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**arfima postestimation** — Postestimation tools for arfima

Postestimation commands predict margins Remarks and examples Methods and formulas References Also see

## Postestimation commands

The following postestimation commands are of special interest after arfima:

Command	Description	
estat acplot	estimate autocorrelations and autocovariances	
irf	create and analyze IRFs	
psdensity	estimate the spectral density	

The following standard postestimation commands are also available:

Command	Description
contrast	contrasts and ANOVA-style joint tests of estimates
*estat ic	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC)
estat summarize	summary statistics for the estimation sample
estat vce	variance-covariance matrix of the estimators (VCE)
estimates	cataloging estimation results
etable	table of estimation results
forecast	dynamic forecasts and simulations
lincom	point estimates, standard errors, testing, and inference for linear combinations of coefficients
lrtest	likelihood-ratio test
*margins	marginal means, predictive margins, marginal effects, and average marginal effects
*marginsplot	graph the results from margins (profile plots, interaction plots, etc.)
*nlcom	point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
predict	linear predictions, innovations, standardized innovations, etc.
*predictnl	point estimates, standard errors, testing, and inference for generalized predictions
pwcompare	pairwise comparisons of estimates
test	Wald tests of simple and composite linear hypotheses
testnl	Wald tests of nonlinear hypotheses

 $<sup>^{*}</sup>$ estat ic, margins, marginsplot, nlcom, and predictnl are not appropriate after arfima, mpl.

# predict

### **Description for predict**

predict creates a new variable containing predictions such as expected values, fractionally differenced series, and innovations. All predictions are available as static one-step-ahead predictions, and the dependent variable is also available as a dynamic multistep prediction.

## Menu for predict

Statistics > Postestimation

### Syntax for predict

statistic	Description	
Main		
хb	predicted values; the default	
<u>r</u> esiduals	predicted innovations	
<u>rsta</u> ndard	standardized innovations	
<u>fdif</u> ference	fractionally differenced series	

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

options	Description
Options	
rmse([type] newvar)	put the estimated root mean squared error of the predicted statistic in a new variable; only permitted with options xb and residuals
<u>dyn</u> amic( <i>datetime</i> )	forecast the time series starting at <i>datetime</i> ; only permitted with option xb

datetime is a # or a time literal, such as td(1jan1995) or tq(1995q1); see [D] Datetime.

# **Options for predict**

\_\_\_\_ Main

xb, the default, calculates the predictions for the level of depvar.

residuals calculates the predicted innovations.

rstandard calculates the standardized innovations.

fdifference calculates the fractionally differenced predictions of depvar.

```
Options
```

rmse([type] newvar) puts the root mean squared errors of the predicted statistics into the specified
new variables. The root mean squared errors measure the variances due to the disturbances but do
not account for estimation error. rmse() is only permitted with the xb and residuals options.

dynamic (datetime) specifies when predict starts producing dynamic forecasts. The specified datetime must be in the scale of the time variable specified in tsset, and the datetime must be
inside a sample for which observations on the dependent variables are available. For example, dynamic(tq(2008q4)) causes dynamic predictions to begin in the fourth quarter of 2008, assuming
that your time variable is quarterly; see [D] **Datetime**. If the model contains exogenous variables,
they must be present for the whole predicted sample. dynamic() may only be specified with xb.

# margins

### **Description for margins**

margins estimates margins of response for expected values.

## Menu for margins

Statistics > Postestimation

## Syntax for margins

```
margins [marginlist] [, options]
margins [marginlist] , predict(statistic ...) [options]

statistic Description

xb predicted values; the default
residuals not allowed with margins
rstandard not allowed with margins
fdifference not allowed with margins
```

Statistics not allowed with margins are functions of stochastic quantities other than e(b).

For the full syntax, see [R] margins.

# Remarks and examples

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Remarks are presented under the following headings:

Forecasting after ARFIMA IRF results for ARFIMA

### Forecasting after ARFIMA

We assume that you have already read [TS] **arfima**. In this section, we illustrate some of the features of predict after fitting an ARFIMA model using arfima.

### Example 1

We have monthly data on the one-year Treasury bill secondary market rate imported from the Federal Reserve Bank (FRED) database using import fred; see [D] import fred. Below we fit an ARFIMA model with two autoregressive terms and one moving-average term to the data.

```
. use https://www.stata-press.com/data/r18/tb1yr
(FRED, 1-year Treasury bill; secondary market rate, monthly 1959-2001)
. arfima tb1yr, ar(1/2) ma(1)
Iteration 0: Log likelihood = -235.31856
Iteration 1:
              Log likelihood = -235.26104
                                            (backed up)
              Log likelihood = -235.25974
Iteration 2:
                                            (backed up)
Iteration 3:
             Log likelihood = -235.2544
                                            (backed up)
Iteration 4: Log likelihood = -235.13355
Iteration 5: Log likelihood = -235.13064
Iteration 6: Log likelihood = -235.12108
Iteration 7:
              Log likelihood = -235.11917
              Log likelihood = -235.11869
Iteration 8:
              Log likelihood = -235.11868
Iteration 9:
Refining estimates:
Iteration 0:
              Log likelihood = -235.11868
Iteration 1:
              Log likelihood = -235.11868
ARFIMA regression
Sample: 1959m7 thru 2001m8
                                                        Number of obs =
                                                                             506
                                                        Wald chi2(4) = 1864.16
Log likelihood = -235.11868
                                                        Prob > chi2
                                                                          0.0000
       tb1yr
               Coefficient
                            Std. err.
                                            z
                                                 P>|z|
                                                            [95% conf. interval]
tb1yr
                 5.496708
                              2.92038
                                          1.88
                                                 0.060
       _cons
                                                          -.2271321
                                                                        11.22055
ARFIMA
          ar
                                          2.05
                                                 0.041
         L1.
                 .2326101
                             .1136625
                                                            .0098357
                                                                        .4553845
         L2.
                 .3885209
                              .083565
                                          4.65
                                                 0.000
                                                            .2247365
                                                                        .5523054
          ma
                                         11.58
                                                 0.000
         L1.
                 .7755849
                             .0669559
                                                            .6443537
                                                                         .906816
                 .4606495
                             .0646499
                                          7.13
                                                 0.000
                                                            .3339381
                                                                        .5873609
     /sigma2
                 .1466495
                              .009232
                                         15.88
                                                 0.000
                                                            .1285551
                                                                        .1647438
```

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

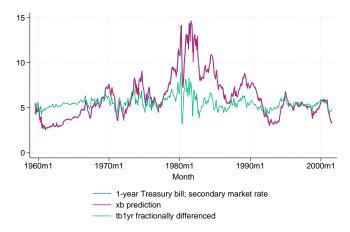
All the parameters are statistically significant at the 5% level, and they indicate a high degree of dependence in the series. In fact, the confidence interval for the fractional-difference parameter d indicates that the series may be nonstationary. We will proceed as if the series is stationary and suppose that it is fractionally integrated of order 0.46.

We begin our postestimation analysis by predicting the series in sample:

```
. predict ptb
(option xb assumed)
```

We continue by using the estimated fractional-difference parameter to fractionally difference the original series and by plotting the original series, the predicted series, and the fractionally differenced series. See [TS] **arfima** for a definition of the fractional-difference operator.

- . predict fdtb, fdifference
- . tsline tb1yr ptb fdtb, legend(cols(1))



The above graph shows that the in-sample predictions appear to track the original series well and that the fractionally differenced series looks much more like a stationary series than does the original.

### Example 2

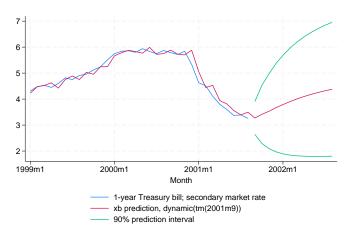
In this example, we use the above estimates to produce a dynamic forecast and a confidence interval for the forecast for the one-year treasury bill rate and plot them.

We begin by extending the dataset and using predict to put the dynamic forecast in the new ftb variable and the root mean squared error of the forecast in the new rtb variable. (As discussed in *Methods and formulas*, the root mean squared error of the forecast accounts for the idiosyncratic error but not for the estimation error.)

- . tsappend, add(12)
- . predict ftb, xb dynamic(tm(2001m9)) rmse(rtb)

Now we compute a 90% confidence interval around the dynamic forecast and plot the original series, the in-sample forecast, the dynamic forecast, and the confidence interval of the dynamic forecast.

```
. scalar z = invnormal(0.95)
. generate lb = ftb - z*rtb if month>=tm(2001m9)
(506 missing values generated)
. generate ub = ftb + z*rtb if month>=tm(2001m9)
(506 missing values generated)
. tsline tblyr ftb if month>tm(1998m12) ||
> tsrline lb ub if month>=tm(2001m9),
> legend(cols(1) label(3 "90% prediction interval"))
```



#### IRF results for ARFIMA

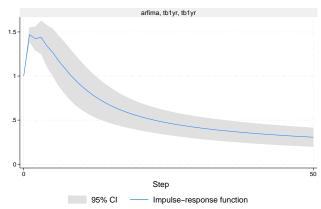
We assume that you have already read [TS] **irf** and [TS] **irf create**. In this section, we illustrate how to calculate the impulse–response function (IRF) of an ARFIMA model.

1

## Example 3

Here we use the estimates obtained in example 1 to calculate the IRF of the ARFIMA model; see [TS] **irf** and [TS] **irf create** for more details about IRFs.

```
. irf create arfima, step(50) set(myirf)
(file myirf.irf created)
(file myirf.irf now active)
(file myirf.irf updated)
. irf graph irf
```



Graphs by irfname, impulse variable, and response variable

The figure shows that a shock to tb1yr causes an initial spike in tb1yr, after which the impact of the shock starts decaying slowly. This behavior is characteristic of long-memory processes.

#### 4

## Methods and formulas

Denote  $\gamma_h$ ,  $h = 1, \dots, t$ , to be the autocovariance function of the ARFIMA(p, d, q) process for two observations,  $y_t$  and  $y_{t-h}$ , h time periods apart. The covariance matrix V of the process of length T has a Toeplitz structure of

$$\mathbf{V} = \begin{pmatrix} \gamma_0 & \gamma_1 & \gamma_2 & \dots & \gamma_{T-1} \\ \gamma_1 & \gamma_0 & \gamma_1 & \dots & \gamma_{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{T-1} & \gamma_{T-2} & \gamma_{T-3} & \dots & \gamma_0 \end{pmatrix}$$

where the process variance is  $\gamma_0 = \text{Var}(y_t)$ . We factor  $\mathbf{V} = \mathbf{LDL'}$ , where  $\mathbf{L}$  is lower triangular and  $\mathbf{D} = \operatorname{Diag}(\nu_t)$ . The structure of  $\mathbf{L}^{-1}$  is of importance.

$$\mathbf{L}^{-1} = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -\tau_{1,1} & 1 & 0 & \dots & 0 & 0 \\ -\tau_{2,2} & -\tau_{2,1} & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\tau_{T-1,T-1} & -\tau_{T-1,T-2} & -\tau_{T-1,T-2} & \dots & -\tau_{T-1,1} & 1 \end{pmatrix}$$

$$v_{t} - \mathbf{x}_{t} \boldsymbol{\beta}. \text{ The best linear predictor of } z_{t+1} \text{ based on } z_{1}, z_{2}, \dots, z_{t}$$

Let  $z_t = y_t - \mathbf{x}_t \boldsymbol{\beta}$ . The best linear predictor of  $z_{t+1}$  based on  $z_1, z_2, \ldots, z_t$  is  $\hat{z}_{t+1} = z_t$  $\sum_{k=1}^t \tau_{t,k} z_{t-k+1}. \text{ Define } -\tau_t = (-\tau_{t,t}, -\tau_{t,t-1}, \dots, -\tau_{t-1,1}) \text{ to be the } t \text{th row of } \mathbf{L}^{-1} \text{ up to, but}$ not including, the diagonal. Then  $\tau_t = \mathbf{V}_t^{-1} \gamma_t$ , where  $\mathbf{V}_t$  is the  $t \times t$  upper left submatrix of  $\mathbf{V}$  and  $\gamma_t = (\gamma_1, \gamma_2, \dots, \gamma_t)'$ . Hence, the best linear predictor of the innovations is computed as  $\hat{\epsilon} = \mathbf{L}^{-1}\mathbf{z}$ , and the one-step predictions are  $\hat{\mathbf{y}} = \hat{\boldsymbol{\epsilon}} + \mathbf{X}\hat{\boldsymbol{\beta}}$ . In practice, the computation is

$$\widehat{\mathbf{y}} = \widehat{\mathbf{L}}^{-1} \left( \mathbf{y} - \mathbf{X} \widehat{\boldsymbol{\beta}} \right) + \mathbf{X} \widehat{\boldsymbol{\beta}}$$

where  $\widehat{\mathbf{L}}$  and  $\widehat{\mathbf{V}}$  are computed from the maximum likelihood estimates. We use the Durbin–Levinson algorithm (Palma 2007; Golub and Van Loan 2013) to factor  $\widehat{\mathbf{V}}$ , invert  $\widehat{\mathbf{L}}$ , and scale  $\mathbf{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}$  using only the vector of estimated autocovariances  $\widehat{\boldsymbol{\gamma}}$ .

The prediction error variances of the one-step predictions are computed recursively in the Durbin-Levinson algorithm. They are the  $\nu_t$  elements in the diagonal matrix  $\mathbf{D}$  computed from the Cholesky factorization of  $\mathbf{V}$ . The recursive formula is  $\nu_0 = \gamma_0$ , and  $\nu_t = \nu_{t-1}(1 - \tau_{t,t}^2)$ .

Forecasting is carried out as described by Beran (1994, sec. 8.7),  $\widehat{\mathbf{z}}_{T+k} = \widetilde{\gamma}_k' \widehat{\mathbf{V}}^{-1} \widehat{\mathbf{z}}$ , where  $\widetilde{\gamma}_k = (\widehat{\gamma}_{T+k-1}, \widehat{\gamma}_{T+k-2}, \dots, \widehat{\gamma}_k)$ . The forecast mean squared error is computed as MSE $(\widehat{\mathbf{z}}_{T+k}) = \widehat{\gamma}_0 - \widetilde{\gamma}_k' \widehat{\mathbf{V}}^{-1} \widetilde{\gamma}_k$ . Computation of  $\widehat{\mathbf{V}}^{-1} \widetilde{\gamma}_k$  is carried out efficiently using algorithm 4.7.2 of Golub and Van Loan (2013).

### References

Beran, J. 1994. Statistics for Long-Memory Processes. Boca Raton, FL: Chapman and Hall/CRC.

Golub, G. H., and C. F. Van Loan. 2013. *Matrix Computations*. 4th ed. Baltimore: Johns Hopkins University Press. Palma, W. 2007. *Long-Memory Time Series: Theory and Methods*. Hoboken, NJ: Wiley.

### Also see

- [TS] **arfima** Autoregressive fractionally integrated moving-average models
- [TS] estat acplot Plot parametric autocorrelation and autocovariance functions
- [TS] irf Create and analyze IRFs, dynamic-multiplier functions, and FEVDs
- [TS] psdensity Parametric spectral density estimation after arima, arfima, and ucm
- [U] 20 Estimation and postestimation commands

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