1} &= Y_t - C_t + (1 - \delta) K_t \\
\ln(Z_{t+1}) &= \rho \ln(Z_t) + e_{t+1}
\end{aligned}
$$

贝叶斯 DSGE: 例(随机增长模型)

参数	理论范围	常用范围	先验分布	分布均值

参数	理论范围	常用范围	先验分布	分布均值
折现率β	(0, 1)	(0.95, 0.99)	Beta(95, 5)	0.95
资本份额α	(0, 1)	about 1/3	Beta(10, 20)	0.33
折旧率δ	(0, 1)	(0.025, 0.1)	Beta(7, 120)	0.05
生产率	(0, 1)	close to 1	Beta(90, 10)	0.9

在 Mathematica 中，给定常用取值的均值、下限和上限，计算出近似的(α, β)。

```
low = 0.9; upp = 0.99; mean = 0.95;
NMinimize[{(Quantile[BetaDistribution[a, b], 0.025] - low)^2 + (Quantile[BetaDist
ribution[a, b], 0.975] - upp)^2 + (Mean[BetaDistribution[a, b]] - mean)^2, a > 1,
  b > 1}, {a, b}]
```

贝叶斯 DSGE: 例(随机增长模型)

```
. bayes, prior({beta}, beta( 95, 5)) prior({delta}, beta(7, 120)) ///
>  prior({alpha}, beta(10, 20)) prior({rhoz}, beta(90, 10)) dots rseed(17) : ///
>  dsgenl (1 = {beta}*(c/F.c)*(1 + F.r - {delta})) (y = z*k^({alpha}))  (r = {alp
ha}*y/k) ///
>  (F.k = y - c + (1-{delta})*k) (ln(F.z) = {rhoz}*ln(z)), ///
>  exostate(z) endostate(k) observed(y) unobserved(c r)
note: initial parameter vector set to means of priors.

Burn-in 2500 aaaaaaaaa1000aaaa.....2000..... done
Simulation 10000 .........1000.........2000.........3000.........4000.........500
0.........6000.........7000.........8000.........90
> 00.........10000 done

Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ dsgell({beta},{delta},{alpha},{rhoz},{sd(e.z)})

Priors:
     {beta} ~ beta(95,5)
    {delta} ~ beta(7,120)
    {alpha} ~ beta(10,20)
     {rhoz} ~ beta(90,10)
  {sd(e.z)} ~ igamma(.01,.01)
------------------------------------------------------------------------------

Bayesian first-order DSGE model                  MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
```

```
Sample: 1955q1 thru 2015q4                    Number of obs     =        244
                                              Acceptance rate   =      .2334
                                              Efficiency:   min =     .02516
                                                            avg =     .03615
Log marginal-likelihood = -680.90377                        max =     .04325
```

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]	
beta	.9675309	.0152937	.000749	.9702225	.9315703	.9903865
delta	.0360957	.0135769	.000676	.0342968	.0137134	.0666271
alpha	.300006	.0835242	.005266	.2971842	.1535816	.4834538
rhoz	.7155932	.0482744	.002321	.7126392	.6279212	.8206983
sd(e.z)	3.658618	.1913428	.010979	3.646614	3.315456	4.056209

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