Bayesian Analysis using Stata

Bill Rising
StataCorp LP

2015 Oceania Stata Users Group Meeting
Canberra, ACT
25 September 2015

Contents

1 Introduction

1.1 Goals

Goals

- Learn a little about Bayesian analysis
- Learn the core of how Bayesian analysis are implemented in Stata 14

1.2 Brief Glimpse into Bayesian Analysis

Uncertainty as Probability

- In the frequentist world, probabilities are long-run proportions of repeated identical experiments
    - ◇ In some ways, this means we never know any probabilities of any events
- In the Bayesian world, probabilities are an expression of uncertainty
    - ◇ The advantage of the Bayesian viewpoint is that it allows talking about probabilities for events which cannot be repeated
        - ⋆ What is the chance of a major earthquake in Alaska this year?
        - ⋆ What is the chance that Australia takes the 2015 Rugby World Cup?
    - ◇ The disadvantage is that these probabilities become subjective

---

Bayesian Analysis

- Uncertainty about parameters is expressed via a prior distribution $p(\boldsymbol{\theta})$
  - The prior distribution is necessarily subjective
  - If there is little knowledge about possible values, vague or non-informative priors get used
- The dataset $\mathbf{y}$ is used to update these priors into posterior distributions via Bayes rule

$$p(\boldsymbol{\theta}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{y})}$$

  - $p(\mathbf{y}|\boldsymbol{\theta})$ is the likelihood
  - $p(\mathbf{y})$ is the marginal density of the data

$$p(\mathbf{y}) = \int_{\boldsymbol{\theta}} p(\mathbf{y}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

    - This last integral has been the bugaboo

Advantages and Disadvantages of Bayesian Analysis

- Advantages
  - Theoretically should allow updating knowledge with past experience
  - Can speak directly about probabilities instead of applying long-run proportions to a single event
    - Think of confidence intervals: have long-run chance of catching the parameter value, but know nothing about the current estimate
  - Can choose among multiple competing hypotheses
- Disadvantages
  - Could be worried about subjectivity

Why Has Bayesian Analysis Become More Popular

- Computational speed allows rapid but good approximations of the marginal density of the data
  - Before computational horsepower could be used, only a small set of models could be estimated
- All the magic comes from Markov Chain Monte Carlo (MCMC) methods
  - These sample points from the not-fully-specified density in such a way that if left running forever, the density of simulation points would equal the target density

---

Implementation in Stata 14

- In Stata 14, the estimation portion of Baysian analysis is implented by the `bayesmh` command
  - `mh` for Metropolis-Hastings
- We will see how this works, both via point-and-click and syntactically
- We will look at some diagnostics and other post-estimation tools

---

2 Bayesian Analysis in Stata 14

2.1 Starting Simple

A Simple Story

- We'll work with a very simple dataset measuring counts
- Here is our simulated story:
  - We've collected data from 70 people in Canberra about the number of parking tickets they've gotten in the last year
  - We would like to get some concept of the rate the people get the tickets
  - We will do this based on the rumor that Canberra is particularly finicky about parking
- We'll simulate a dataset as though the true number of parking fines per year per person is 1.3

```
. do makepois

. set more off

. clear

. * pick a seed for reproducibility
. set seed 1800

. * set the number of observations
. set obs 70
number of observations (_N) was 0, now 70

. * create the observations
. gen y = rpoisson(1.3)

. label var y "Parking tickets in Canberra"

. label data "The IKEA of datasets: one variable of counts"

.
. save pois
file pois.dta saved

.
end of do-file
```

- Let's see the mean count for this simulation

```
. sum y

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
           y |         70    1.257143    1.099219          0          3
```

Starting a Bayesian Analysis: the Prior

- We would now like to do a Bayesian investigation of the rate λ of getting fined
  - Suppose we are truly interested whether the rate of fines is over one per year per person
- To start out, we need to specify a prior distribution
- How would this possibly be done?
  - We could try to use a vague prior which has very little information in it
  - We could try to elicit the opinions of experts
- We'll start with a vague prior

Choosing a Vague Prior

- Vague priors are only vaguely defined: they ought to cover all remotely plausible values without favoring any values
- We will choose a flat prior, meaning that all possible ticketing rates have the same probability
  - Because this means that we need a probability density proportional to 1 over the interval 0 to ∞, this is an improper prior
  - Improper priors should typically be avoided, but this will help the exposition here
- So, for us, $p(\lambda) \propto 1$ for $0 < \lambda < \infty$
  - Clearly, like continuous-time white noise, this is impossible but helpful

---

Specifying our Model: the Interface

- We will start by using the point-and-click interface
- There are two ways to access this
  - Either select **Statistics** > **Bayesian analysis** > **Estimation**
  - Or type `db bayesmh` in the command window
- We will choose what we would like to do now, and then come back to the full range of possible models

---

Choosing the Likelihood Model

- We would like a univariate linear model
- Clicking the drop-down menu for the *Dependent variable* and choose `y`
- We have no independent variables
- Choose *Poisson regression* as the ***Likelihood model***
- We can leave the *Exposure variable* blank
- Tick the *Do not exponentiate linear predictor*
  - This will cause our out output to report rates instead of the natural log of rates

---

Specifying the Prior

- Click on the **Create...** button for the ***Priors of model parameters***
- From the *Parameters specification* dropdown, choose `{y:_cons}`
  - This is because we are modeling only the constant term without any covariates
- We will choose the *Flat prior* item
- Click **OK** to dismiss the subdialog

Making Our Computations Reproducible

- We should set a random seed for this MCMC
  - This will make sure that we can show our result in the future
- Click on the ***Simulation*** tab
- We'll put 7434 as the random seed
  - This is an arbitrary non-negative integer

Computing the Posterior

- We are already done specifying this simple model, so click the **Submit** button
- The command gets issued

```
. bayesmh y, likelihood(poisson, noglmtransform) ///
     prior({y:_cons}, flat) rseed(7434)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ poisson({y:_cons})

Prior:
  {y:_cons} ~ 1 (flat)
------------------------------------------------------------------------------

Bayesian Poisson regression                      MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         70
                                                 Acceptance rate  =      .4271
Log marginal likelihood = -102.22367             Efficiency       =      .2315

------------------------------------------------------------------------------
             |                                                Equal-tailed
           y |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
       _cons |  1.274535   .1358713   .002824   1.274437   1.020925   1.548038
------------------------------------------------------------------------------
```

- Stata churns through the MCMC computations to find the posterior distribution
- Stata reports the results

General Notes about the Output

- At the top, you see `Burn In ...` followed by `Simulation ...` as notifications
  - These would be for seeing progress in very computationally intensive models
- We see the two elements we need to specify for any Bayesian analysis: the Likelihood model and the Prior distribution
- There is information about how the MCMC sampling was done
- There is information about summary statistics of the posterior distribution
  - Recall that we are not specifically trying to estimate mean values; we are finding a posterior distrbution

---

Output Specifics: MCMC

- By default, Stata uses a burn-in of 2,500 iterations
  - This is used to tune the adaptive model and to give time for the simulation to reach the main part of the posterior distribution
- By default, Stata runs the MCMC chain for 10,000 iterations
- The acceptance rate is the rate that new picks from the distribution are accepted
- The efficiency is relative to independent samples from the posterior distribution

---

Output Specifics: Regression Table

- The mean of our posterior distribution for the arrival rate is 1.27
- The standard deviation of the posterior distribution is 0.136
- The MCSE of 0.0028 is the standard error of estimation of the mean due to our using MCMC to find the posterior distribution
  - How much the posterior mean would vary from run to run if we used different random seeds
- The median is the median of the posterior distribution
- The probability that the arrival rate is between 1.021 and and 1.548 is 95%
  - Note this is not a trapping probability for unknown future samples

---

Starting with Postestimation

- We can see what postestimation commands are available by typing

```
. db postest
```

- Now click on the disclosure control next to *Bayesian analysis*
- Select the *Graphical summaries and convergence diagnostics* item
- Click on the **Launch** button

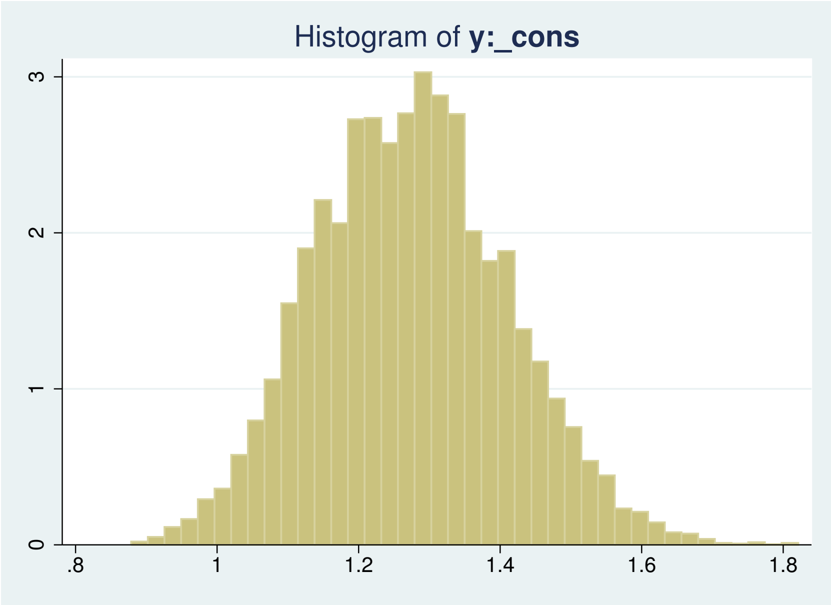
Investigating the Posterior

- We can draw a picture of the posterior distribution in a couple of ways
- To make a histogram, select the *Histograms* graph type
- To make life simple select the *Graphs for all model parameters* radio button
- Click on the **Submit** button

Histogram of the Posterior

- Here is the histogram version of the posterior distribution

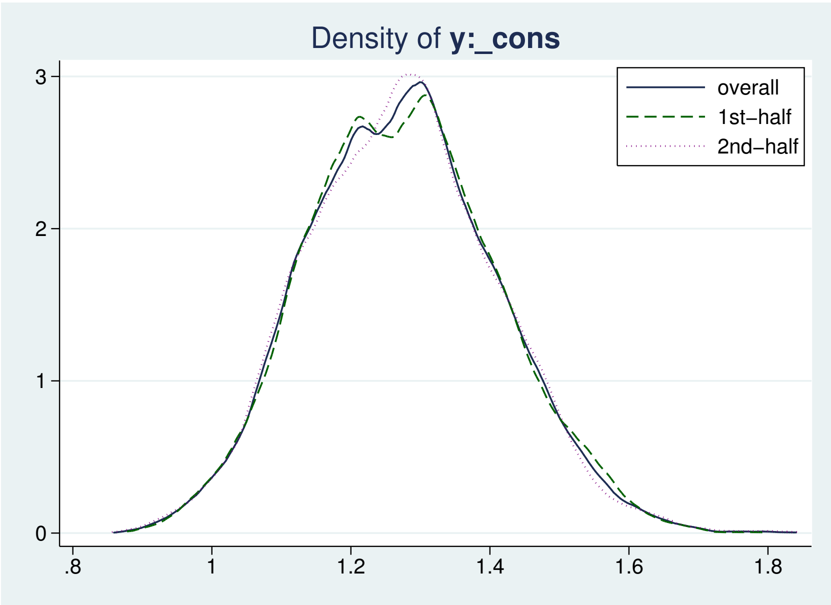
```
. bayesgraph histogram _all
```



Density Plot of the Posterior

- To get a density plot, select the *Density plots* graph type
- Click on the **Submit** button

```
. bayesgraph kdensity _all
```



Finding the Probability the Rate is Larger than 1

- Navigate back to the *Postestimation Selector* dialog box
- Select the *Interval hypothesis testing* menu item
- Choose {y:_cons} parameter from the *Test model parameter* list
- Enter 1 as the *Lower* bound and leave . as the *Upper* bound
- Click the **Submit** button

```
. bayestest interval ({y:_cons}, lower(1))

Interval tests     MCMC sample size =    10,000

       prob1 : {y:_cons} > 1

-----------------------------------------------
             |      Mean    Std. Dev.      MCSE
-------------+---------------------------------
       prob1 |     .9837     0.12663   .0024803
-----------------------------------------------
```

- We can read off the probability as 0.98
  - ◇ This is a true probability
  - ◇ It is a subjective probability based on our flat prior

2.2 Looking More Carefully

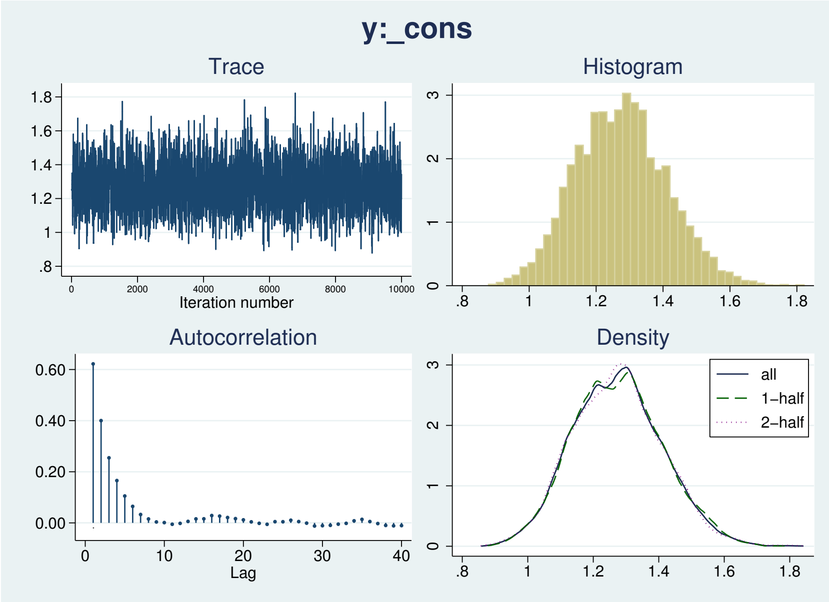
How MCMC Can Break

- There are multiple ways that MCMC can give bad answers
  - ◇ It can mix poorly, meaning either that
    - ⋆ New candidate points for the simulation get rejected too often
    - ⋆ The jumps are too small to cover the distribution
  - ◇ It can have bad initial values
    - ⋆ These should be irrelevant because of the long burn-in sequence
    - ⋆ But... if there is poor mixing this might not be the case
    - ⋆ This leads to what is called 'drift'

MCMC Diagnostics

- There is a simple tool for looking at the standard diagnostics all at once
- Select *Multiple diagnostics in compact form* in the *bayesgraph* dialog, and press **Submit**

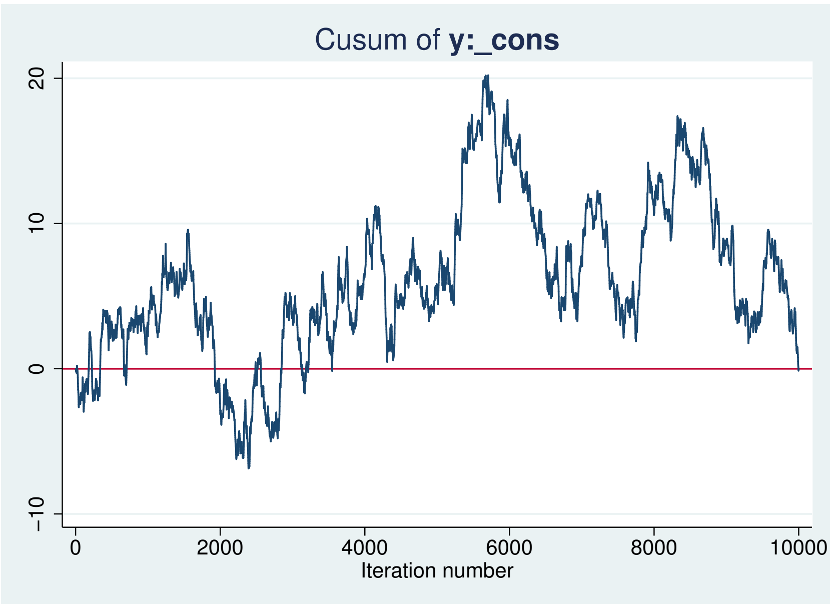
```
. bayesgraph diagnostics _all
```



Looking for Drift

- The cusum (short for cumulative sum) plot is used to look for small step size and drift
- Select *Cumulative sum plots* and press **Submit**

```
. bayesgraph cusum _all
```



Simple Diagnostic Conclusion

- Everything looks fine because there is no sign of bad mixing or drift

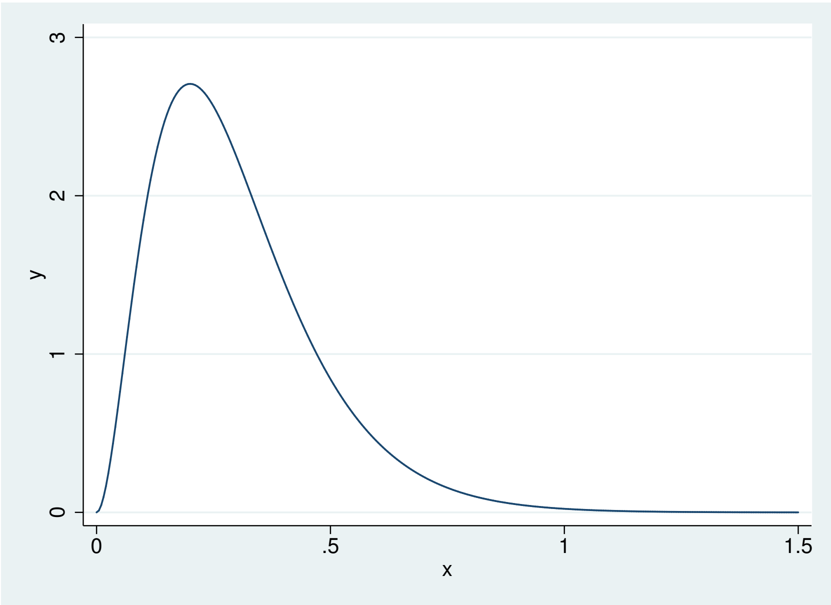
Playing with Different Priors

- Suppose we talk to people in Sydney, Melbourne, Adalaide, and Brisbane
- They all agree that the the rate of fines should be about 1 every 3 years, with little chance of averaging more than 1 fine per year
    - ◇ Thus, they are completely incorrect about Canberra
- Based on this, a good prior would be a Gamma(3, 0.1)

Aside: Graph of the Prior

- Here is a graph of the Gamma(3, 0.1) distribution

```
. twoway function y = gammaden(3,0.1,0,x), range(0 1.5)
```



Specifying a New Prior

- Type `db bayesmh` to get our dialog box back
- Select the *Prior 1* prior
- Click on the **Edit** button
- Choose *Gamma distribution*
- Enter 3 as the *Shape* and 0.1 as the *Scale*
- Click on the `OK` button to dismiss the subdialog

Changing the Seed

- Go to the ***Simulation*** tab
- Change the random seed to some other number, say 9983
- Click on the `Submit` button to run the analysis

```
. bayesmh y, likelihood(poisson, noglmtransform) ///
     prior({y:_cons}, gamma(3,0.1)) rseed(9983)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ poisson({y:_cons})

Prior:
  {y:_cons} ~ gamma(3,0.1)
------------------------------------------------------------------------------

Bayesian Poisson regression                      MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         70
                                                 Acceptance rate  =      .4313
Log marginal likelihood = -107.68681             Efficiency       =      .2345

------------------------------------------------------------------------------
             |                                                Equal-tailed
           y |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
       _cons |  1.136429   .1181007   .002439   1.130957   .9216155   1.380885
------------------------------------------------------------------------------
```

What Happened?

- We can see that the mean of the posterior distribution is smaller
  - ◇ We should, however, be encouraged that the mean is only somewhat smaller despite the very-different prior
- If we now compute our probability that the rate is larger than 1, though: 0.88

```
. bayestest interval ({y:_cons}, lower(1))

Interval tests     MCMC sample size =    10,000

       prob1 : {y:_cons} > 1

-----------------------------------------------
             |      Mean    Std. Dev.      MCSE
-------------+---------------------------------
       prob1 |     .8814     0.32333   .0062655
-----------------------------------------------
```

2.3 Changing the Problem

Specifying Our Own Likelihood

- What if we wanted a likelihood which is not one of the 10 built-in likelihoods?
- We can specify this by using the `likelihood()` option with the `llf()` suboption
- We just need an example to show this...

Changing the Problem

- Suppose now that our sample came just from those who had had a ticket in the last year

```
. drop if y == 0
(23 observations deleted)
```

  - ◇ We've lost quite a bit of our sample
- We cannot use the same likelihood model as we had before
- Instead, we have a truncated Poisson, where the probability of 0 fines has become 0
- Truncated Poisson distributions are not a part of Stata's suite, so we need to do some math

---

Writing Our New Likelihood Model

- Here is the Poisson distribution with parameter λ is

$$p(y) = \frac{\lambda^y \mathrm{e}^{-\lambda}}{y!}; \qquad y = 0, 1, 2 \ldots$$

- If y cannot be zero, we just need to rescale to get the total probability to be 1:

$$p(y) = \frac{\lambda^y \mathrm{e}^{-\lambda}}{y!(1 - \mathrm{e}^{-\lambda})}; \qquad y = 1, 2 \ldots$$

- From this, our log likelihood becomes

$$y \ln(\lambda) - \lambda - \ln(y!) - \ln(1 - \mathrm{e}^{-\lambda})$$

Substitutable Expressions

- The way we tell Stata to use the log-likelihood function is by using a *substitutable expression*
- We just need to replace
  - ◇ Symbols with the variables that represent them
  - ◇ Coefficient names to replace parameters

Specifying Our New Likelihood Model

- In our case
  - ◇ `y` (the variable) replaces y the count symbol
  - ◇ `{y:_cons}` replaces λ
- This gives the straightforward but unwieldy expression

```
y*ln({y:_cons})-{y:_cons}-lngamma(y+1)-ln(1-exp(-{y:_cons}))
```

Working from Do-files

- Now the commands are becoming complicated enough that typing as we go will be unhelpful
- Let's open up a project file for this talk

```
. projman bayes
```

Finally: Analyzing the Truncated Gamma

- We can run our analysis with this do-file

```
. do trunc_pois

. ** truncated poisson estimation
. bayesmh y, prior({y:_cons}, flat) ///
>   rseed(3772) saving(trunc_pois)  ///
>   likelihood(llf(y*ln({y:_cons})-{y:_cons}-lngamma(y+1)-ln(1-exp(-{y:_cons})))
> )

Burn-in ...
note: invalid initial state
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ logdensity(y*ln({y:_cons})-{y:_cons}-lngamma(y+1)-ln(1-exp(-{y:_cons})))

Prior:
  {y:_cons} ~ 1 (flat)
------------------------------------------------------------------------------

Bayesian regression                              MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         47
                                                 Acceptance rate  =      .4315
Log marginal likelihood = -56.819423             Efficiency       =      .2378

------------------------------------------------------------------------------
             |                                                Equal-tailed
           y |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
       _cons |  1.444531   .2067589    .00424   1.435181   1.055125   1.881356
------------------------------------------------------------------------------

file trunc_pois.dta saved

.
. ** storing the model for later
. est store trunc_pois

.
end of do-file
```

  - The `saving()` option has been added because we will need it if we would like to compare this model to another later
  - We stored the model for later comparisons

- The mean from our posterior distribution now overshoots the true mean
  - This could happen because there were too many 0-valued observations in the original dataset

---

Truncated Gamma Notes

- Notice the `note: invalid initial state` warning under `Burn in ...`:
  - This happened here because Stata started λ at 0, which is not a valid rate
  - This should only worry us if the efficiencies are low or if the chain did not converge

- Just as before, we can look at the diagnostics (not shown)
- Here is the probability that the rate of fines is over 1

```
. bayestest interval ({y:_cons}, lower(1))

Interval tests     MCMC sample size =    10,000

       prob1 : {y:_cons} > 1

-----------------------------------------------
             |      Mean    Std. Dev.      MCSE
-------------+---------------------------------
       prob1 |     .9894     0.10241   .0017583
-----------------------------------------------
```

A Competing Likelihood

- Suppose we suspect that there could be overdispersion or underdispersion for our model
- We could try specifying a likelihood which accommodates this: the generalized Poisson distribution
- Here is one parameterization

$$p(y) = \frac{1}{y!}\left(\frac{\mu}{1+\alpha\mu}\right)^{y}(1-\alpha y)^{y-1}\exp\left\{-\frac{\mu(1+\alpha y)}{1+\alpha\mu}\right\}; y = 0, 1, 2, ...$$

- This distribution has mean μ and variance $\mu(1+\alpha\mu)^2$
  - ◇ Thus, if $\alpha > 0$ there is overdispersion; if $\alpha < 0$ there is underdispersion

---

Estimating This Competing Likelihood

- We can once again specify our own log likelihood:

$$\begin{aligned} llf(y) = &-\ln(y!) + y\left(\ln(\mu) - \ln(1+\alpha\mu)\right) \\ &+ (y-1)\ln(1+\alpha y) - \frac{\mu(1+\alpha y)}{1+\alpha\mu} \\ &- \ln\left(1 - \exp(-\frac{\mu}{1+\alpha\mu})\right) \end{aligned}$$

  - ◇ The last term comes from rescaling because the distribution is truncated

- Luckily, this mess has been put in a do-file

```
. do trunc_gpois

. ** truncated gen'l poisson estimation
. ** specified nocons, so that the two parameters {mu} and {alpha}
. **   could both be specified by name
. bayesmh y, nocons prior({mu}, uniform(0,100)) prior({alpha}, flat) ///
>   rseed(40213) saving(trunc_gpois) ///
>   likelihood(llf(-lngamma(y+1) + y*(ln({mu}) - ln(1 +{alpha}*{mu})) ///
>     + (y-1)*ln(1 + {alpha}*y) - ({mu}*(1 + {alpha}*y))/(1 +{alpha}*{mu}) ///
>   - ln(1 - exp(-{mu}/(1+{alpha}*{mu})))))

Burn-in ...
note: invalid initial state
Simulation ...

Model summary
------------------------------------------------------------------------------
```

```
Likelihood:
  y ~ logdensity(<expr1>)

Priors:
     {mu} ~ uniform(0,100)
  {alpha} ~ 1 (flat)

Expression:
  expr1 : -lngamma(y+1)+y*(ln({mu}) - ln(1 +{alpha}*{mu}))+(y-1)*ln(1 +{alpha}
          *y)-({mu}*(1 +{alpha}*y))/(1 +{alpha}*{mu})-ln(1 - exp(-{mu}/(1+{alp
          ha}*{mu})))
------------------------------------------------------------------------------

Bayesian regression                              MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         47
                                                 Acceptance rate  =      .2366
                                                 Efficiency:  min =     .05308
                                                              avg =     .08066
Log marginal likelihood = -59.917961                          max =      .1082

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
          mu |  1.683539   .1867278   .005675   1.693644   1.290789   1.999938
       alpha | -.1618633   .0711474   .003088    -.17427  -.2490146   -.001503
------------------------------------------------------------------------------

file trunc_gpois.dta saved

.
. ** storing the model for later
. est store trunc_gpois

.
end of do-file
```

Uh oh! Bad Efficiency

- If we look at the efficiencies, we can see that one of the parameters probably has high autocorrelations
- First, let's see which parameter had which efficiency by looking at effective sample sizes

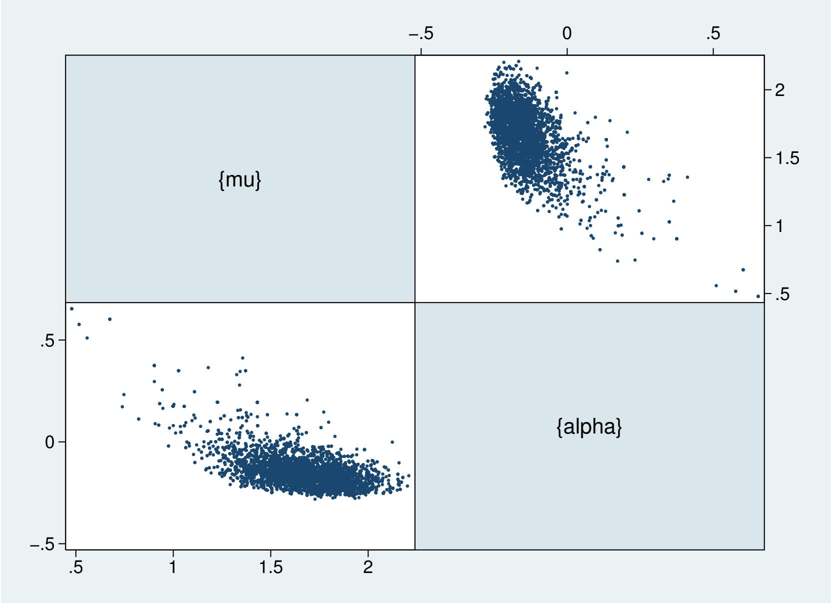
```
. bayesstats ess _all

Efficiency summaries    MCMC sample size =    10,000

----------------------------------------------------
             |        ESS   Corr. time    Efficiency
-------------+--------------------------------------
          mu |    1082.48         9.24        0.1082
       alpha |     530.78        18.84        0.0531
----------------------------------------------------
```

- We should investigate this

Plotting Simulations

- We can make a scatterplot matrix of the simulation values

```
. bayesgraph matrix {mu} {alpha}
```



Correlated Simulations

- Correlated MCMC simulation values slow down the MCMC chain, as do possibly illegal values
- One solution we could try here would be to transform the parameters to make their range extend over the whole real line
  - ◇ This is hard here, because the range of α depends on μ
- We might also try specifying legal initial values

```
. do trunc_gpois2

. ** truncated gen'l poisson estimation
. ** specified nocons, so that the two parameters {mu} and {alpha}
. **   could both be specified by name
. bayesmh y, nocons prior({mu}, uniform(0,100)) prior({alpha}, flat) ///
>   rseed(40213) saving(trunc_gpois2) ///
>   likelihood(llf(-lngamma(y+1) + y*(ln({mu}) - ln(1 +{alpha}*{mu})) ///
>     + (y-1)*ln(1 + {alpha}*y) - ({mu}*(1 + {alpha}*y))/(1 +{alpha}*{mu}) ///
>   - ln(1 - exp(-{mu}/(1+{alpha}*{mu}))))) ///
>   initial({mu} 1 {alpha} 0)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ logdensity(<expr1>)

Priors:
     {mu} ~ uniform(0,100)
  {alpha} ~ 1 (flat)

Expression:
  expr1 : -lngamma(y+1)+y*(ln({mu}) - ln(1 +{alpha}*{mu}))+(y-1)*ln(1 +{alpha}
          *y)-({mu}*(1 +{alpha}*y))/(1 +{alpha}*{mu})-ln(1 - exp(-{mu}/(1+{alp
          ha}*{mu})))
```

```
------------------------------------------------------------------------------

Bayesian regression                              MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         47
                                                 Acceptance rate  =      .2634
                                                 Efficiency:  min =     .09876
                                                              avg =      .1121
Log marginal likelihood = -60.126325                          max =      .1255

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
          mu |  1.685297   .1746314   .005557   1.694533   1.324338   2.001587
       alpha | -.1648068   .0583825   .001648  -.1739788  -.2474469  -.0151704
------------------------------------------------------------------------------

file trunc_gpois2.dta saved

.
. ** storing the model for later
. est store trunc_gpois

.
end of do-file
```

- This seemed to help
  - Try experimenting with other starting values if you like

Extending the Chain

- If we would like to get an effective sample size which is close to what we had for the truncated poisson model, we need to extend the chain
- The `mcmcsize(25000)` option does this

```
. do trunc_gpois3

. ** truncated gen'l poisson estimation
. ** specified nocons, so that the two parameters {mu} and {alpha}
. **   could both be specified by name
. bayesmh y, nocons prior({mu}, uniform(0,100)) prior({alpha}, flat) ///
>   rseed(40213) saving(trunc_gpois3) ///
>   likelihood(llf(-lngamma(y+1) + y*(ln({mu}) - ln(1 +{alpha}*{mu})) ///
>     + (y-1)*ln(1 + {alpha}*y) - ({mu}*(1 + {alpha}*y))/(1 +{alpha}*{mu}) ///
>   - ln(1 - exp(-{mu}/(1+{alpha}*{mu}))))) ///
>   initial({mu} 1 {alpha} 0) ///
>   mcmcsize(25000)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  y ~ logdensity(<expr1>)

Priors:
     {mu} ~ uniform(0,100)
  {alpha} ~ 1 (flat)
```

```
Expression:
  expr1 : -lngamma(y+1)+y*(ln({mu}) - ln(1 +{alpha}*{mu}))+(y-1)*ln(1 +{alpha}
          *y)-({mu}*(1 +{alpha}*y))/(1 +{alpha}*{mu})-ln(1 - exp(-{mu}/(1+{alp
          ha}*{mu})))
------------------------------------------------------------------------------

Bayesian regression                              MCMC iterations  =     27,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     25,000
                                                 Number of obs    =         47
                                                 Acceptance rate  =      .2641
                                                 Efficiency:  min =      .1003
                                                              avg =      .1026
Log marginal likelihood = -60.079039                          max =      .1049

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
          mu |  1.685861   .1765273   .003525   1.695722   1.304009   1.999089
       alpha | -.1642611   .0613568   .001198  -.1746719  -.2463381  -.0211519
------------------------------------------------------------------------------

file trunc_gpois3.dta saved

.
. ** storing the model for later
. est store trunc_gpois

.
end of do-file
```

Comparing Competing Models

- We can now see which of the two models we prefer
- This is done using the `bayestest model` command
- Being Bayesians, we assign prior probabilities to each of the models, and then compute their posterior probabilities given our data
- We have no reason to think one model is better than the other so we'll use the default of equally likely

```
. bayestest model trunc*

Bayesian model tests

---------------------------------------------
             |   log(ML)       P(M)     P(M|y)
-------------+-------------------------------
  trunc_pois |  -56.8194     0.5000     0.9630
 trunc_gpois |  -60.0790     0.5000     0.0370
---------------------------------------------
Note: Marginal likelihood (ML) is computed using
      Laplace-Metropolis approximation.
```

- We now think that there is a 96% chance that simple truncated poisson is true

---

Aside: Bayesian Hypothesis Testing

- One wonderful part of the Bayesian world is that more than two models may be compared
- One must take care that hypotheses are plausible
  - ◇ No point values for continuous variables, for example, unless they are 0 values for something that might not exist
- Sometimes it makes sense to have prior distributions which are not evenly distributed
  - ◇ There can be a decision-theoretic reason for this, for example different costs associated with falsely conclusions
- This is far more flexible than the typical us-versus-them hypothesis testing

---

Information Criteria

- We can also compare models using the deviance information criterion (DIC) and Bayes factors

```
. bayesstats ic trunc*

Bayesian information criteria

----------------------------------------------
             |       DIC    log(ML)    log(BF)
-------------+--------------------------------
  trunc_pois |  114.3289  -56.81942          .
 trunc_gpois |  109.3351  -60.07904  -3.259617
----------------------------------------------
Note: Marginal likelihood (ML) is computed
      using Laplace-Metropolis approximation.
```

- The smaller DIC for the `trunc_gpois` model says that it should do a better job producing a similar dataset
- The log(BF) column gives the log of odds that the `trunc_gpois` model is true
  - ◇ Here: `ln(0.0370/0.9630)`
- The Bayes factor will always give the same subjective result as assuming equal prior probabilities for models

---

2.4 Something A Little More Complex

Linear Regression

- All we've been doing is looking at a dataset of counts

```
. save pois_plus, replace
```

- Now let's try playing with linear regressions
- Open up the `autometric` dataset

```
. use autometric

(auto data in metric units)
```

  - ◇ Made for all countries except the US, Liberia, and Myanmar

---

Modeling Energy Usage

- We'd like to measure energy usage of these cars
- Perhaps: regressing `lp100km` on `weight`, `displacement` and `foreign`
- Let's go back to the dialog box for teaching purposes
  - ◇ Reset the dialog box by clicking the big **R** button

Filling in the Dialog Box

- This will take a little effort, but specify
  - ◇ `{var}` as the variance for the likelihood
  - ◇ Normals with large variances for the coefficients
  - ◇ Jeffries prior for the prior of `{var}`
  - ◇ A random seed of 142857
- Click on `OK` to submit and close

```
. do reg

. * using centering
. bayesmh lp100km weight displacement foreign, ///
>   likelihood(normal({var})) ///
>   prior({weight},       normal(0,1000)) ///
>   prior({displacement}, normal(0,1000)) ///
>   prior({foreign},      normal(0,1000)) ///
>   prior({_cons},        normal(0,1000)) ///
>   prior({var},     igamma(0.001,0.001)) ///
>   rseed(142857)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  lp100km ~ normal(xb_lp100km,{var})

Priors:
  {lp100km:weight displacement foreign _cons} ~ normal(0,1000)             (1)
                                        {var} ~ igamma(0.001,0.001)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_lp100km.

Bayesian normal regression                       MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         74
                                                 Acceptance rate  =      .3087
                                                 Efficiency:  min =     .03667
                                                              avg =     .04561
Log marginal likelihood =  -164.5299                          max =     .06078

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
lp100km      |
      weight |   .007643   .0010869   .000053   .0076434   .0054059   .0098129
displacement |  .2117928   .2616287   .010612   .2086352  -.2788908   .7602025
```

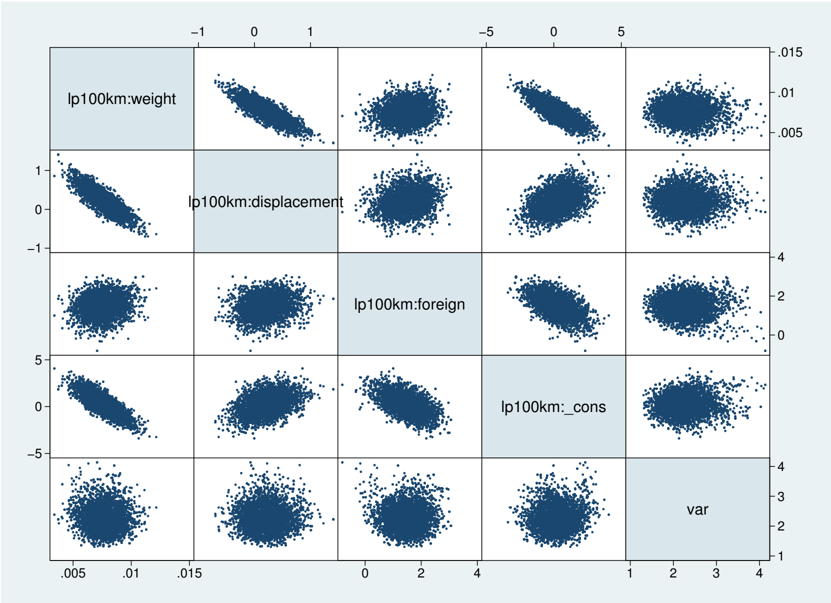
```
     foreign |  1.483588    .490692   .022521   1.480786   .4803052   2.441043
       _cons |  .2130119     .98965   .048576   .2051924    -1.7418   2.230979
-------------+----------------------------------------------------------------
         var |  2.214069   .3922478   .020485   2.163138   1.587671   3.160582
------------------------------------------------------------------------------


.
end of do-file
```

- The model converges, but not at all efficiently

Looking at the Problem

- Draw a graph matrix to see the problems

```
. bayesgraph matrix _all
```



Partial Fix Number 1

- If we mean center the weight and the displacement, we'll get rid of some of the correlation between their simulated values and those of the intercept

```
. sum weight displacement

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
      weight |         74    1369.527    352.5243        800       2195
displacement |         74    3.233919    1.505725       1.29       6.96
```

- While we're at it, let's make weight no so big

```
. gen wt1300 = (weight-1300)/1000
. gen displacement3 = displacement - 3
```

- Now let's see what happened

```
. do regcent
```

```
. * using centering
. bayesmh lp100km wt1300 displacement3 foreign, ///
>   likelihood(normal({var})) ///
>   prior({wt1300},        normal(0,1000)) ///
>   prior({displacement3}, normal(0,1000)) ///
>   prior({foreign},       normal(0,1000)) ///
>   prior({_cons},         normal(0,1000)) ///
>   prior({var},     igamma(0.001,0.001)) ///
>   rseed(142857)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  lp100km ~ normal(xb_lp100km,{var})

Priors:
  {lp100km:wt1300 displacement3 foreign _cons} ~ normal(0,1000)            (1)
                                         {var} ~ igamma(0.001,0.001)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_lp100km.

Bayesian normal regression                       MCMC iterations  =     12,500
Random-walk Metropolis-Hastings sampling         Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         74
                                                 Acceptance rate  =      .2936
                                                 Efficiency:  min =      .0276
                                                              avg =     .05888
Log marginal likelihood = -157.72151                          max =      .1017

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
lp100km      |
      wt1300 |   7.59653   1.104404    .03831   7.618547   5.458599   9.690238
displaceme~3 |  .2263987   .2707287   .008489   .2208552  -.2853743   .7609745
     foreign |  1.521567   .4884013   .021159   1.525246   .5444562   2.451707
       _cons |   10.7747   .2496765   .014735   10.76647   10.31451   11.29672
-------------+----------------------------------------------------------------
         var |  2.254268   .4067513   .024484   2.190814   1.600719   3.180133
------------------------------------------------------------------------------

.
end of do-file
```

Partial Fix Number 2

- We've chosen very special prior distributions for our model
  - ◇ Normal priors for a normal regression are semi conjugate
  - ◇ This means that they produce normal posterior distributions
    - ⋆ This means we know the posterior distrobution explicity
- So... we can use Gibbs sampling here
  - ◇ This is a special case of Metropolis-Hastings which exploits knowledge fo the closed form
- As a side effect, we will estimate each of the predictors separately

◇ The default is to estimate them all at once

Result of Gibbs Sampling

- Here is our Gibbs sampler

```
. do reggibbs

. * using centering
. bayesmh lp100km wt1300 displacement3 foreign, ///
>   likelihood(normal({var})) ///
>   prior({wt1300},        normal(0,1000)) ///
>   prior({displacement3}, normal(0,1000)) ///
>   prior({foreign},       normal(0,1000)) ///
>   prior({_cons},         normal(0,1000)) ///
>   prior({var},       igamma(.001,.001)) ///
>   block({lp100km:wt1300},         gibbs) ///
>   block({lp100km:displacement3},  gibbs) ///
>   block({lp100km:foreign},        gibbs) ///
>   block({lp100km:_cons} ,         gibbs) ///
>   block({var},                    gibbs) ///
>   rseed(142857)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  lp100km ~ normal(xb_lp100km,{var})

Priors:
  {lp100km:wt1300 displacement3 foreign _cons} ~ normal(0,1000)            (1)
                                         {var} ~ igamma(.001,.001)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_lp100km.

Bayesian normal regression                       MCMC iterations  =     12,500
Gibbs sampling                                   Burn-in          =      2,500
                                                 MCMC sample size =     10,000
                                                 Number of obs    =         74
                                                 Acceptance rate  =          1
                                                 Efficiency:  min =     .07728
                                                              avg =      .2942
Log marginal likelihood = -157.63634                          max =      .7768

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
lp100km      |
      wt1300 |  7.500904   1.137687   .040773   7.502938   5.242152   9.722559
displaceme~3 |  .2416701   .2732162   .009828    .238976  -.2873393   .7886288
     foreign |  1.479528   .4995871   .009963   1.473448    .494879   2.487035
       _cons |  10.78787   .2489216   .004643   10.78923    10.2879   11.27402
-------------+----------------------------------------------------------------
         var |  2.231845   .3881057   .004403   2.189157   1.596965   3.094858
------------------------------------------------------------------------------

.
end of do-file
```

- This has helped a bunch with everything except the correlated predictors

- So: collinearity is a problem here, too!
- Our only solution is to run the chain much longer

```
. do reggibbs2

. * using centering
. bayesmh lp100km wt1300 displacement3 foreign, ///
>   likelihood(normal({var})) ///
>   prior({wt1300},       normal(0,1000)) ///
>   prior({displacement3}, normal(0,1000)) ///
>   prior({foreign},      normal(0,1000)) ///
>   prior({_cons},        normal(0,1000)) ///
>   prior({var},       igamma(.001,.001)) ///
>   block({lp100km:wt1300},         gibbs) ///
>   block({lp100km:displacement3},  gibbs) ///
>   block({lp100km:foreign},        gibbs) ///
>   block({lp100km:_cons} ,         gibbs) ///
>   block({var},                    gibbs) ///
>   mcmcsize(50000)                        ///
>   rseed(142857)

Burn-in ...
Simulation ...

Model summary
------------------------------------------------------------------------------
Likelihood:
  lp100km ~ normal(xb_lp100km,{var})

Priors:
  {lp100km:wt1300 displacement3 foreign _cons} ~ normal(0,1000)           (1)
                                         {var} ~ igamma(.001,.001)
------------------------------------------------------------------------------
(1) Parameters are elements of the linear form xb_lp100km.

Bayesian normal regression                       MCMC iterations  =     52,500
Gibbs sampling                                   Burn-in          =      2,500
                                                 MCMC sample size =     50,000
                                                 Number of obs    =         74
                                                 Acceptance rate  =          1
                                                 Efficiency:  min =       .102
                                                              avg =      .3223
Log marginal likelihood = -157.64735                          max =      .8371

------------------------------------------------------------------------------
             |                                                Equal-tailed
             |      Mean   Std. Dev.     MCSE     Median  [95% Cred. Interval]
-------------+----------------------------------------------------------------
lp100km      |
      wt1300 |  7.504571   1.111813   .015286   7.500416   5.313615   9.693149
displaceme~3 |  .2415545   .2662422   .003729   .2390393  -.2807387   .7687533
     foreign |  1.484437    .484902   .004194   1.484105   .5278635    2.43858
       _cons |  10.78488   .2444281   .001997   10.78407   10.30452    11.2654
-------------+----------------------------------------------------------------
         var |  2.228945   .3880054   .001897   2.185259   1.592776   3.106856
------------------------------------------------------------------------------

.
end of do-file
```

3 Conclusion

3.1 Conclusion

What We Have Seen

- Use of part of the GUI for Bayesian analysis in Stata
- Specification of a non-standard likelihood
- Specification of priors
- Basic Bayesian estimation
- Basic Baysian model comparison
- Gibbs samplers
- Centering

What We Have Not Seen

- Complex models
  - There are many many examples in the manuals
- Writing our own evaluators
  - If you have a likelihood function which is not the sum of the likelihoods for each of the observations, you can write a specially-formed evaluator program
    - This is similar in kind to the `ml` command

Conclusion

- We've just touched on what can be done
- I hope this has been somewhat informative

Index