dfgls — DF-GLS unit-root test

Description Options Acknowledgments Quick start Remarks and examples References Menu Stored results Also see Syntax Methods and formulas

Description

dfgls performs a modified Dickey-Fuller t test for a unit root in which the series has been transformed by a generalized least-squares regression.

Quick start

Modified Dickey-Fuller unit-root test for y1 using GLS-transformed series using tsset data dfgls y1

Same as above, for series y2 that has no linear time trend dfgls y2, notrend

Same as above, but with at most 2 lags dfgls y2, notrend maxlag(2)

Menu

Statistics > Time series > Tests > DF-GLS test for a unit root

Syntax

dfgls	varname	[<i>if</i>]	[<i>in</i>]	[,	options]
options]	Descri	ption		

Main	
<pre>maxlag(#)</pre>	use # as the highest lag order for Dickey-Fuller GLS regressions
<u>not</u> rend	series is stationary around a mean instead of around a linear time trend
ers	present interpolated critical values from Elliott, Rothenberg, and Stock (1996)

You must tsset your data before using dfgls; see [TS] tsset. *varname* may contain time-series operators; see [U] 11.4.4 Time-series varlists. collect is allowed; see [U] 11.1.10 Prefix commands.

Options

_ Main)

- maxlag(#) sets the value of k, the highest lag order for the first-differenced, detrended variable in the Dickey-Fuller regression. By default, dfgls sets k according to the method proposed by Schwert (1989); that is, dfgls sets $k_{\text{max}} = \text{floor}[12\{(T+1)/100\}^{0.25}]$.
- notrend specifies that the alternative hypothesis be that the series is stationary around a mean instead of around a linear time trend. By default, a trend is included.
- ers specifies that dfgls should present interpolated critical values from tables presented by Elliott, Rothenberg, and Stock (1996), which they obtained from simulations. See *Critical values* under *Methods and formulas* for details.

Remarks and examples

stata.com

dfgls tests for a unit root in a time series. It performs the modified Dickey-Fuller t test (known as the DF-GLS test) proposed by Elliott, Rothenberg, and Stock (1996). Essentially, the test is an augmented Dickey-Fuller test, similar to the test performed by Stata's dfuller command, except that the time series is transformed via a generalized least squares (GLS) regression before performing the test. Elliott, Rothenberg, and Stock and later studies have shown that this test has significantly greater power than the previous versions of the augmented Dickey-Fuller test.

dfgls performs the DF-GLS test for the series of models that include 1 to k lags of the firstdifferenced, detrended variable, where k can be set by the user or by the method described in Schwert (1989). Stock and Watson (2019, 622–624) provide an excellent discussion of the approach.

As discussed in [TS] **dfuller**, the augmented Dickey–Fuller test involves fitting a regression of the form

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \zeta_1 \Delta y_{t-1} + \zeta_2 \Delta y_{t-2} + \dots + \zeta_k \Delta y_{t-k} + \epsilon_t$$

and then testing the null hypothesis H_0 : $\beta = 0$. The DF-GLS test is performed analogously but on GLS-detrended data. The null hypothesis of the test is that y_t is a random walk, possibly with drift. There are two possible alternative hypotheses: y_t is stationary about a linear time trend or y_t is stationary with a possibly nonzero mean but with no linear time trend. The default is to use the former. To specify the latter alternative, use the notrend option.

▷ Example 1

Here we use the German macroeconomic dataset and test whether the natural log of investment exhibits a unit root. We use the default options with dfgls.

. use https://www.stata-press.com/data/r18/lutkepohl2 (Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)					
. dfgls ln_inv					
DF-GLS test fo Variable: ln_i			Number of	obs = 80	
Lag selection: Schwert criterion			Maximum 1	ag = 11	
		C:	ritical value		
[lags] D	F-GLS tau	1%	5%	10%	
11	-2.925	-3.610	-2.763	-2.489	
10	-2.671	-3.610	-2.798	-2.523	
9	-2.766	-3.610	-2.832	-2.555	
8	-3.259	-3.610	-2.865	-2.587	
7	-3.536	-3.610	-2.898	-2.617	
6	-3.115	-3.610	-2.929	-2.646	
5	-3.054	-3.610	-2.958	-2.674	
4	-3.016	-3.610	-2.986	-2.699	
3	-2.071	-3.610	-3.012	-2.723	
2	-1.675	-3.610	-3.035	-2.744	
1	-1.752	-3.610	-3.055	-2.762	
Min SIC = -6 .	rron seq t) = 7 169137 at lag 4 136692 at lag 3	l with RMSE	= .0398949		

The null hypothesis of a unit root is not rejected for lags 1-3, it is rejected at the 10% level for lags 9-10, and it is rejected at the 5% level for lags 4-8 and 11. For comparison, we also test for a unit root in log of investment by using dfuller with two different lag specifications. We need to use the trend option with dfuller because it is not included by default.

. ururrer r	n_inv, lag(4) t	rend		
Augmented D:	ickey-Fuller te	est for unit	root	
Variable: 1	n_inv			obs = 87 lags = 4
HO: Random	walk with or wi	thout drift		
	Test statistic	1%	Dickey-Fuller critical value 5%	
Z(t)	-3.133	-4.069	-3.463	-3.158
Mackinnon a	pproximate <i>p-</i> va	lue for Z(t) = 0.0987.	
	n_inv, lag(7) t ickey-Fuller te	rend		
	ickey-Fuller te	rend	root Number of	obs = 84 lags = 7
Augmented D Variable: 1	ickey-Fuller te	crend est for unit	root Number of	
Augmented D Variable: 1	ickey-Fuller te n_inv	crend est for unit	root Number of	lags = 7

MacKinnon approximate p-value for Z(t) = 0.0090.

The critical values and the test statistic produced by dfuller with 4 lags do not support rejecting the null hypothesis, although the MacKinnon approximate p-value is less than 0.1. With 7 lags, the critical values and the test statistic reject the null hypothesis at the 5% level, and the MacKinnon approximate p-value is less than 0.01.

That the dfuller results are not as strong as those produced by dfgls is not surprising because the DF-GLS test with a trend has been shown to be more powerful than the standard augmented Dickey–Fuller test.

4

Stored results

If maxlag(0) is specified, dfgls stores the following in r():

Scalars

r(N)	number of observations
r(rmse0)	RMSE
r(dft0)	DF-GLS statistic
r(cv_1)	1% critical value
r(cv_5)	5% critical value
r(cv_10)	10% critical value

Otherwise, dfgls stores the following in r():

Scalars	
r(maxlag)	highest lag order k
r(N)	number of observations
r(siclag)	lag chosen by SIC
r(maiclag)	lag chosen by modified AIC
r(optlag)	lag chosen by sequential-t
r(siclag)	lag chosen by SIC

r(maiclag) r(optlag)	lag chosen by modified AIC lag chosen by sequential- t
r(sicmin)	minimum SIC
r(maicmin)	minimum MAIC
r(optrmse)	RMSE for sequential- t method
r(sicrmse)	RMSE for SIC method
r(maicrmse)	RMSE for MAIC method
Matrices	
r(results) r(cvalues)	<i>k</i> , MAIC, SIC, RMSE, and DF-GLS statistics <i>k</i> , DF-GLS statistics, and critical values

Methods and formulas

dfgls tests for a unit root. There are two possible alternative hypotheses: y_t is stationary around a linear trend or y_t is stationary with no linear time trend. Under the first alternative hypothesis, the DF-GLS test is performed by first estimating the intercept and trend via GLS. The GLS estimation is performed by generating the new variables, \tilde{y}_t , x_t , and z_t , where

and $\alpha^* = 1 - (13.5/T)$. An OLS regression is then estimated for the equation

$$\widetilde{y}_t = \delta_0 x_t + \delta_1 z_t + \epsilon_t$$

The OLS estimators $\hat{\delta}_0$ and $\hat{\delta}_1$ are then used to remove the trend from y_t ; that is, we generate

$$y^* = y_t - (\widehat{\delta}_0 + \widehat{\delta}_1 t)$$

Finally, we perform an augmented Dickey–Fuller test on the transformed variable by fitting the OLS regression

$$\Delta y_t^* = \alpha + \beta y_{t-1}^* + \sum_{j=1}^k \zeta_j \Delta y_{t-j}^* + \epsilon_t \tag{1}$$

and then test the null hypothesis H_0 : $\beta = 0$ by using tabulated critical values.

To perform the DF-GLS test under the second alternative hypothesis, we proceed as before but define $\alpha^* = 1 - (7/T)$, eliminate z from the GLS regression, compute $y^* = y_t - \delta_0$, fit the augmented Dickey-Fuller regression by using the newly transformed variable, and perform a test of the null hypothesis that $\beta = 0$ by using the tabulated critical values.

dfgls reports the DF-GLS statistic and its critical values obtained from the regression in (1) for $k \in \{1, 2, \dots, k_{\max}\}$. By default, dfgls sets $k_{\max} = \text{floor}[12\{(T+1)/100\}^{0.25}]$ as proposed by Schwert (1989), although you can override this choice with another value. The sample size available with k_{max} lags is used in all the regressions. Because there are k_{max} lags of the first-differenced series, $k_{max} + 1$ observations are lost, leaving $T - k_{max}$ observations. dfgls requires that the sample of T + 1 observations on $y_t = (y_0, y_1, \dots, y_T)$ have no gaps.

dfgls reports the results of three different methods for choosing which value of k to use. These are method 1, the Ng-Perron sequential t; method 2, the minimum Schwarz information criterion (SIC); and method 3, the Ng-Perron modified Akaike information criterion (MAIC). Although the SIC has a long history in time-series modeling, the Ng-Perron sequential t was developed by Ng and Perron (1995), and the MAIC was developed by Ng and Perron (2000).

The SIC can be calculated using either the log likelihood or the sum-of-squared errors from a regression; dfgls uses the latter definition. Specifically, for each k

$$\text{SIC} = \ln(\widehat{\text{rmse}}^2) + (k+1)\frac{\ln(T-k_{\max})}{(T-k_{\max})}$$

where

$$\widehat{\text{rmse}} = \frac{1}{(T - k_{\max})} \sum_{t = k_{\max} + 1}^{T} \widehat{e}_t^2$$

dfgls reports the value of the smallest SIC and the k that produced it.

Ng and Perron (1995) derived a sequential-t algorithm for choosing k:

- i. Set n = 0 and run the regression in method 2 with all $k_{\max} n$ lags. If the coefficient on $\beta_{k_{\max}}$ is significantly different from zero at level α , choose k to k_{\max} . Otherwise, continue to ii.
- ii. If $n < k_{\text{max}}$, set n = n + 1 and continue to iii. Otherwise, set k = 0 and stop.
- iii. Run the regression in method 2 with $k_{\max} n$ lags. If the coefficient on $\beta_{k_{\max}-n}$ is significantly different from zero at level α , choose k to $k_{\max} n$. Otherwise, return to ii.

Per Ng and Perron (1995), dfgls uses $\alpha = 10\%$. dfgls reports the k selected by this sequential-t algorithm and the rmse from the regression.

Method 3 is based on choosing k to minimize the MAIC. The MAIC is calculated as

$$MAIC(k) = \ln(\widehat{rmse}^2) + \frac{2\{\tau(k) + k\}}{T - k_{max}}$$

where

$$\tau(k) = \frac{1}{\widehat{\mathrm{rmse}}^2} \widehat{\beta}_0^2 \sum_{t=k_{\max}+1}^T \widetilde{y}_t^2$$

and \tilde{y} was defined previously.

Critical values

By default, dfgls uses the 5% and 10% critical values computed from the response surface analysis of Cheung and Lai (1995). Because Cheung and Lai (1995) did not present results for the 1% case, the 1% critical values are always interpolated from the critical values presented by ERS.

ERS presented critical values, obtained from simulations, for the DF-GLS test with a linear trend and showed that the critical values for the mean-only DF-GLS test were the same as those for the ADF test. If dfgls is run with the ers option, dfgls will present interpolated critical values from these tables. The method of interpolation is standard. For the trend case, below 50 observations and above 200 there is no interpolation; the values for 50 and ∞ are reported from the tables. For a value Nthat lies between two values in the table, say, N_1 and N_2 , with corresponding critical values CV₁ and CV₂, the critical value

$$cv = cv_1 + \frac{N - N_1}{N_1}(cv_2 - cv_1)$$

is presented. The same method is used for the mean-only case, except that interpolation is possible for values between 50 and 500.

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Also see

- [TS] dfuller Augmented Dickey–Fuller unit-root test
- [TS] **pperron** Phillips–Perron unit-root test
- [TS] tsset Declare data to be time-series data
- [XT] **xtunitroot** Panel-data unit-root tests

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