跨度回归，偏度回归与峰度回归及Stata应用

Spread Regression, Skewness Regression and Kurtosis Regression with Applications in Stata

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Abstract

- Quantile regression provides a powerful tool to study the effects of covariates on key quantiles of conditional distribution. Yet we often lack a general picture about how covariates affect the overall shape of conditional distribution.

- We propose quantile-based spread regression, skewness regression and kurtosis regression to quantify the effects of covariates on the spread, skewness and kurtosis of conditional distribution.

Abstract (cont.)

- This methodology is then applied to U.S. census data with substantive findings.

- We demonstrate the implementation of spread, skewness and kurtosis regressions with official Stata command **iqreg**, and two user-written commands **skewreg** and **kurtosisreg**.

Outline

1. Introduction
2. Quantile-based Measures of Conditional Spread, Skewness and Kurtosis
3. The Spread Regression
4. The Skewness Regression
5. The Kurtosis Regression
6. An Application to the U.S. Wage Data
7. Stata Application

1. Introduction

- Quantile regression provides a powerful tool to study the effects of covariates on key quantiles of conditional distribution of dependent variable given covariates.

- But there are (too) many regression quantiles…

- How do covariates affect the overall shape of conditional distribution?

How to Characterize Distribution

- A simple way to characterize conditional distribution by looking at summary statistics:

- Location (mean, median)
- Scale (variance, spread, or interquartile range)
- Asymmetry (skewness)
- Fat tails or tail risk (kurtosis)

Quantile-based Measures

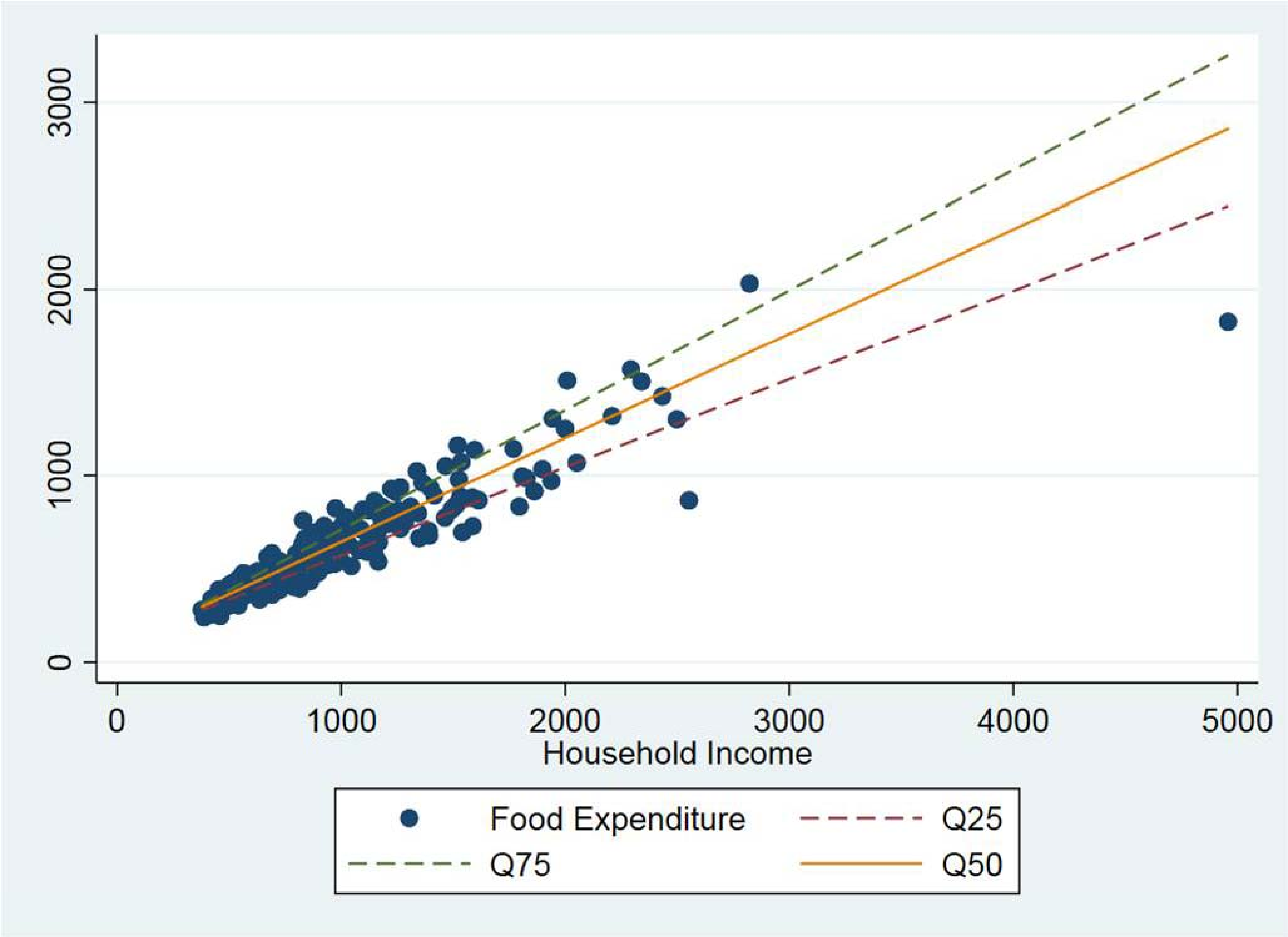
- Median
- Spread (e.g. Interquartile Range)
- Skewness (defined by quantiles)
- Kurtosis (defined by quantiles)

Advantages of Quantile-based Measures

- Moment-based measures may not exist, whereas quantile-based measures are always well defined
- Moment-based measures are sensitive to outliers, whereas quantile-based measures are robust to outliers
- Quantile-based measures can easily connect with quantile regression.

A Motivating Example

- Take a look at the classic Engel (1857) dataset
- Food expenditure is regressed on household income


0
1000
2000
3000
0
1000
2000
3000
4000
5000
Household Income
Food Expenditure
Q25
Q75
Q50

How Much Can We See

- The spread increases with the only covariate household income. But by how much, and is it statistically significant?

- How about the effect of household income on the conditional skewness and conditional kurtosis of food expenditure?

Our Contributions

- We propose a model that examines the impact of covariates on important properties conditional distribution, such as quantile-based measures of spread, skewness, and kurtosis.

- Estimated conditional spread, skewness and kurtosis functions are of additional interests

2. Quantile-based Measures of Conditional Spread, Skewness and Kurtosis

- Consider a random variable y and covariates $\mathbf{x}$ (p-dim vector), denote the distribution function of y conditional on $\mathbf{x}$ as $F(y \mid \mathbf{x})$ and the quantile function of y conditional on $\mathbf{x}$ is $Q_Y(\tau \mid \mathbf{x})$

- We want to study how the distributional properties of y (spread, skewness and kurtosis) vary with $\mathbf{x}$

The Spread Regression

- Let SP_y be a measure of the spread of y given $\mathbf{x}$, then SP_y is varying with $\mathbf{x}$, and suppose this relationship is captured by the functional relationship

$$SP_y = m(\mathbf{x})$$

- We call this relationship as the “spread regression” relationship.

The Skewness Regression

- Similarly, let SK_y be a measure of the skewness of y given $\mathbf{x}$, then SK_y is varying with $\mathbf{x}$, and suppose this relationship is captured by the functional relationship

$$SK_y = s(\mathbf{x})$$

- We call this relationship as the "skewness regression" relationship.

The Kurtosis Regression

- Let KUR_y be a measure of the kurtosis of y given $\mathbf{x}$, then KUR_y is varying with $\mathbf{x}$, and suppose this relationship is captured by the functional relationship

$$KUR_y = k(\mathbf{x})$$

- We call this relationship as the "kurtosis regression" relationship.

Quantile-based Measurements

- The properties of $m(\mathbf{x})$, $s(\mathbf{x})$ and $k(\mathbf{x})$ are dependent on how we measure the spread, skewness and kurtosis.

- We consider quantile-based measures for the spread, skewness and kurtosis, and study the relationship between spread, skewness, kurtosis and useful covariates $\mathbf{x}$.

Measurement of Spread

- A widely used robust measure of the spread is the Interquartile Range (IQR), then

$$SP_y = m(\mathbf{x}) = Q_Y(0.75 \mid \mathbf{x}) - Q_Y(0.25 \mid \mathbf{x})$$

- In general, for appropriate chosen τ , we may measure the spread of y given $\mathbf{x}$ by

$$SP_y = m(\tau, \mathbf{x}) = Q_Y(1-\tau \mid \mathbf{x}) - Q_Y(\tau \mid \mathbf{x})$$

- For example, τ = 0.25 or 0.1

Measurement of Skewness

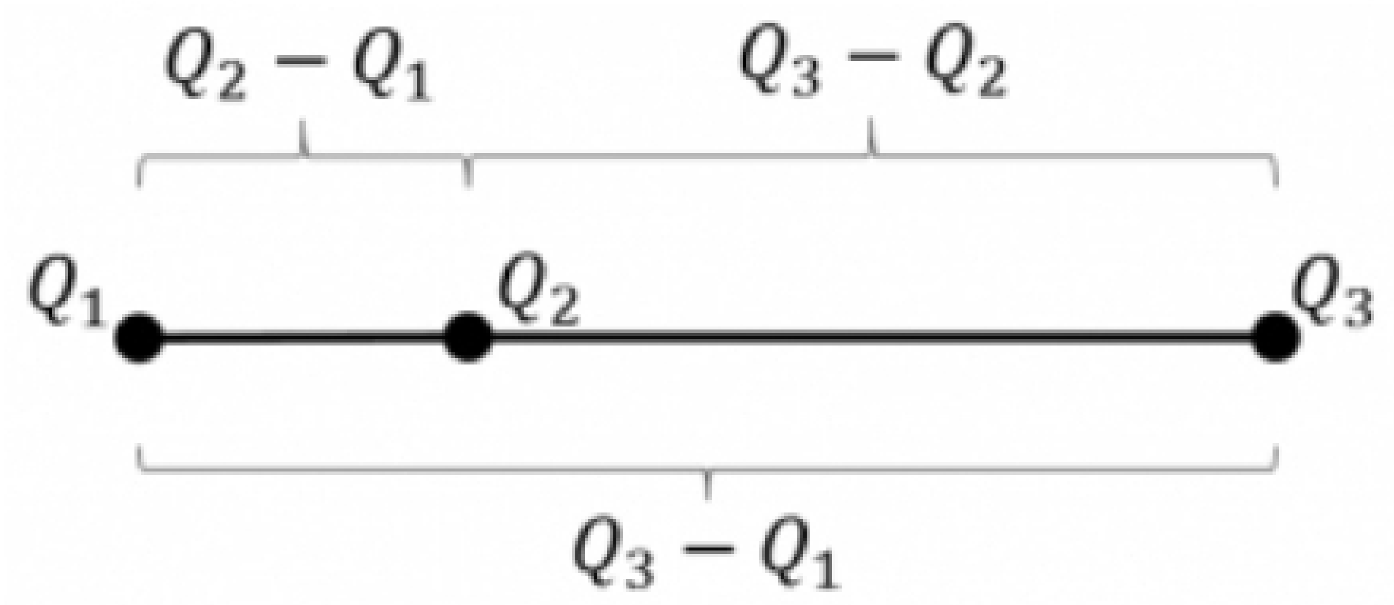
- Consider the following robust measure of skewness based on quantiles (Bowley,1920) :

$$SK_y = s(\mathbf{x}) = \frac{[Q_Y(0.75|\mathbf{x}) - Q_Y(0.5|\mathbf{x})] - [Q_y(0.5|\mathbf{x}) - Q_y(0.25|\mathbf{x})]}{Q_Y(0.75|\mathbf{x}) - Q_y(0.25|\mathbf{x})}$$

- In general, for appropriate chosen τ, we may measure the skewness of y given $\mathbf{x}$ by

$$SK_y = s(\tau, \mathbf{x}) = \frac{[Q_y(1-\tau|\mathbf{x}) - Q_y(0.5|\mathbf{x})] - [Q_y(0.5|\mathbf{x}) - Q_y(\tau|\mathbf{x})]}{Q_y(1-\tau|\mathbf{x}) - Q_y(\tau|\mathbf{x})}$$

Intuition of Skewness Measure



Measurement of Kurtosis

- Consider the following robust measure of kurtosis based on quantiles (Moors, 1988):

$$KUR_y = k(\mathbf{x}) = \frac{\left[Q_y(7/8|\mathbf{x}) - Q_y(5/8|\mathbf{x})\right] + \left[Q_y(3/8|\mathbf{x}) - Q_y(1/8|\mathbf{x})\right]}{Q_y(6/8|\mathbf{x}) - Q_y(2/8|\mathbf{x})}$$

- Moors (1988) shows that the conventional moment-based measure of kurtosis can be interpreted as a measure of the dispersion of a distribution around the two values $\mu \pm \sigma$.

Quantile Regression

- We consider the linear quantile regression model, which assumes that the conditional quantile functions of y given $\mathbf{z} = (1\ \mathbf{x}')'$ are linear in covariates:

$$Q_y(\tau \mid \mathbf{x}) = \boldsymbol{\theta}(\tau)'\mathbf{z}$$

- Extensions to other quantile regression models can also be analyzed

Estimation of Quantile Regression

- The quantile regression estimator solves

$$\hat{\boldsymbol{\theta}}(\tau) = \underset{\boldsymbol{\theta} \in \mathbb{R}^{p+1}}{\operatorname{argmin}} \sum_{t=1}^{n} \rho_\tau (y_t - \mathbf{z}_t' \boldsymbol{\theta})$$

- $\rho_\tau(u) = u\left(\tau - I(u < 0)\right)$ is the check function

- The estimated conditional quantile function:

$$\hat{Q}_{y_t}\left(\tau \mid \mathbf{z}_t\right) = \mathbf{z}_t' \hat{\boldsymbol{\theta}}(\tau)$$

3. The Spread Regression

- Suppose that the conditional quantiles $Q_y(1-\tau\,|\,\mathbf{x})$ and $Q_y(\tau\,|\,\mathbf{x})$ are estimated by appropriate quantile regressions, and denote the estimators by $\hat{Q}_y(1-\tau\,|\,\mathbf{x})$ and $\hat{Q}_y(\tau\,|\,\mathbf{x})$, then SP_y can be estimated via

$$\hat{m}(\tau,\mathbf{x}) = \hat{Q}_y(1-\tau\,|\,\mathbf{x}) - \hat{Q}_y(\tau\,|\,\mathbf{x})$$

The Spread Estimator

- Recall conditional spread is given by

$$SP_y = m(\tau, \mathbf{x}) = \boldsymbol{\theta}(1-\tau)'\mathbf{z} - \boldsymbol{\theta}(\tau)'\mathbf{z}$$

$$\hat{m}(\tau, \mathbf{x}) = \hat{\boldsymbol{\theta}}(1-\tau)'\mathbf{z} - \hat{\boldsymbol{\theta}}(\tau)'\mathbf{z}$$

- Consistency and asymptotic normality:

$$\sqrt{n}\left(\hat{m}(\tau, \mathbf{x}) - m(\tau, \mathbf{x})\right) \xrightarrow{d} N(0, V)$$

Reporting the Spread Regression

- The estimated coefficients of spread regression, as well as their corresponding standard errors and statistical significance, can be reported just like a typical linear regression.

- In this way, applied researchers can easily make inference about the effect of covariates $\mathbf{x}$ on the scale or dispersion of the conditional distribution of y given $\mathbf{x}$.

The Estimated Conditional Spread

- Sometimes we are interested in the estimated conditional spread $\hat{m}(\tau, \mathbf{x})$ as well.

- For example, we could use $\hat{m}(\tau, \mathbf{x})$ as a regressor in another regression, in the same spirit as estimated conditional heteroskedasticity from an ARCH or GARCH model is often further used as an input in a another regression.

4. The Skewness Regression

- Suppose that the conditional quantiles $Q_y(1-\tau\,|\,\mathbf{x})$ and $Q_y(\tau\,|\,\mathbf{x})$ are estimated by appropriate quantile regressions, and denote the estimators by $\hat{Q}_y(1-\tau\,|\,\mathbf{x})$ and $\hat{Q}_y(\tau\,|\,\mathbf{x})$, then SK_y can be estimated via

$$\hat{s}(\tau,\mathbf{x})=\frac{\left[\hat{Q}_y(1-\tau\,|\,\mathbf{x})-\hat{Q}_y(0.5\,|\,\mathbf{x})\right]-\left[\hat{Q}_y(0.5\,|\,\mathbf{x})-\hat{Q}_y(\tau\,|\,\mathbf{x})\right]}{\hat{Q}_y(1-\tau\,|\,\mathbf{x})-\hat{Q}_y(\tau\,|\,\mathbf{x})}$$

The Skewness Estimator

- Recall

$$SK_y = s(\tau, \mathbf{x}) = \frac{\left(\boldsymbol{\theta}(1-\tau) + \boldsymbol{\theta}(\tau) - 2\boldsymbol{\theta}(0.5)\right)' \mathbf{z}}{\left(\boldsymbol{\theta}(1-\tau) - \boldsymbol{\theta}(\tau)\right)' \mathbf{z}}$$

$$\hat{s}(\tau, \mathbf{x}) = \frac{\left(\hat{\boldsymbol{\theta}}(1-\tau) + \hat{\boldsymbol{\theta}}(\tau) - 2\hat{\boldsymbol{\theta}}(0.5)\right)' \mathbf{z}}{\left(\hat{\boldsymbol{\theta}}(1-\tau) - \hat{\boldsymbol{\theta}}(\tau)\right)' \mathbf{z}}$$

- which is a nonlinear function

Reporting the Skewness Regression

- One way to report the results of skewness regression is to report the regression coefficients and associated standard errors for the numerator and denominator separately.
- However, this would not be very helpful, since simultaneous movements in the numerator and denominator could cancel each other out.

Average Marginal Effects (AME)

- The AME of x_j on conditional skewness $\hat{s}(\tau, \mathbf{x})$ can be written as

$$AME_{skew,j} = \frac{1}{n}\sum_{t=1}^{n} \left. \frac{\partial \hat{s}(\tau, \mathbf{x})}{\partial x_j} \right|_{\mathbf{x}=\mathbf{x}_t}$$

- We may estimate the standard error of AME by the Delta Method.

The Estimated Conditional Skewness

- Sometimes, we are interested in using the estimated conditional skewness as an input in another regression (e.g., estimating a three-moment asset pricing model).

- It can be shown that $\hat{s}(\tau, \mathbf{x})$ is a consistent estimator of $s(\tau, \mathbf{x})$, and it is also asymptotically normal.

5. The Kurtosis Regression

- Suppose that the conditional quantiles $Q_y(1-\tau\,|\,\mathbf{x})$ and $Q_y(\tau\,|\,\mathbf{x})$ are estimated by appropriate quantile regressions, and denote the estimators by $\hat{Q}_y(1-\tau\,|\,\mathbf{x})$ and $\hat{Q}_y(\tau\,|\,\mathbf{x})$, then KUR_y can be estimated via

$$\hat{k}(\mathbf{x}) = \frac{\left[\hat{Q}_y(7/8\,|\,\mathbf{x}) - \hat{Q}_y(5/8\,|\,\mathbf{x})\right] + \left[\hat{Q}_y(3/8\,|\,\mathbf{x}) - \hat{Q}_y(1/8\,|\,\mathbf{x})\right]}{\hat{Q}_y(6/8\,|\,\mathbf{x}) - \hat{Q}_y(2/8\,|\,\mathbf{x})}$$

The Kurtosis Estimator

- By the plug-in principle,

$$\widehat{KUR}_y = \hat{k}(\mathbf{x}) = \frac{\left[\hat{\boldsymbol{\theta}}(7/8\,|\,\mathbf{x}) - \hat{\boldsymbol{\theta}}(5/8\,|\,\mathbf{x}) + \hat{\boldsymbol{\theta}}(3/8\,|\,\mathbf{x}) - \hat{\boldsymbol{\theta}}(1/8\,|\,\mathbf{x})\right]' \mathbf{z}}{\left[\hat{\boldsymbol{\theta}}(6/8\,|\,\mathbf{x}) - \hat{\boldsymbol{\theta}}(2/8\,|\,\mathbf{x})\right]' \mathbf{z}}$$

- which is a nonlinear function

Average Marginal Effects (AME)

- The AME of x_j on conditional kurtosis $\hat{k}(\mathbf{x})$ can be written as

$$AME_{kurtosis,j} = \frac{1}{n}\sum_{t=1}^{n} \frac{\partial \hat{s}(\mathbf{x})}{\partial x_j}\bigg|_{\mathbf{x}=\mathbf{x}_t}$$

- We may estimate the standard error of AME by the Delta Method.

The Estimated Conditional Kurtosis

- Sometimes, we are interested in using the estimated conditional kurtosis as an input in another regression (e.g., estimating a four-moment asset pricing model).

- It can be shown that $\hat{k}(\mathbf{x})$ is a consistent estimator of $k(\mathbf{x})$, and it is also asymptotically normal.

6. An Application to the U.S. Wage Data

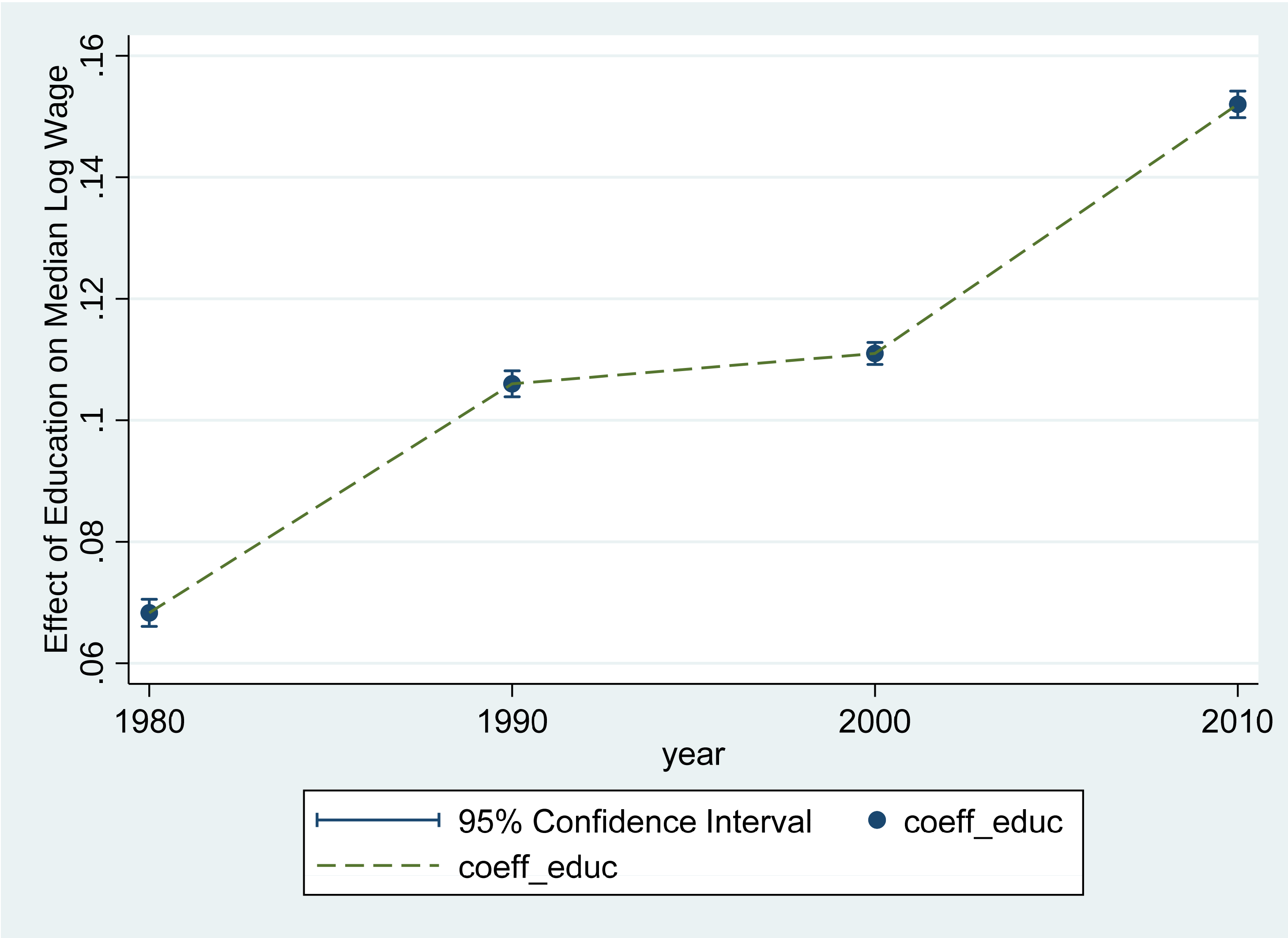
- Following Angrist, Chernozhukov and Fernandez-Val (2006, Econometrica), we use 1% US Census data in 1980, 1990, 2000, 2010 to study the effect of covariates on the median, spread, skewness and kurtosis of the conditional distribution of log real weekly wage.

- **Sample**: U.S.-born black and white men aged 40-49 with at least five years of education, with positive annual earnings and hours worked in the year preceding the census.

Results from Median Regressions: Coefficient Estimates

	(1) 1980	(2) 1990	(3) 2000	(4) 2010
educ	0.0683***	0.106***	0.111***	0.152***
	(0.00114)	(0.00109)	(0.000923)	(0.00111)
black	-0.248***	-0.191***	-0.234***	-0.273***
	(0.0114)	(0.00867)	(0.00857)	(0.00748)
exper	0.0278***	0.0568***	-0.0108	0.0418***
	(0.00444)	(0.00495)	(0.00703)	(0.00604)
exper2	-0.000460***	-0.000828***	0.000266*	-0.000638***
	(0.0000866)	(0.000104)	(0.000144)	(0.000119)
_cons	5.206***	4.166***	5.074***	4.063***
	(0.0644)	(0.0643)	(0.0887)	(0.0831)
N	65023	86785	97397	130956

Effects of Education on Median Log Wage



Interpretation of Median Regression

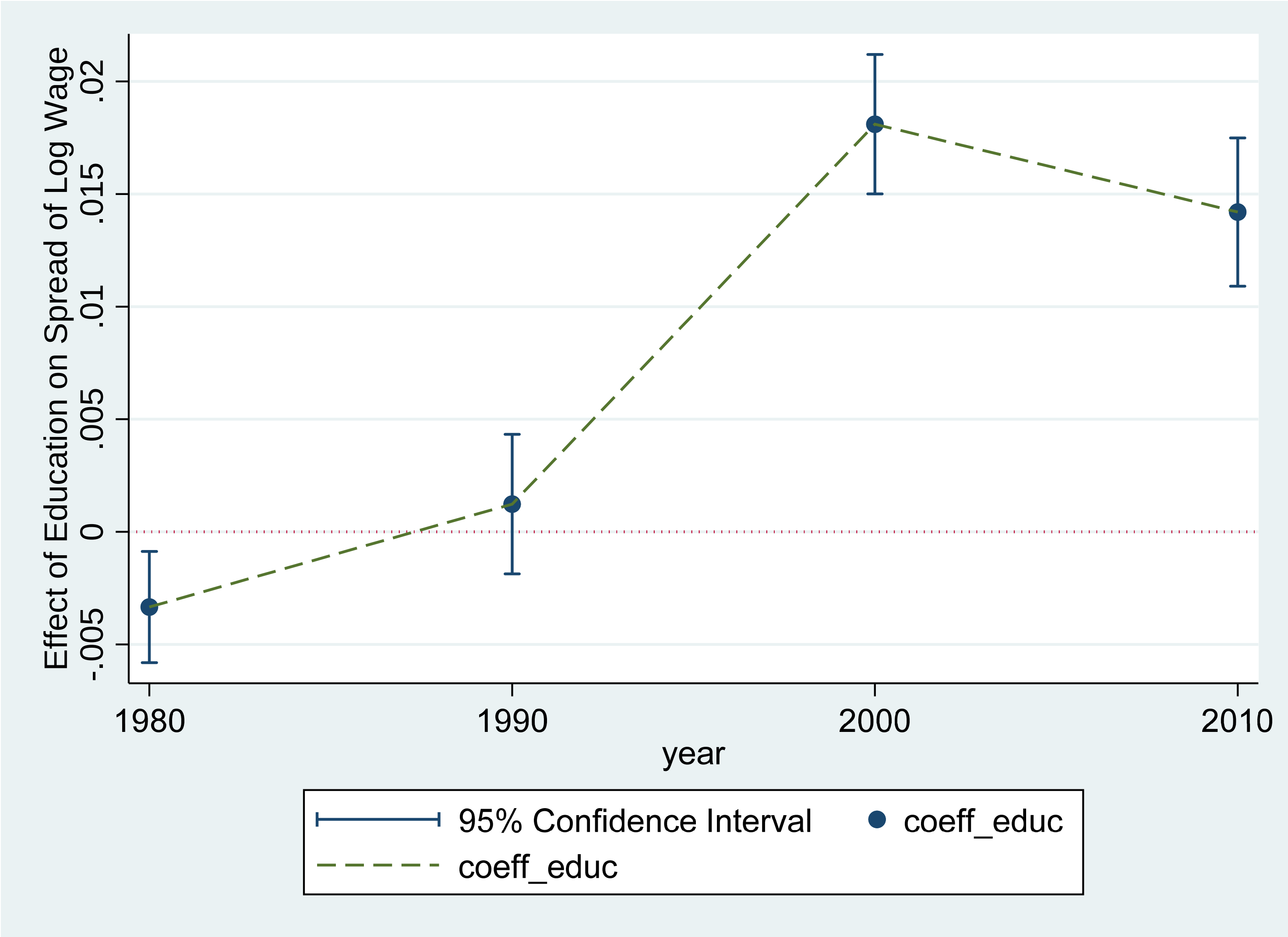
- The returns to education rose sharply from 1980 to 1990, stabilized during 1990-2000, and picked up steam again from 2000 to 2010.

- The confidence bands are very narrow, since the returns to education are estimated quite precisely given the large sample sizes.

Results from Spread Regressions: Coefficient Estimates (.75 - .25)

	(1) 1980	(2) 1990	(3) 2000	(4) 2010
educ	-0.00334***	0.00123	0.0181***	0.0142***
	(0.00126)	(0.00158)	(0.00158)	(0.00168)
black	0.111***	0.0658***	0.0543***	0.108***
	(0.0121)	(0.0124)	(0.00886)	(0.0113)
exper	-0.0478***	-0.0128	-0.0319***	-0.0146*
	(0.00641)	(0.00814)	(0.00900)	(0.00847)
exper2	0.000950***	0.000303*	0.000677***	0.000349**
	(0.000123)	(0.000163)	(0.000181)	(0.000174)
_cons	1.170***	0.742***	0.795***	0.700***
	(0.0895)	(0.109)	(0.117)	(0.110)
N	65023	86785	97397	130956

Effects of Education on Spread of Log Wage



Interpretation of Spread Regression

- While the average marginal effect of education on the spread was negatively significant in 1980, it turned positive but insignificant in 1990, and became positively significant in 2000 and 2010.

- The reversal of sign and significance implies that while more education mildly reduced the dispersion or inequality of wage income in 1980, this effect disappeared in 1990, whereas in 2000 and 2010, more education increased the inequality of income instead.

Results from Skewness Regressions: Average Marginal Effects

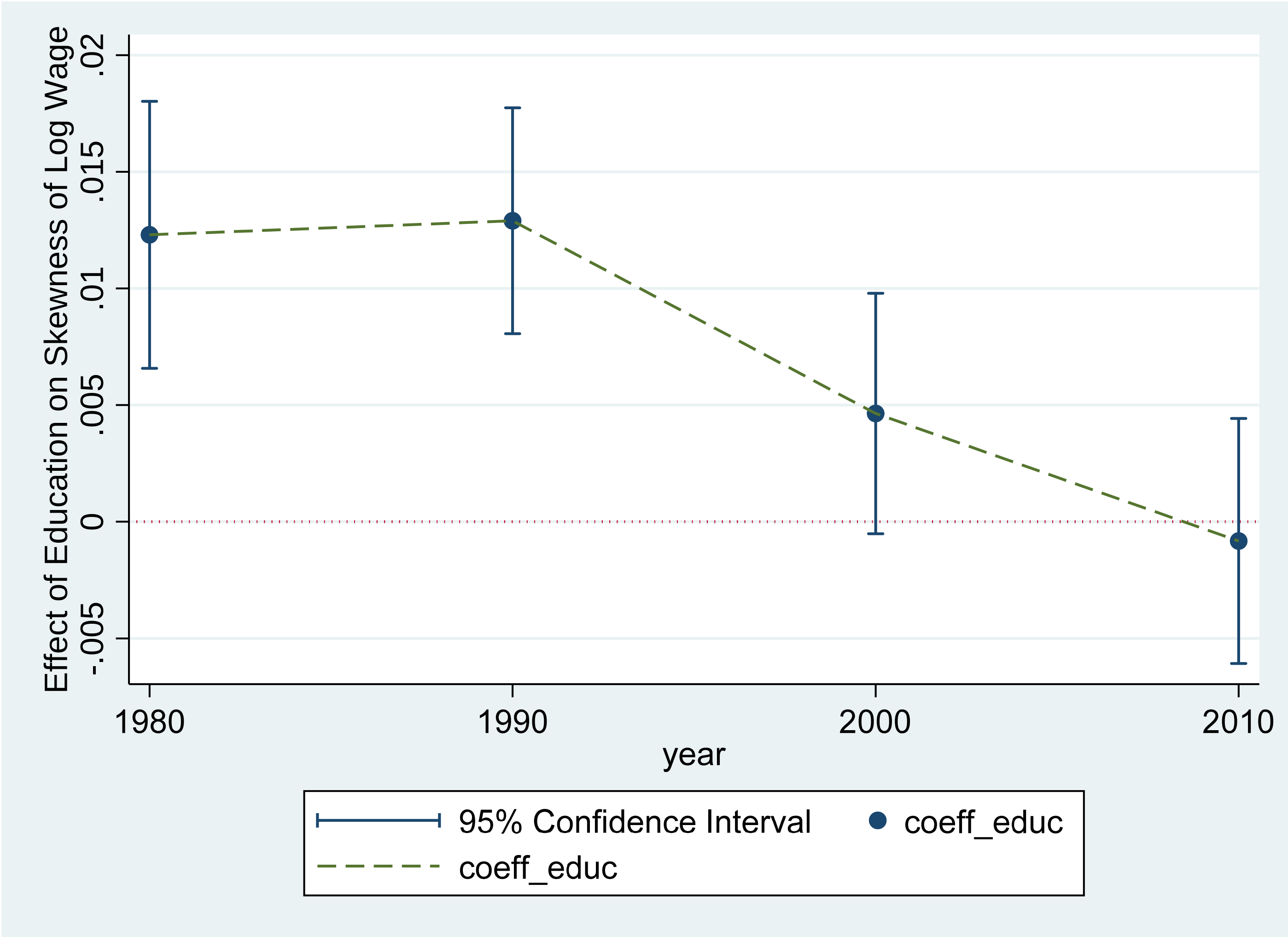
	(1) 1980	(2) 1990	(3) 2000	(4) 2010
educ	0.0123***	0.0129***	0.00464*	-0.000827
	(0.00292)	(0.00247)	(0.00263)	(0.00268)
black	-0.0312	-0.0576***	-0.00974	-0.000700
	(0.0234)	(0.0183)	(0.0179)	(0.0139)
exper	0.00423**	-0.000493	-0.00162	-0.00109
	(0.00191)	(0.00172)	(0.00191)	(0.00157)
N	65023	86785	97397	130956

Interpretation of Skewness Regression

- In 1980 and 1990, the average marginal effect of education on the skewness was positively significant at the 1% level, i.e., more education made the conditional distribution of log real wage skewed to the right.

- However, this effect was only positively significant at the 10% level in 2000, and turned negative although insignificant in 2010. In other words, more education ceased to contribute to the right skewness of income distribution in 2010.

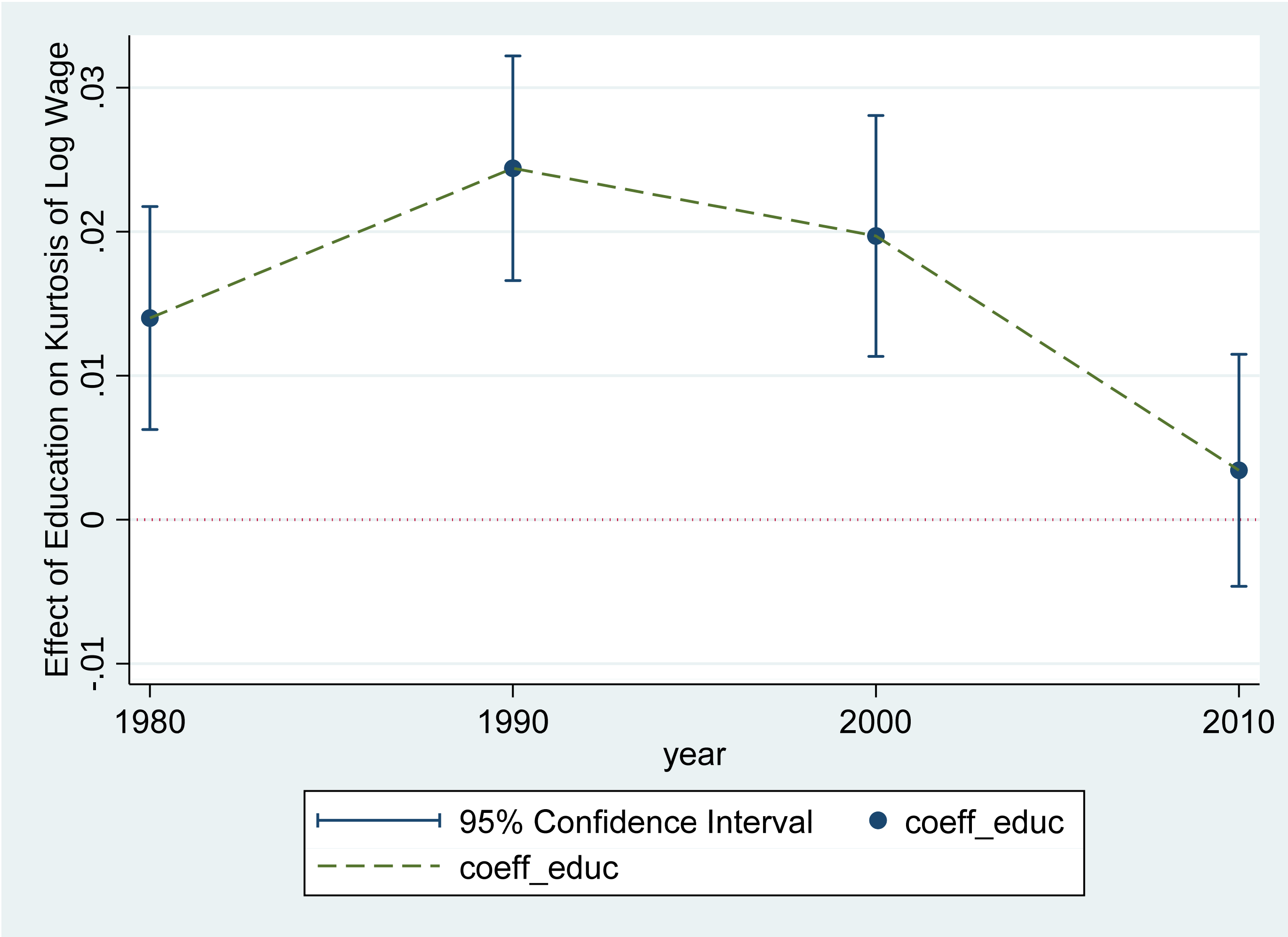
Average Marginal Effects of Education on Skewness of Log Wage



Results from Kurtosis Regressions: Average Marginal Effects

	(1) 1980	(2) 1990	(3) 2000	(4) 2010
educ	0.0140***	0.0244***	0.0197***	0.00343
	(0.00395)	(0.00398)	(0.00427)	(0.00411)
black	-0.146***	-0.0954***	0.0121	-0.0275
	(0.0274)	(0.0266)	(0.0267)	(0.0218)
exper	0.00315	0.00266	0.00552**	-0.000998
	(0.00289)	(0.00251)	(0.00269)	(0.00275)
N	65023	86785	97397	130956

Average Marginal Effects of Education on Kurtosis of Log Wage



Interpretation of Kurtosis Regression

- Throughout 1980-2000, the average marginal effect of education on kurtosis was positively significant at the 1% level, i.e., more education increased fat tails or tail risk in the income distribution.

- The magnitude of this effect changed over time. From 1980 to 1990, the positive effect of education on kurtosis increased.

- But the positive effect of education on kurtosis declined during 1990-2010, and became insignificant in 2010.

7. Applications in Stata

- Spread regression can be implemented by official Stata command **iqreg** (interquantile regression)

- Skewness and kurtosis regressions can be implemented by user-written commands **skewreg** and **kurtosisreg**

安装skewreg与kurtosisreg命令

- `ssc install skewreg`
  (同时安装skewreg与kurtosisreg，也可使用命令 `net install skewreg`)

- `net get skewreg`
  (获取示例数据集census80.dta)

注：若下载数据集超时，可使用命令“`set timeout2 1000`”将超时上限设为1000秒（默认180秒）

help skewreg

help skewreg

Title

skewreg — Skewness Regression

Syntax

skewreg *depvar* [*indepvars*] [*if*] [*in*] [, *options*]

options	Description
Model	
quantile(#)	specify quantile of interest; default is **quantile(.25)**
reps(#)	specify number of bootstrap replications; default is **reps(50)**
seed(#)	set random seed; default is **seed(1)**
Reporting	
detail	show detailed results
graph	graph coefficients and confidence intervals
level(#)	set confidence level; default is **level(95)**
predict(*string*)	predict conditional skewness

indepvars may contain factor variables; see fvvarlist.
by and **bysort** are allowed; see prefix.

help kurtosis

help kurtosisreg

Title

kurtosisreg — Kurtosis Regression

Syntax

kurtosisreg *depvar* [*indepvars*] [*if*] [*in*] [, *options*]

options	Description
Model	
reps(#)	specify number of bootstrap replications; default is **reps(50)**
seed(#)	set random seed; default is **seed(1)**
Reporting	
detail	show detailed results
graph	graph coefficients and confidence intervals
level(#)	set confidence level; default is **level(95)**
predict(*string***)**	predict conditional kurtosis

indepvars may contain factor variables; see fvvarlist.
by and **bysort** are allowed; see prefix.

Example Data Set

- `sysuse census80,clear`

- `describe`

```
Contains data from ./census80.dta
  obs:        65,023
 vars:             7                          24 Jul 2020 08:10
                                              (_dta has notes)
---------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
---------------------------------------------------------------------------------
age             float   %9.0g                 Age in Years
educ            float   %9.0g                 Years of Schooling
logwk           float   %9.0g                 Average Log Weekly Wage in 1989
                                                Dollars
perwt           float   %9.0g                 Person Weight
exper           float   %9.0g                 Potential Experience (age - educ - 6)
exper2          float   %9.0g                 Square of exper
black           float   %9.0g                 Black or African American
---------------------------------------------------------------------------------
```

Notes about Data

- notes

```
_dta:
  1.  1% US census data in 1980 obtained from Angrist, Chernozhukov and
      Fernandez-Val(2006) for U.S.-born black and white men aged 40-49 with five or
      more years of education, positive annual earnings, and positive hours worked in
      the year preceding the census. Individuals with imputed values for age,
      education, earnings or weeks worked were also excluded from the sample.
```

Spread Regression

- `set seed 123`
- `iqreg logwk educ black exper c.exper#c.exper,nolog reps(50)`

- Note: Use `c.exper#c.exper` instead of `exper2` to get correct marginal effects.

```
.75-.25 Interquantile regression                    Number of obs =     65,023
  bootstrap(50) SEs                                 .75 Pseudo R2 =     0.1004
                                                    .25 Pseudo R2 =     0.0797
```

logwk	Coef.	Bootstrap Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	-.0033427	.0013555	-2.47	0.014	-.0059994	-.000686
black	.1109744	.0132168	8.40	0.000	.0850695	.1368793
exper	-.0478427	.0079419	-6.02	0.000	-.0634088	-.0322766
c.exper#c.exper	.0009504	.0001483	6.41	0.000	.0006597	.0012411
_cons	1.170364	.115405	10.14	0.000	.9441702	1.396558

Average Marginal Effects

- `margins,dydx(*)`

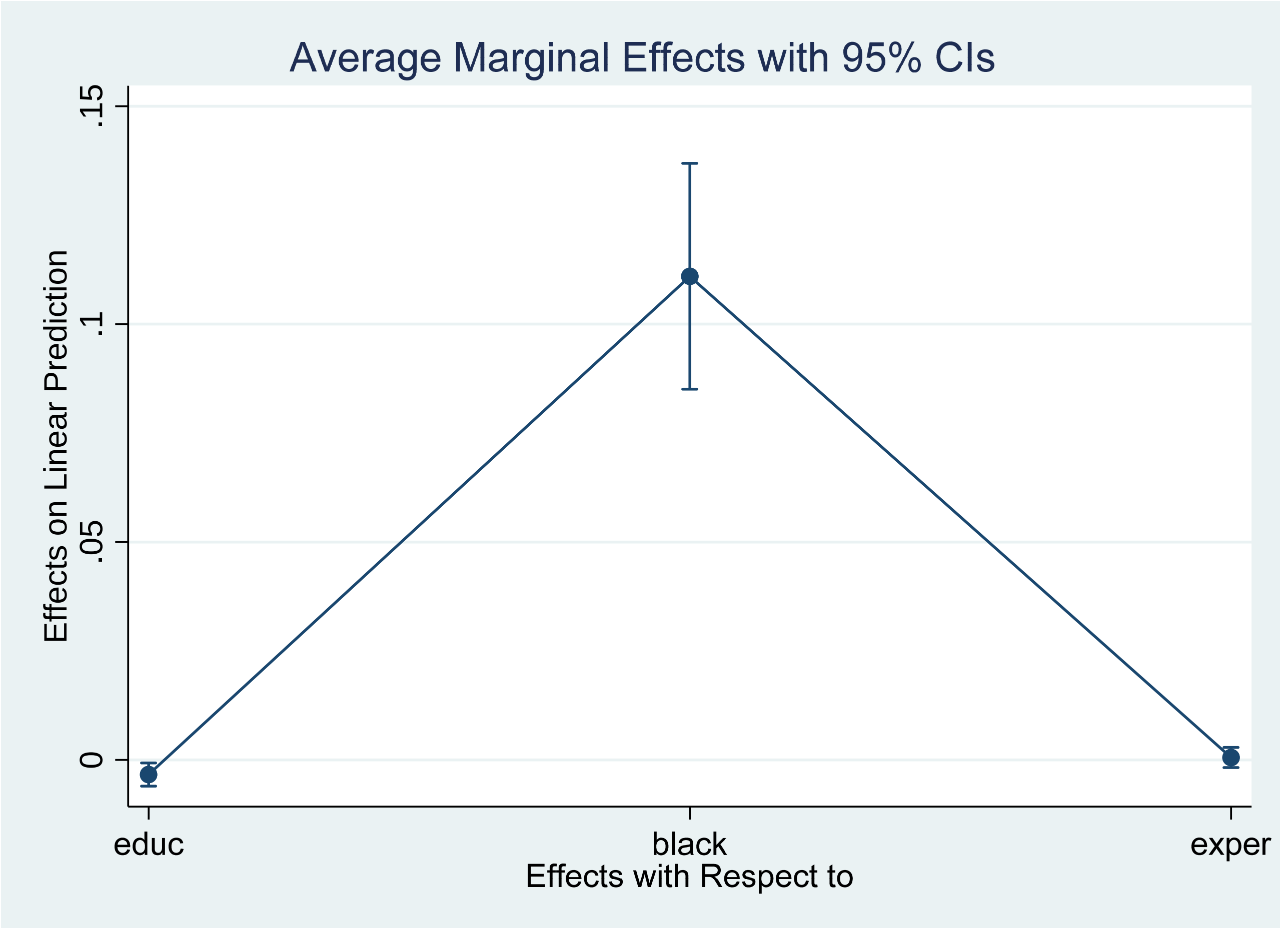
```
Average marginal effects                        Number of obs     =     65,023
Model VCE    : Bootstrap

Expression   : Linear prediction, predict()
dy/dx w.r.t. : educ black exper
```

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
educ	-.0033427	.0013555	-2.47	0.014	-.0059994	-.000686
black	.1109744	.0132168	8.40	0.000	.08507	.1368788
exper	.0005575	.0011797	0.47	0.637	-.0017546	.0028697

Visualize Average Marginal Effects

- `marginsplot`



Skewness Regression

- `skewreg logwk educ i.black exper c.exper#c.exper,seed(123) reps(50) graph predict(skewness)`

- Note: Use `i.black` instead of `black` for correct computation of average marginal effects.

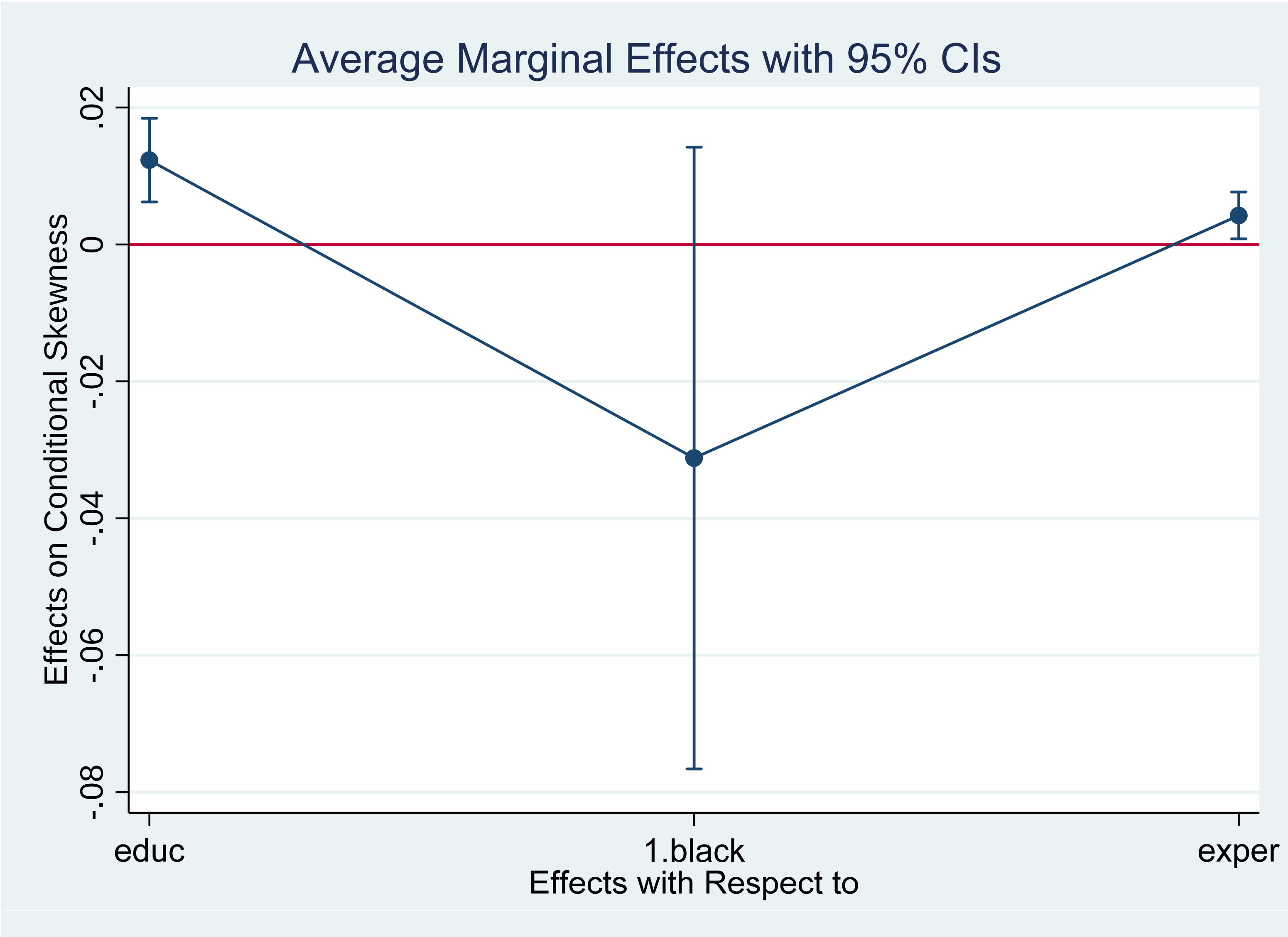
Skewness regression: Average marginal effects

$$\frac{[Q(.75|x)-Q(0.5|x)]-[Q(0.5|x)-Q(.25|x)]}{Q(.75|x)-Q(.25|x)}$$

Number of obs = 65,023
Random seed = 123
Number of reps = 50

Skewness	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	.0123256	.0031234	3.95	0.000	.0062038	.0184475
1.black	-.0311925	.0231731	-1.35	0.178	-.0766118	.0142268
exper	.0042327	.0017501	2.42	0.016	.0008025	.007663

Note: Std. Err. computed by the delta method from bootstrap standard errors.


Average Marginal Effects with 95% CIs
Effects on Conditional Skewness
.02
0
-.02
-.04
-.06
-.08
educ
1.black
exper
Effects with Respect to

Skewness Regression at a Different Quantile with Detailed Results

- `skewreg logwk educ i.black exper c.exper#c.exper,seed(123) reps(50) quantile(0.1) detail`

```
Simultaneous quantile regression                    Number of obs =     65,023
  bootstrap(50) SEs                                 .10 Pseudo R2 =     0.0528
                                                    .50 Pseudo R2 =     0.0878
                                                    .90 Pseudo R2 =     0.1100
```

logwk	Coef.	Bootstrap Std. Err.	t	P>|t|	[95% Conf.	Interval]
q10						
educ	.0734956	.0028306	25.96	0.000	.0679475	.0790436
1.black	-.3296203	.0246119	-13.39	0.000	-.3778597	-.2813809
exper	.0469789	.0121046	3.88	0.000	.0232539	.0707039
c.exper#c.exper	-.0008963	.0002348	-3.82	0.000	-.0013564	-.0004362
_cons	4.275216	.1709273	25.01	0.000	3.940198	4.610233
q50						
educ	.0683212	.001133	60.30	0.000	.0661006	.0705418
1.black	-.2483907	.0104287	-23.82	0.000	-.268831	-.2279504
exper	.0277656	.0041316	6.72	0.000	.0196677	.0358635
c.exper#c.exper	-.00046	.0000787	-5.85	0.000	-.0006143	-.0003058
_cons	5.206376	.0628533	82.83	0.000	5.083184	5.329569
q90						
educ	.0790741	.0017326	45.64	0.000	.0756782	.08247
1.black	-.2130949	.0095937	-22.21	0.000	-.2318985	-.1942912
exper	-.0387191	.0080783	-4.79	0.000	-.0545525	-.0228857
c.exper#c.exper	.0008646	.0001461	5.92	0.000	.0005783	.001151
_cons	6.391171	.1217469	52.50	0.000	6.152547	6.629795

Skewness regression: The numerator part
[Q(.9|x)-Q(0.5|x)]-[Q(0.5|x)-Q(.1|x)]

Number of obs = 65,023
Random seed = 123
Number of reps = 50

Numerator	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	.0159273	.0033627	4.74	0.000	.0093364	.0225182
1.black	-.0459338	.0266477	-1.72	0.085	-.0981634	.0062957
exper	-.0020287	.0019909	-1.02	0.308	-.0059308	.0018733

Skewness regression: The denominator part
[Q(.9|x)-Q(.1|x)]
(same as spread/interquantile regression)

Number of obs = 65,023
Random seed = 123
Number of reps = 50

Denominator	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	.0055785	.0029505	1.89	0.059	-.0002046	.0113616
1.black	.1165254	.0276245	4.22	0.000	.0623814	.1706695
exper	.0039807	.001516	2.63	0.009	.0010094	.006952

```
Skewness regression: Average marginal effects      Number of obs  =     65,023
 [Q(.9|x)-Q(0.5|x)]-[Q(0.5|x)-Q(.1|x)]                Random seed  =        123
________________________________________           Number of reps =         50
            Q(.9|x)-Q(.1|x)
```

Skewness	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	.0140413	.0027322	5.14	0.000	.0086863	.0193964
1.black	-.0236819	.0186098	-1.27	0.203	-.0601572	.0127934
exper	-.000852	.0016073	-0.53	0.596	-.0040024	.0022983

Note: Std. Err. computed by the delta method from bootstrap standard errors.

Kurtosis Regression

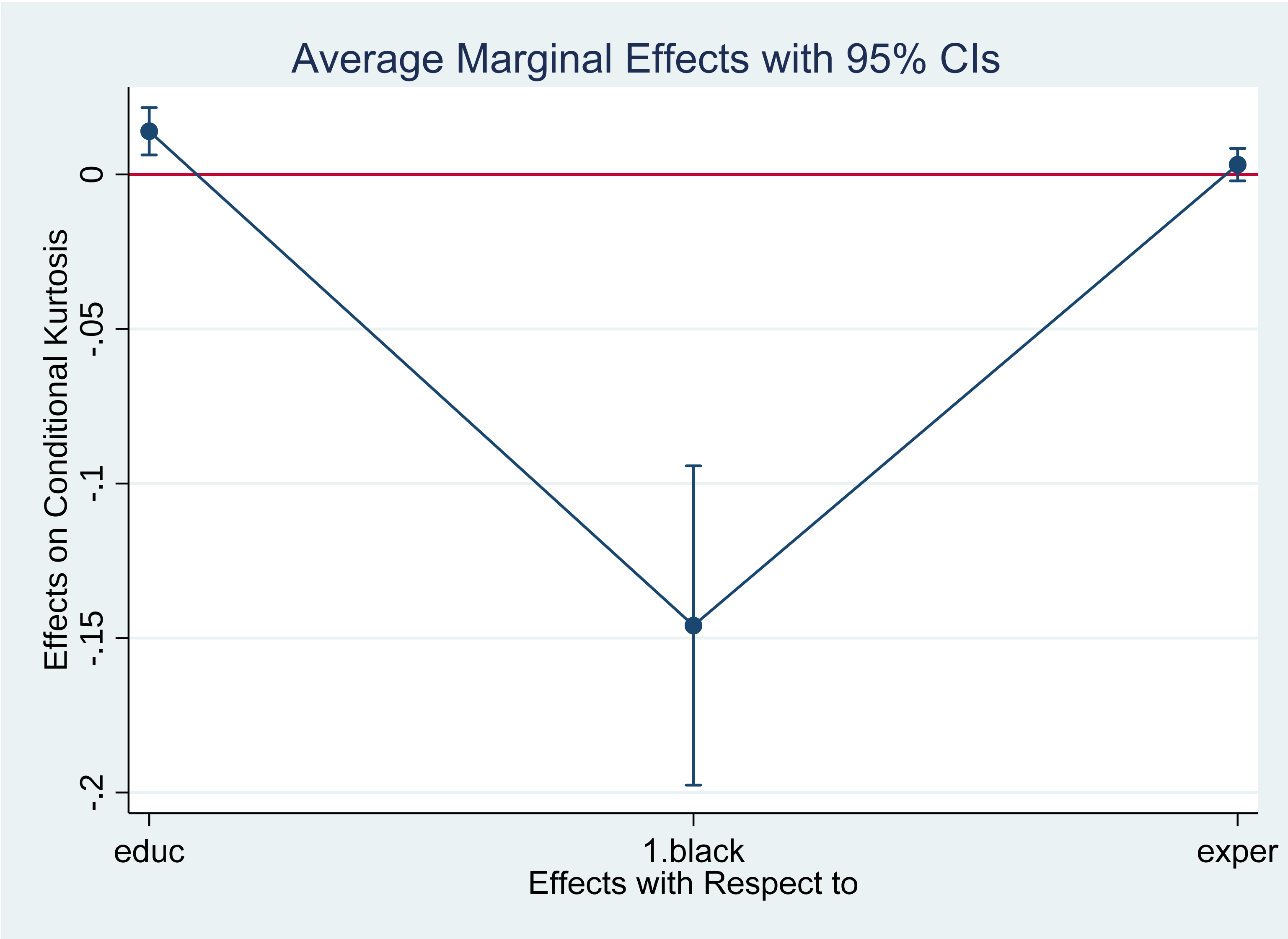
- `kurtosisreg logwk educ i.black exper c.exper#c.exper,seed(123) reps(50) graph predict(kurtosis)`

- Note: Use `i.black` instead of `black` for correct computation of average marginal effects.

```
Kurtosis regression: Average marginal effects       Number of obs  =     65,023
 [Q(7/8|x)-Q(5/8|x)]-[Q(3/8|x)-Q(1/8|x)]             Random seed  =        123
 -----------------------------------------          Number of reps =         50
            Q(6/8|x)-Q(2/8|x)
```

Kurtosis	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	.0139546	.0039017	3.58	0.000	.0063073	.0216019
1.black	-.1459633	.0263496	-5.54	0.000	-.1976084	-.0943181
exper	.0031493	.0026774	1.18	0.239	-.0020984	.0083971

Note: Std. Err. computed by the delta method from bootstrap standard errors.


Average Marginal Effects with 95% CIs
Effects on Conditional Kurtosis
0
-.05
-.1
-.15
-.2
educ
1.black
exper
Effects with Respect to

Welcome feedbacks!
Thank you ☺