合成控制法 (SCM) 的安慰剂检验、稳健性检验及可视化操作

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Contents

1 Introduction

Synthetic Control Method (SCM)

- Proposed by Abadie and Gardeazabal (2003)
- Popular method for policy evaluation in panel data with a single treated unit
- Classical example:
  - Terrorist conflict in the Basque Country (Abadie and Gardeazabal 2003)
  - Tobacco control program in California (Abadie, Diamond, and Hainmueller 2010)
  - German reunification (Abadie, Diamond, and Hainmueller 2015)
- Stata command:
  - `synth` (Abadie, Diamond, and Hainmueller 2011)
  - `synth_runner` (Galiani and Quistorff 2017)
  - `synth2` (Yan and Chen 2021)

1 Introduction

Advantages

- Understand the relative importance of each candidate
- Post-intervention outcomes are not used in synthetic.
- Maximize similarity between control and treatment cases

Disadvantages

- Hard to argue for the weights used to create the "synthetic control"

2 Synthetic Control Method (SCM)

Setting:

- $i = 1, \ldots, N+1$ units in periods $t = 1, \ldots, T_0, T_0+1, \ldots, T$
- First unit $(i = 1)$ is the treated one during $t = T_0 + 1, \ldots, T$
- Other N units $(i = 2, \ldots, N+1)$ are called a donor pool (捐赠池)
- y_{it}^1 and y_{it}^0 be the outcome of unit i in period t with and without intervention, respectively

2 Synthetic Control Method (SCM)

The observed outcome y_{it} can be expressed as

$$\begin{aligned} y_{it} &= y_{it}^1 D_{it} + y_{it}^0 (1 - D_{it}) \\ &= y_{it}^0 + \alpha_{it} D_{it}, \end{aligned} \tag{1}$$

- D_{it} is a treatment indicator (**处理指示器**)
- $D_{it} = 1$ if unit i is treated in period t, and $D_{it} = 0$ otherwise
- $\alpha_{it} = y_{it}^1 - y_{it}^0$ denotes the treatment effect for unit i at time t.

2 Synthetic Control Method (SCM)

Suppose that y_{it}^0 is generated by a factor model:

$$y_{it}^0 = \delta_t + \boldsymbol{\theta}_t' \boldsymbol{z}_i + \boldsymbol{\lambda}_t' \boldsymbol{\mu}_i + \varepsilon_{it}, \tag{2}$$

- δ_t is a time fixed effect
- $\boldsymbol{z}_i$ is a $(K \times 1)$ vector of observed covariates
- $\boldsymbol{\theta}_t$ is a $(K \times 1)$ vector of unknown coefficients
- $\boldsymbol{\lambda}_t$ is a vector of unobserved common factors
- $\boldsymbol{\mu}_i$ is a vector of unknown factor loadings
- ε_{it} is an idiosyncratic shock with zero mean for all i

2 Synthetic Control Method (SCM)

SCM seeks to approximate the unknown y_{1t}^0 $(t = T_0 + 1, \ldots, T)$ by a weighted average of the observed outcomes for control units in the donor pool, and the treatment effects are estimated accordingly by

$$\widehat{\Delta}_{1t} = y_{1t} - \widehat{y}_{1t}^0 = y_{1t} - \sum_{i=2}^{N+1} w_i y_{it} \quad (t = T_0 + 1, \cdots, T), \tag{3}$$

- $\mathrm{w} = (w_2, \ldots, w_{J+1})'$ is a $(N \times 1)$ vector of weights (a potential synthetic control) such that $0 \leq w_i \leq 1$ for $i = 2, \ldots, N+1$ and $\sum_{i=2}^{N+1} w_i = 1$.

Question

How to calculate w?

2 Synthetic Control Method (SCM)

Setting:

- x_1 be the $(K \times 1)$ vector containing the pre-treatment covariates of the treated unit
- X_0 be the $(K \times N)$ matrix containing the pre-treatment covariates of the N control units

The optimal synthetic control $\mathrm{w}^*(\mathrm{V})$ is obtained by solving the following minimization problem:

$$\mathrm{w}^*(\mathrm{V}) = \arg\min_{\mathrm{w}} \sqrt{(\mathrm{x}_1 - \mathrm{X}_0\mathrm{w})' \, \mathrm{V} \, (\mathrm{x}_1 - \mathrm{X}_0\mathrm{w})} \tag{4}$$

- V is a $(K \times K)$ diagonal matrix with nonnegative elements on its diagonal
- V contains covariate weights measuring the importance of each covariate in predicting the outcome.

Note

The optimal unit weights $\mathrm{w}^*(\mathrm{V})$ is a function of V, any choice of V are valid for inferential procedures

2 Synthetic Control Method (SCM)

Setting:

- z_1 be the $(T_0 \times 1)$ vector of pre-treatment outcomes for the treated unit
- Z_0 be the $(T_0 \times N)$ matrix of pre-treatment outcomes for the N control units

SCM use a data-driven procedure to choose the optimal V^*, which minimizes the mean squared prediction error (MSPE) of the outcome variable for the pre-treatment periods:

$$\mathrm{V}^* = \arg\min_{\mathrm{V}} \sqrt{\left(\mathrm{z}_1 - \mathrm{Z}_0 \mathrm{w}^*(\mathrm{V})\right)' \left(\mathrm{z}_1 - \mathrm{Z}_0 \mathrm{w}^*(\mathrm{V})\right)} \tag{5}$$

Summary

$$\mathrm{V}^* \Longrightarrow \mathrm{w}^* = \mathrm{w}^*\left(\mathrm{V}^*\right) \Longrightarrow \widehat{y}_{1t}^0 \Longrightarrow \widehat{\Delta}_{1t} = y_{1t} - \widehat{y}_{1t}^0$$

3.1 In-space placebo test

- Use "fake treatment units" for statistical inference
- Assign the treatment to control units in the donor pool iteratively, and estimate placebo effects in each iteration
- Null hypothesis of "no treatment effects" for each post-treatment period
- Right-sided p-values for "unusually large"
- Left-sided p-values for "unusually small"
- Two-sided p-values for "unusually large in absolute values"

3.1 In-space placebo test

$$\text{two-sided } p\text{-value}(t) = \frac{1}{N+1}\sum_{i=1}^{N+1} 1\left(\left|\widehat{\Delta}_{it}\right| \geq \left|\widehat{\Delta}_{1t}\right|\right), \quad t = T_0+1,\ldots,T, \tag{6}$$

$$\text{right-sided } p\text{-value}(t) = \frac{1}{N+1}\sum_{i=1}^{N+1} 1\left(\widehat{\Delta}_{it} \geq \widehat{\Delta}_{1t}\right), \quad t = T_0+1,\ldots,T, \tag{7}$$

$$\text{left-sided } p\text{-value}(t) = \frac{1}{N+1}\sum_{i=1}^{N+1} 1\left(\widehat{\Delta}_{it} \leq \widehat{\Delta}_{1t}\right), \quad t = T_0+1,\ldots,T, \tag{8}$$

- $\widehat{\Delta}_{1t}$ is the treatment effect
- $\widehat{\Delta}_{it}$ is the placebo effect for unit $i \neq 1$
- $1(\cdot)$ is the indicator function, which equals 1 if the expression inside is true, and 0 otherwise

3.1 In-space placebo test

Specifically, the probability (i.e., *p*-value) of obtaining a post/pre MSPE ratio as large as that of the treated unit is calculated as follows

$$\frac{1}{N+1}\sum_{i=1}^{N+1} 1\left(\frac{\text{MSPE}_{i,post}}{\text{MSPE}_{i,pre}} \geq \frac{\text{MSPE}_{1,post}}{\text{MSPE}_{1,pre}}\right), \tag{9}$$

- $\text{MSPE}_{i,post}$ and $\text{MSPE}_{i,pre}$ are post-treatment MSPE and pre-treatment MSPE for unit *i* respectively

Examples

If the post/pre MSPE ratio for the treated unit is larger than all other control units, then the corresponding *p*-value is $\frac{1}{N+1}$

.

3.2 In-time placebo test

- Makes use of a fake treatment time before the treatment actually starts
- Assign the treatment to periods from $\widetilde{T}_0$ on, where no treatment actually occurred during the periods $\left[\widetilde{T}_0, T_0\right]$
- The estimated placebo effects during the periods $\left[\widetilde{T}_0, T_0\right]$ shuold not be "significant" or "large" in some sense
- No p-value is computed for in-time placebo test

3.3 Mixed placebo test

- To get around the issue that in-time placebo test does not produce *p*-values
- Use a fake treatment time and fake treatment units simultaneously
- Given a fake treatment time $\widetilde{T}_0$, conduct an in-space placebo test using fake treatment units
- Produces *p*-values for the fake treatment periods $\left[\widetilde{T}_0, T_0\right]$

4 Robustness Test

- Concern: the treatment effects estimated by SCM may be disproportionally driven by just a single control unit with a nonzero weight
- Sensitivity check: re-estimates the model by constructing a synthetic control omitting in each iteration one of control units with a nonzero weight (leave-one-out robustness test, 留一稳健性检验)
- Robustness result: outcomes and treatment effects of leave-one-out synthetic controls are similar to those of synthetic control with all control units

5 The synth2 command

The Stata command synth2 can be installed from the SSC:

```
. ssc install synth2, all replace
```

synth2 *depvar* [*indepvars*] , *trunit*(#) *trperiod*(#) [ctrlunit(*numlist*) preperiod(*numlist*) postperiod(*numlist*) xperiod(*numlist*) mspeperiod(*numlist*) customV(*numlist*) nested allopt margin(*real*) maxiter(#) sigf(#) bound(#) placebo([{unit|unit(*numlist*)} period(*numlist*) cutoff($\#_c$)]) loo frame(*framename*) nofigure]

- xtset *panelvar timevar* must be used to declare a balanced panel dataset in the usual long form; see [XT] **xtset**
- *depvar* and *indepvars* must be numeric variables, and abbreviations are not allowed

6 Examples

Classic example: the effect of California's tobacco control program (Proposition 99, went into effect in January 1989) on cigarette sales (Abadie,Diamond, and Hainmueller 2010)

6 Examples

The dataset `smoking.dta` includes the following variables for 39 US States from 1970 to 2000:

- `cigsale` (outcome variable, cigarette sale per capita in packs)
- `lnincome` (covariate, logged per-capita state personal income)
- `age15to24` (covariate, percentage of the population aged 15-24)
- `retprice` (covariate, annual state-level values of average retail price of cigarettes)
- `beer` (covariate, per-capita beer consumption).

6 Examples

- Use the command "`label list`" to find the unit number for the treated unit California:

```
. use smoking, clear
(Tobacco Sales in 39 US States)

. xtset state year

Panel variable: state (strongly balanced)
 Time variable: year, 1970 to 2000
         Delta: 1 unit

. label list
state:
           1 Alabama
           2 Arkansas
           3 California
           4 Colorado
           5 Connecticut
           6 Delaware
           7 Georgia
           8 Idaho
           9 Illinois
          10 Indiana
          11 Iowa
          12 Kansas
          13 Kentucky
          14 Louisiana
          15 Maine
          16 Minnesota
  (output omitted)
```

6.1 Prediction

- Use the synth2 command to replicate the results of Abadie, Diamond, and Hainmueller (2010):

```
. synth2 cigsale lnincome age15to24 retprice beer cigsale(1988) cigsale(1980) ci
> gsale(1975), trunit(3) trperiod(1989) xperiod(1980(1)1988)
```

Options

`cigsale(1988) cigsale(1980) cigsale(1975)`: the values of `cigsale` in 1988, 1980 and 1975 respectively

`trunit(3)`: specify California as the treated unit

`trperiod(1989)`: specify 1989 as the treatment time

`xperiod(1980(1)1988)`: average the covariates over the 1980-1988 periods

6.1 Prediction

Fitting results in the pre-treatment periods:

Treated Unit	: California		Treatment Time	:	1989
Mean Absolute Error	= 1.69684		Number of Control Units	=	38
Mean Squared Error	= 4.15283		Number of Covariates	=	7
Root Mean Squared Error	= 2.03785		R-squared	=	0.96292

Predictor balance in the pre-treatment periods:

Covariate	V.weight	Treated	Synthetic Control		Average Control	
			Value	Bias	Value	Bias
lnincome	0.0024	10.0766	9.9048	-1.70%	9.8292	-2.45%
age15to24	0.0003	0.1735	0.1754	1.07%	0.1725	-0.59%
retprice	0.0021	89.4222	89.0013	-0.47%	87.2661	-2.41%
beer	0.0021	24.2800	23.2645	-4.18%	23.6553	-2.57%
cigsale(1988)	0.0684	90.1000	92.0883	2.21%	113.8237	26.33%
cigsale(1980)	0.3979	120.2000	120.7221	0.43%	138.0895	14.88%
cigsale(1975)	0.5267	127.1000	126.3862	-0.56%	136.9316	7.74%

Note: "V.weight" is the optimal covariate weight in the diagonal of V matrix.
"Synthetic Control" is the weighted average of control units in the donor pool with optimal weights.
"Average Control" is the simple average of control units in the donor pool with equal weights.

6.1 Prediction

Optimal Unit Weights:

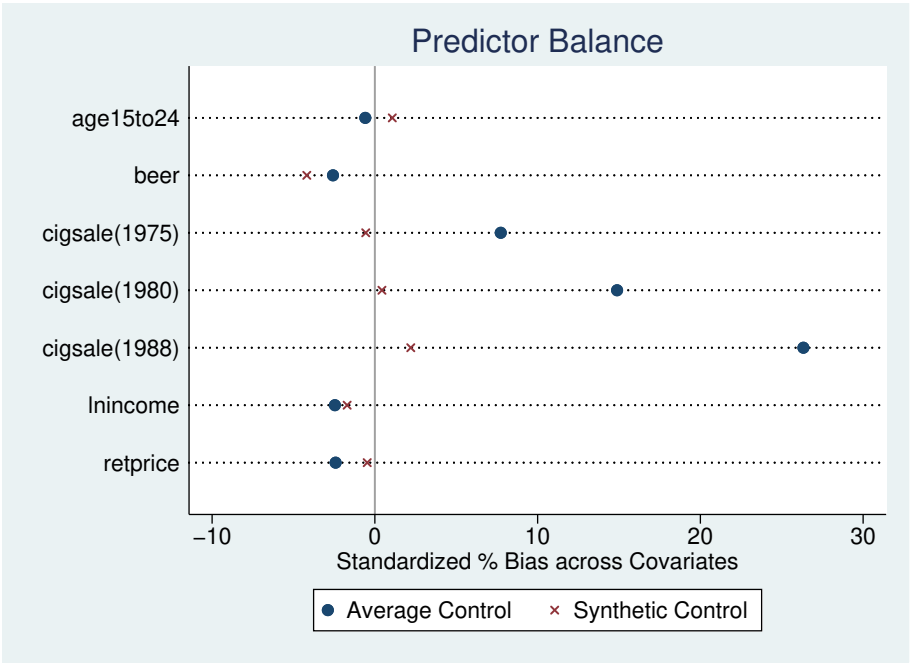
Unit	U.weight
Colorado	0.3560
Utah	0.3440
Nevada	0.2170
Connecticut	0.0830

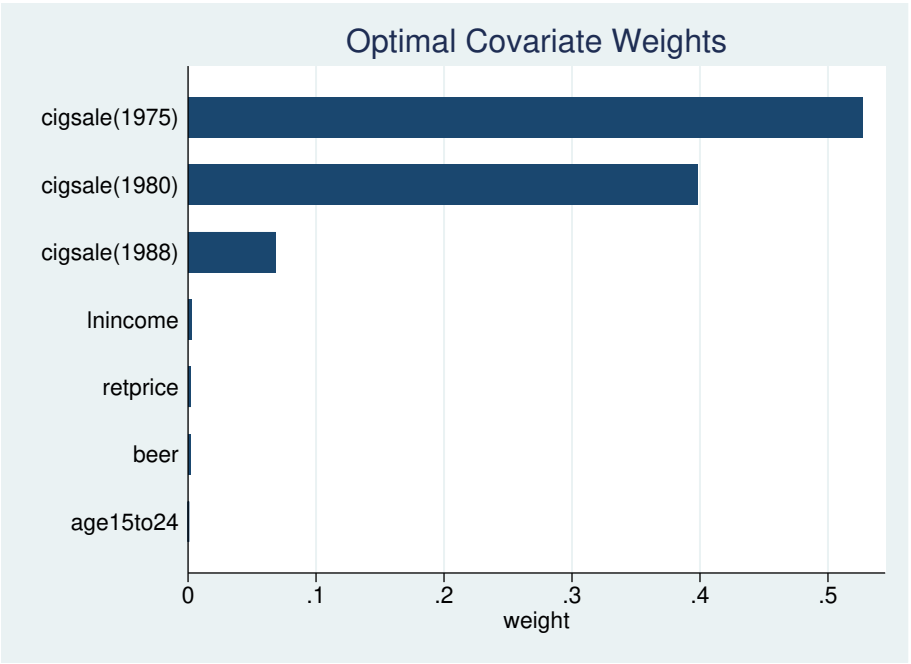
Note: The unit Alabama Arkansas Delaware Georgia Idaho Illinois Indiana Iowa Kansas Kentucky Louisiana Maine Minnesota Mississippi Missouri Montana Nebraska NewHampshire NewMexico NorthCarolina NorthDakota Ohio Oklahoma Pennsylvania RhodeIsland SouthCarolina SouthDakota Tennessee Texas Vermont Virginia WestVirginia Wisconsin Wyoming in the donor pool get a weight of 0.

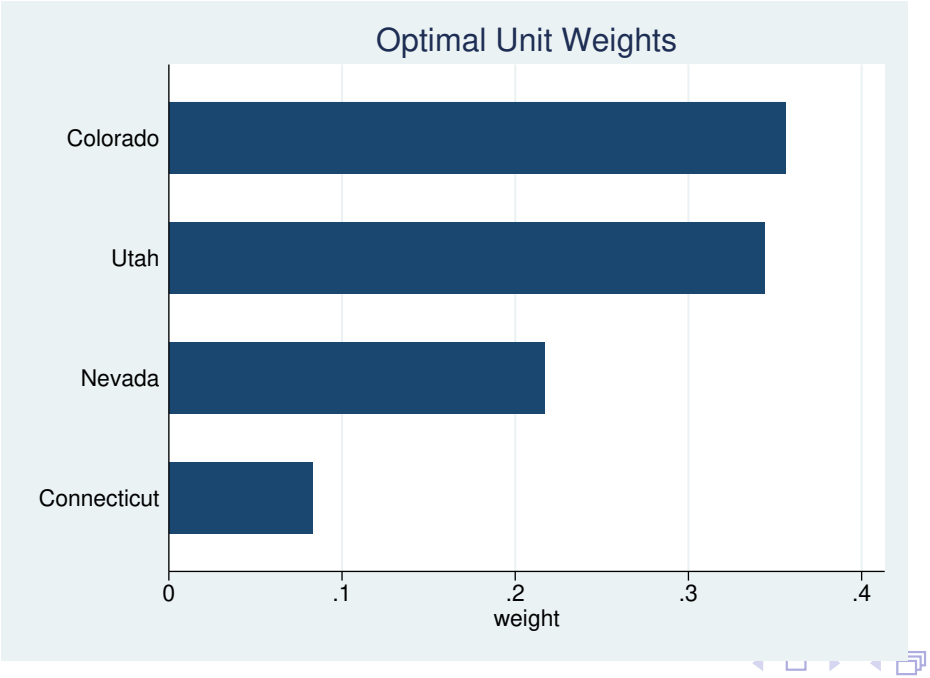
Prediction results in the post-treatment periods:

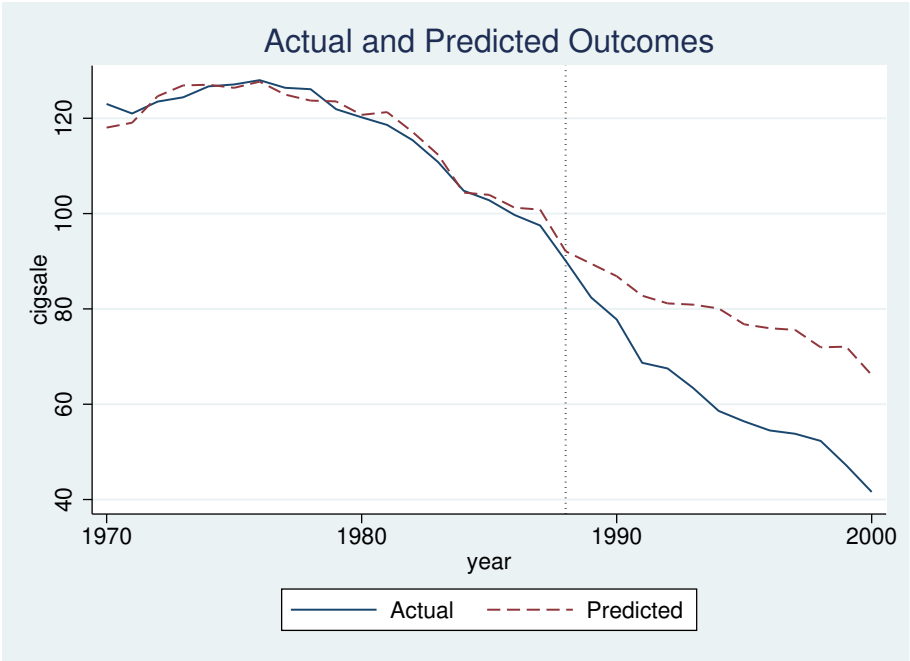
Time	Actual Outcome	Predicted Outcome	Treatment Effect
1989	82.4000	89.4949	-7.0949
1990	77.8000	86.8726	-9.0726
(*output omitted*)			
1997	53.8000	75.5957	-21.7957
1998	52.3000	71.9457	-19.6457
1999	47.2000	72.0777	-24.8777
2000	41.6000	66.1394	-24.5394
Mean	60.3500	78.3144	-17.9644

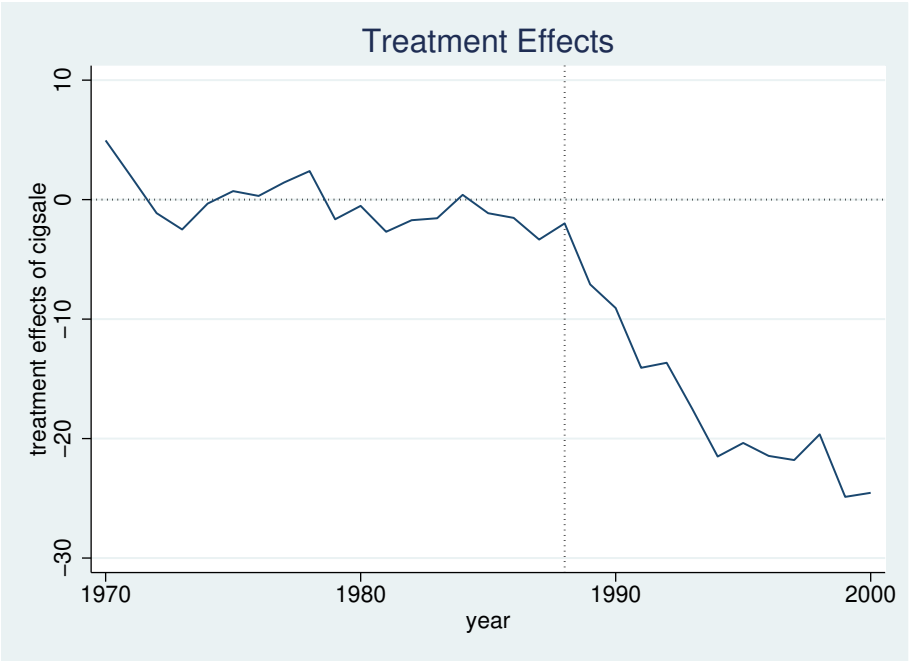
Note: The average treatment effect over the post-treatment periods is -17.9644.











6.2 In-space placebo test

```
. synth2 cigsale lnincome age15to24 retprice beer cigsale(1988) cigsale(1980) ci
> gsale(1975), trunit(3) trperiod(1989) xperiod(1980(1)1988) placebo(unit cut(2)
> )
```

Options

`placebo(unit cut(2))`: request an in-space placebo test using all fake treatment units, but exclude those units with pre-treatment MSPE 2 times larger than that of the treated unit.

6.2 In-space placebo test

(*output omitted*)

```
Implementing placebo test using fake treatment unit Alabama...Arkansas...Colorad
> o...Connecticut...Delaware...Georgia...Idaho...Illinois...Indiana...Iowa...Kan
> sas...Kentucky...Louisiana...Maine...Minnesota...Mississippi...Missouri...Mont
> ana...Nebraska...Nevada...NewHampshire...NewMexico...NorthCarolina...NorthDako
> ta...Ohio...Oklahoma...Pennsylvania...RhodeIsland...SouthCarolina...SouthDakot
> a...Tennessee...Texas...Utah...Vermont...Virginia...WestVirginia...Wisconsin..
> .Wyoming...
```

Placebo test results using fake treatment units:

Unit	Pre MSPE	Post MSPE	Post/Pre MSPE	Pre MSPE of Fake Unit/ Pre MSPE of Treated Unit
California	4.1528	353.5604	85.1371	1.0000
Alabama	7.3449	8.3739	1.1401	1.7686
Arkansas	6.3106	38.8037	6.1490	1.5196
Colorado	36.5014	31.7510	0.8699	8.7895
(*output omitted*)				
WestVirginia	10.2962	288.0129	27.9727	2.4793
Wisconsin	8.7471	37.3961	4.2753	2.1063
Wyoming	114.1686	25.9510	0.2273	27.4917

Note: (1) The probability of obtaining a post/pre-treatment MSPE ratio as large as California´s is 0.0256.
(2) Total 25 units with pre-treatment MSPE 2 times larger than the treated unit are excluded in computing pointwise p-values, including Colorado Connecticut Delaware Illinois Indiana Iowa Kansas Kentucky Maine Minnesota Nevada NewHampshire NorthCarolina NorthDakota Ohio Oklahoma Pennsylvania RhodeIsland SouthDakota Utah Vermont Virginia WestVirginia Wisconsin Wyoming.

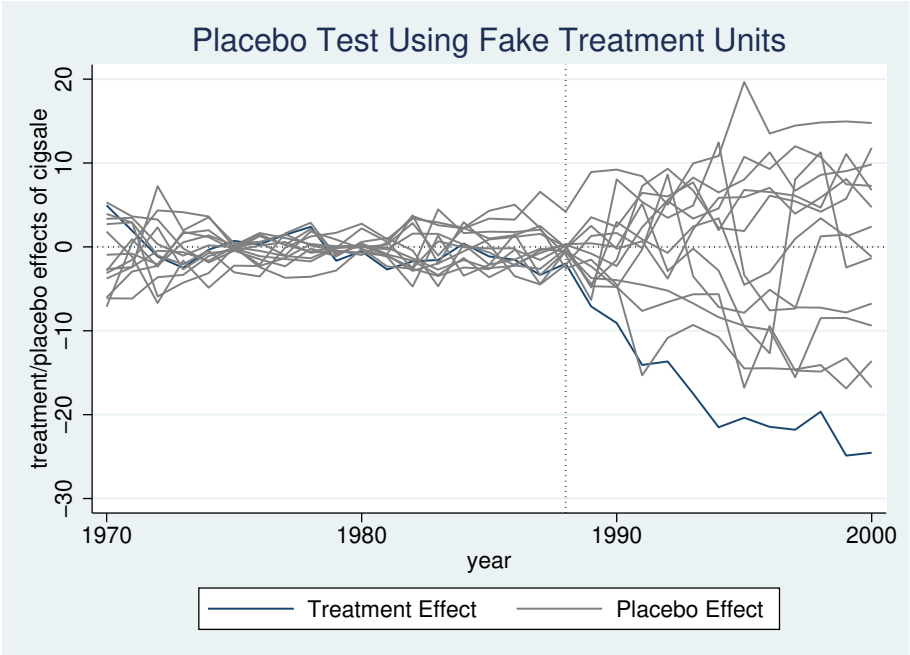
6.2 In-space placebo test

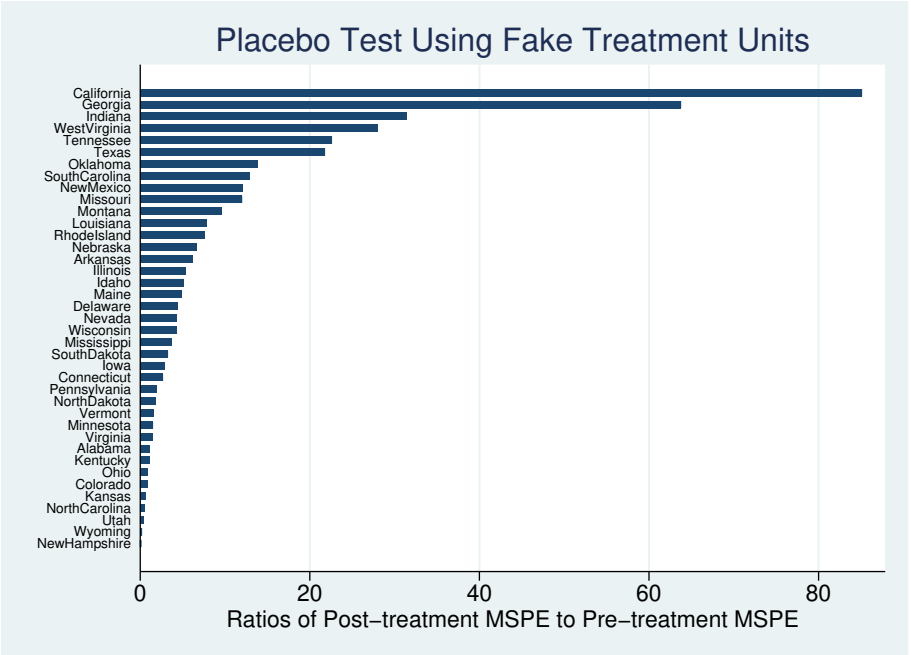
Placebo test results using fake treatment units (continued, cutoff = 2):

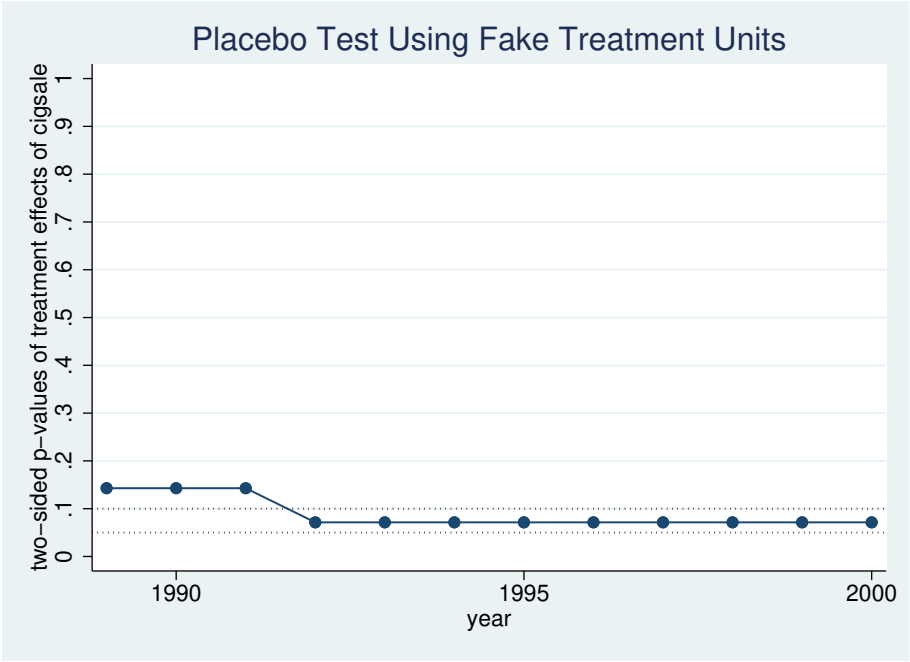
Time	Treatment Effect	p-value of Treatment Effect		
		Two-sided	Right-sided	Left-sided
1989	-7.0949	0.1429	1.0000	0.0714
1990	-9.0726	0.1429	1.0000	0.0714
1991	-14.0748	0.1429	0.9286	0.1429
1992	-13.6553	0.0714	1.0000	0.0714
1993	-17.4974	0.0714	1.0000	0.0714
1994	-21.5010	0.0714	1.0000	0.0714
1995	-20.3676	0.0714	1.0000	0.0714
1996	-21.4506	0.0714	1.0000	0.0714
1997	-21.7957	0.0714	1.0000	0.0714
1998	-19.6457	0.0714	1.0000	0.0714
1999	-24.8777	0.0714	1.0000	0.0714
2000	-24.5394	0.0714	1.0000	0.0714

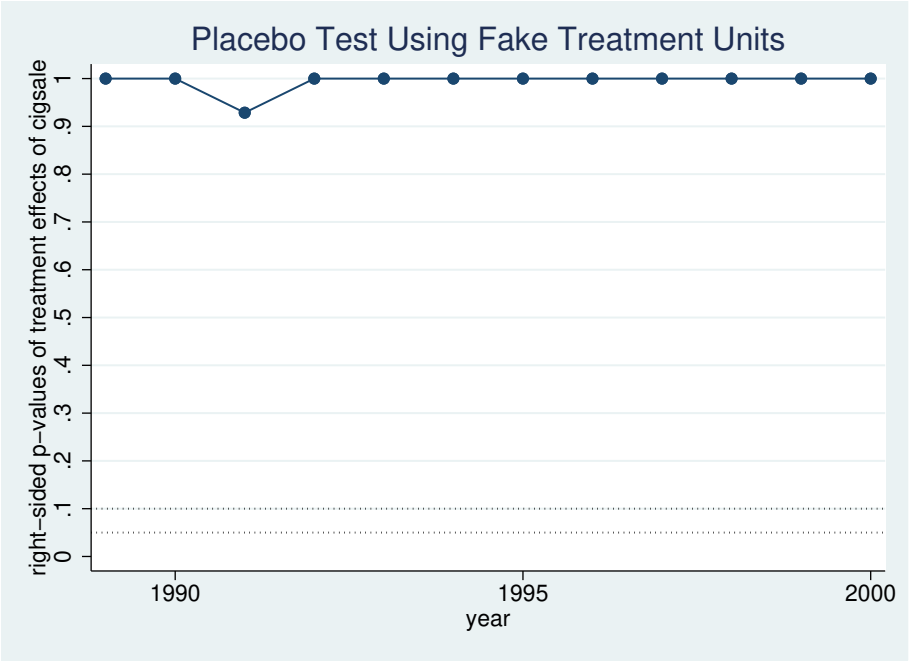
Note: (1) The two-sided p-value of the treatment effect for a particular period is defined as the frequency that the absolute values of the placebo effects are greater than or equal to the absolute value of treatment effect.
(2) The right-sided (left-sided) p-value of the treatment effect for a particular period is defined as the frequency that the placebo effects are greater (smaller) than or equal to the treatment effect.
(3) If the treatment effects are mostly positive, then the right-sided p-values are recommended; whereas the left-sided p-values are recommended if the treatment effects are mostly negative.

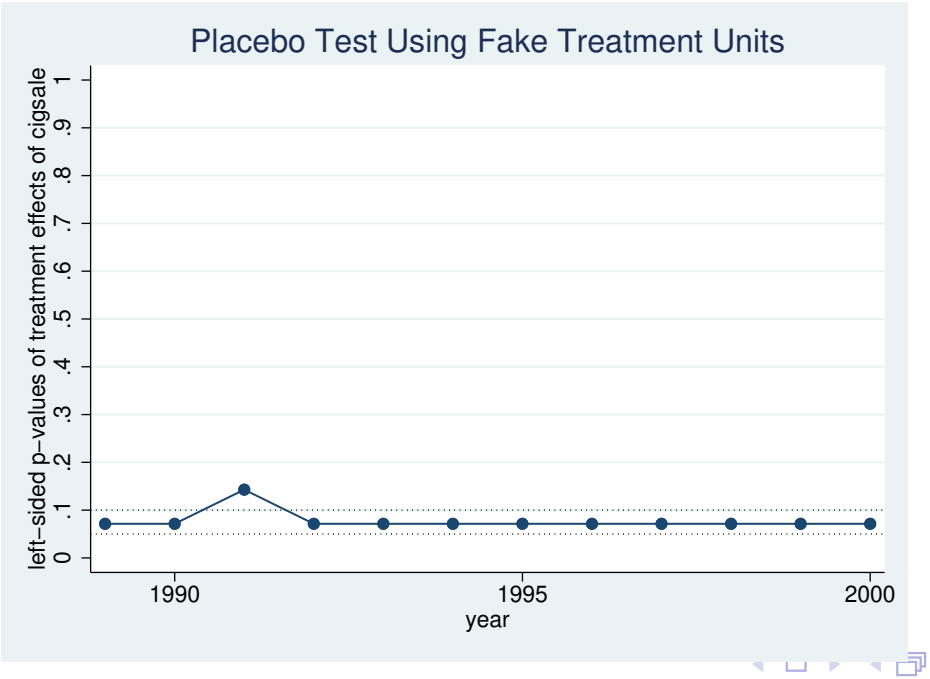
Finished.









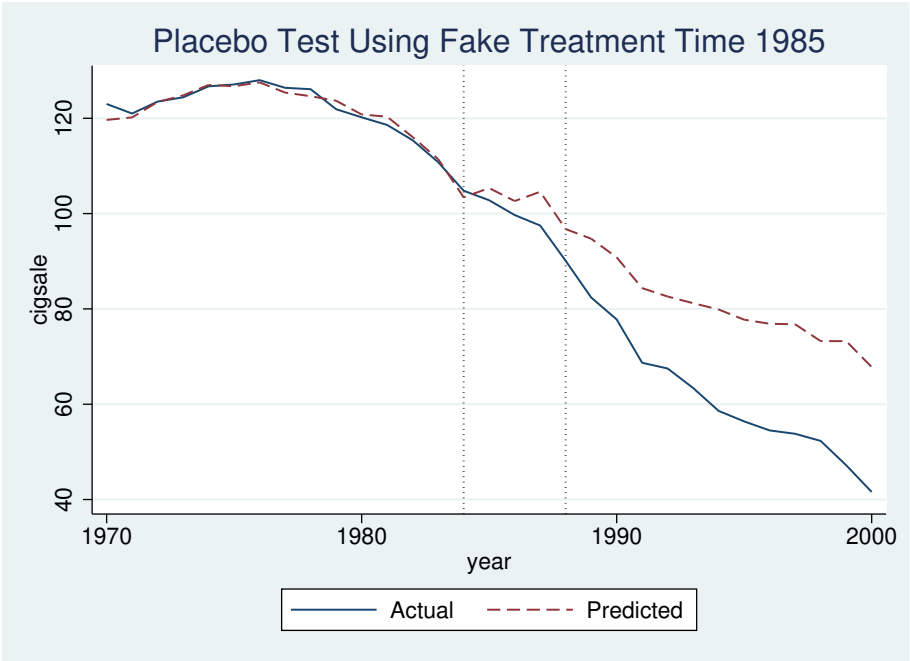


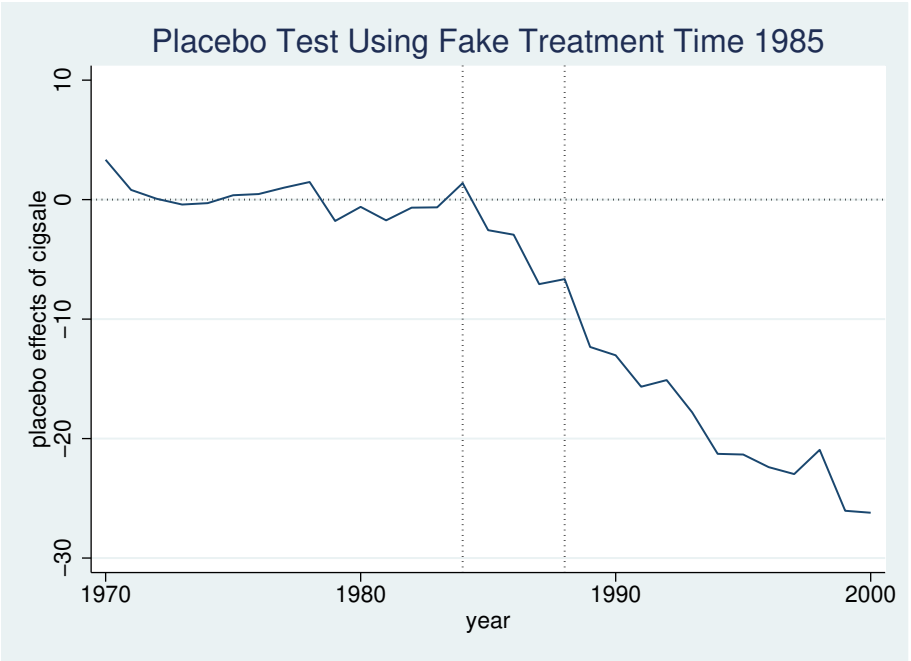
6.3 In-time placebo test

```
. synth2 cigsale lnincome age15to24 retprice beer cigsale(1980) cigsale(1975), t
> runit(3) trperiod(1989) xperiod(1980(1)1984) placebo(period(1985))
  (output omitted)
Implementing placebo test using fake treatment time 1985...

Placebo test results using fake treatment time 1985:
```

Time	Actual Outcome	Predicted Outcome	Treatment Effect
1985	102.8000	105.3568	-2.5568
1986	99.7000	102.6356	-2.9356
1987	97.5000	104.5710	-7.0710
1988	90.1000	96.7533	-6.6533
1989	82.4000	94.7325	-12.3325
1990	77.8000	90.8295	-13.0295
1991	68.7000	84.3586	-15.6586
1992	67.5000	82.6040	-15.1040
1993	63.4000	81.2039	-17.8039
1994	58.6000	79.8776	-21.2776
1995	56.4000	77.7349	-21.3349
1996	54.5000	76.8896	-22.3896
1997	53.8000	76.7704	-22.9704
1998	52.3000	73.2427	-20.9427
1999	47.2000	73.2426	-26.0426
2000	41.6000	67.8051	-26.2051
Mean	69.6437	85.5380	-15.8943

```
Note: The average treatment effect over the post-treatment periods is
      -15.8943.

Finished.
```





6.4 Mixed placebo test

```
. synth2 cigsale lnincome age15to24 retprice beer cigsale(1980) cigsale(1975), t
> runit(3) trperiod(1985) xperiod(1980(1)1984) placebo(unit cut(10))
  (output omitted)
Placebo test results using fake treatment units (continued, cutoff = 10):
```

Time	Treatment Effect	p-value of Treatment Effect		
		Two-sided	Right-sided	Left-sided
1985	-2.5568	0.5217	0.7391	0.3043
1986	-2.9356	0.5652	0.6957	0.3478
1987	-7.0710	0.3043	0.8696	0.1739
1988	-6.6533	0.3913	0.8261	0.2174
1989	-12.3325	0.1739	0.9130	0.1304
1990	-13.0295	0.1304	0.9130	0.1304
1991	-15.6586	0.1739	0.9130	0.1304
1992	-15.1040	0.1739	0.9130	0.1304
1993	-17.8039	0.1304	0.9130	0.1304
1994	-21.2776	0.0870	0.9565	0.0870
1995	-21.3349	0.1304	0.9565	0.0870
1996	-22.3896	0.1304	0.9565	0.0870
1997	-22.9704	0.1739	0.9565	0.0870
1998	-20.9427	0.1304	0.9565	0.0870
1999	-26.0426	0.1304	0.9565	0.0870
2000	-26.2051	0.0870	1.0000	0.0435

```
  (output omitted)

Finished.
```

6.5 Robustness test

```
. synth2 cigsale lnincome age15to24 retprice beer cigsale(1988) cigsale(1980) ci
> gsale(1975), trunit(3) trperiod(1989) xperiod(1980(1)1988) loo frame(californi
> a)
  (output omitted)
Robustness test results in the post-treatment periods:
```

Time	Outcome		Outcome (LOO)	
	Actual	Predicted	Min	Max
1989	82.4000	89.4949	87.4874	91.6284
1990	77.8000	86.8726	86.2213	87.2073
1991	68.7000	82.7748	82.3889	86.0438
1992	67.5000	81.1553	81.2195	84.8336
1993	63.4000	80.8974	81.0339	84.5925
1994	58.6000	80.1010	80.4438	83.9316
1995	56.4000	76.7676	77.1617	80.7926
1996	54.5000	75.9506	76.4011	79.3867
1997	53.8000	75.5957	75.9921	78.6489
1998	52.3000	71.9457	72.4120	77.2199
1999	47.2000	72.0777	72.4941	74.3663
2000	41.6000	66.1394	66.2970	68.9745

```
Note: The last two columns report the minimum and maximum outcomes when one
      control unit with a nonzero weight is excluded at a time.
```

Options

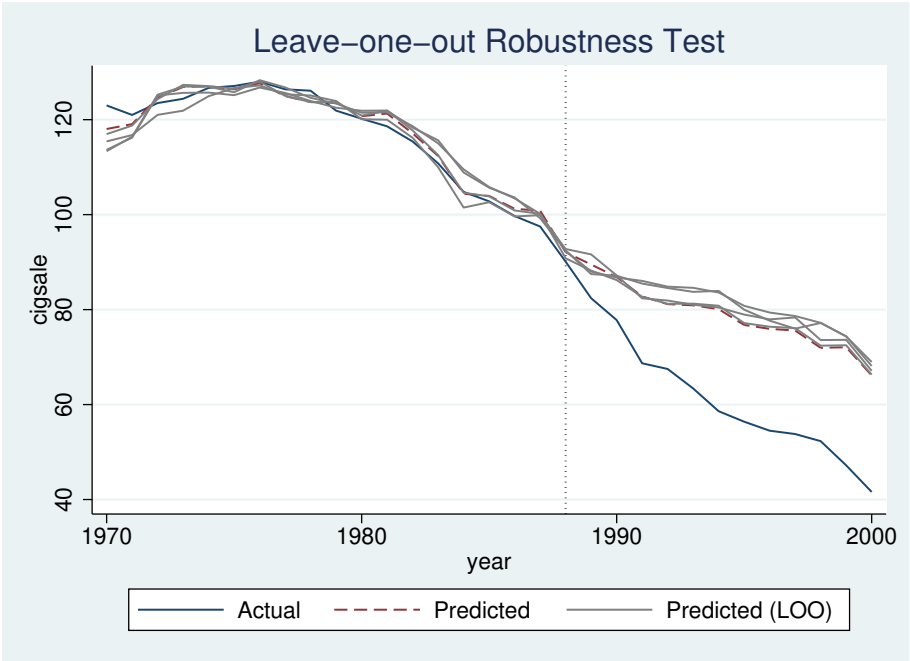
`loo`: iteratively remove one of the control units with positive weights, and then re-estimates the model by SCM.

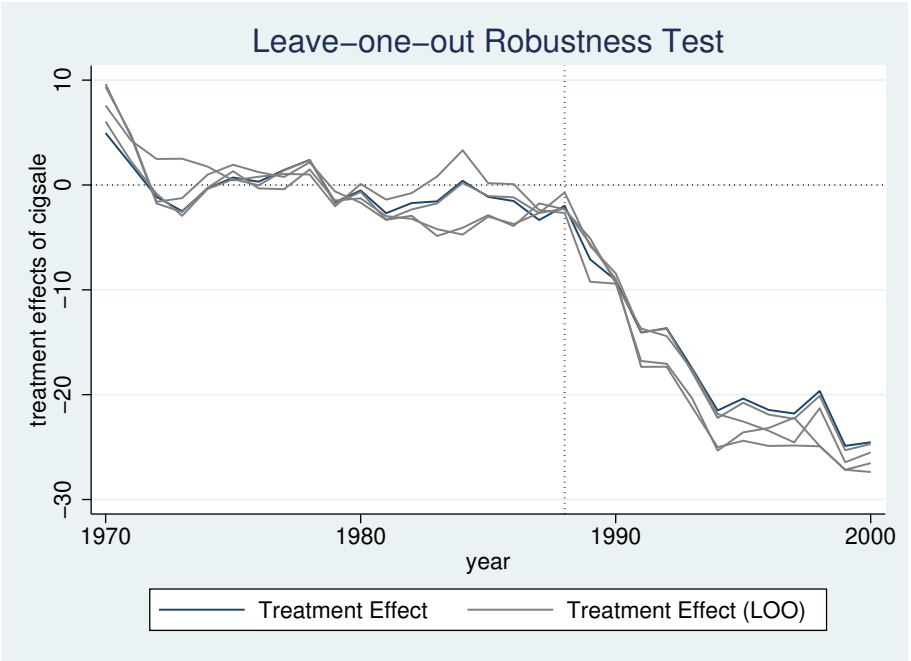
6.5 Robustness test

Time	Treatment Effect	Treatment Effect (LOO) Min	Max
1989	-7.0949	-9.2284	-5.0874
1990	-9.0726	-9.4073	-8.4213
1991	-14.0748	-17.3438	-13.6889
1992	-13.6553	-17.3336	-13.7195
1993	-17.4974	-21.1925	-17.6339
1994	-21.5010	-25.3316	-21.8438
1995	-20.3676	-24.3926	-20.7617
1996	-21.4506	-24.8867	-21.9011
1997	-21.7957	-24.8489	-22.1921
1998	-19.6457	-24.9199	-20.1120
1999	-24.8777	-27.1663	-25.2941
2000	-24.5394	-27.3745	-24.6970

Note: The last two columns report the minimum and maximum treatment effects
when one control unit with a nonzero weight is excluded at a time.

Finished.





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Thank you!