Example 30g - Two-level measurement model (multilevel, generalized response)

Description Remarks and examples References Also see

## Description

We demonstrate a multilevel measurement model with the same data used in [SEM] Example 29g:
. use https://www.stata-press.com/data/r18/gsem_cfa
(Fictional math abilities data)
. summarize

| Variable | Obs | Mean | Std. dev. | Min | Max |
| ---: | ---: | ---: | ---: | ---: | ---: |
| school | 500 | 10.5 | 5.772056 | 1 | 20 |
| id | 500 | 50681.71 | 29081.41 | 71 | 100000 |
| q1 | 500 | .506 | .5004647 | 0 | 1 |
| q2 | 500 | .394 | .4891242 | 0 | 1 |
| q3 | 500 | .534 | .4993423 | 0 | 1 |
| q4 | 500 | .424 | .4946852 | 0 | 1 |
| q5 | 500 | .49 | .5004006 | 0 | 1 |
| q6 | 500 | .434 | .4961212 | 0 | 1 |
| q7 | 500 | .52 | .5001002 | 0 | 1 |
| q8 | 500 | .494 | .5004647 | 0 | 1 |
| att1 | 500 | 2.946 | 1.607561 | 1 | 5 |
| att2 | 500 | 2.948 | 1.561465 | 1 | 5 |
| att3 | 500 | 2.84 | 1.640666 | 1 | 5 |
| att4 | 500 | 2.91 | 1.566783 | 1 | 5 |
| att5 | 500 | 3.086 | 1.581013 | 1 | 5 |
| test1 | 500 | 75.548 | 5.948653 | 55 | 93 |
| test2 | 500 | 80.556 | 4.976786 | 65 | 94 |
| test3 | 500 | 75.572 | 6.677874 | 50 | 94 |
| test4 | 500 | 74.078 | 8.845587 | 43 | 96 |

. notes
_dta:

1. Fictional data on math ability and attitudes of 500 students from 20 schools.
2. Variables q1-q8 are incorrect/correct (0/1) on individual math questions.
3. Variables att1-att5 are items from a Likert scale measuring each student's attitude toward math.
4. Variables test1-test4 are test scores from tests of four different aspects of mathematical abilities. Range of scores: 0-100.

These data record results from a fictional instrument measuring mathematical ability. Variables q1 through q8 are the items from the instrument.

For discussions of multilevel measurement models, including extensions beyond the example we present here, see Mehta and Neale (2005) and Skrondal and Rabe-Hesketh (2004).

See Single-factor measurement models and Multilevel mixed-effects models in [SEM] Intro 5 for background.

## Remarks and examples

Remarks are presented under the following headings:
Fitting the two-level model
Fitting the variance-components model
Fitting the model with the Builder

## Fitting the two-level model

We extend the measurement model fit in [SEM] Example 29g to better account for our (fictional) data. In the data, students are nested within school, but we have ignored that so far. In this example, we include a latent variable at the school level to account for possible school-by-school effects.

The model we wish to fit is


The double-ringed $s c h o o l_{1}$ is new. That new component of the path diagram is saying, "I am a latent variable at the school level-meaning I am constant within school and vary across schools-and I correspond to a latent variable named M1"; see Specifying generalized SEMs: Multilevel mixed effects (2 levels) in [SEM] Intro 2. This new variable will account for the effect, if any, of the identity of the school.

To fit this model without this new, school-level component in it, we would type

```
. gsem (MathAb -> q1-q8), logit
```

To include the new school-level component, we add M1 [school] to the exogenous variables:

```
. gsem (MathAb M1[school] -> q1-q8), logit
Fitting fixed-effects model:
Iteration 0: Log likelihood = -2750.3114
Iteration 1: Log likelihood = -2749.3709
Iteration 2: Log likelihood = -2749.3708
Refining starting values:
Grid node 0: Log likelihood = -2649.0033
Fitting full model:
Iteration 0: Log likelihood = -2649.0033 (not concave)
Generalized structural equation model
Number of obs = 500
Response: q1
Family: Bernoulli
Link: Logit
Response: q2
Family: Bernoulli
Link: Logit
Response: q3
Family: Bernoulli
Link: Logit
Response: q4
Family: Bernoulli
Link: Logit
Response: q5
Family: Bernoulli
Link: Logit
Response: q6
Family: Bernoulli
Link: Logit
Response: q7
Family: Bernoulli
Link: Logit
Response: q8
Family: Bernoulli
Link: Logit
Log likelihood = -2630.2063
    ( 1) [q1]M1[school] = 1
    ( 2) [q2]MathAb = 1
```

|  | Coefficient | Std. err. | z | $P>\|z\|$ | [95\% con | interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1 |  |  |  |  |  |  |
| M1 [school] |  | (constrain |  |  |  |  |
| MathAb | 2.807515 | . 9468682 | 2.97 | 0.003 | . 9516878 | 4.663343 |
| _cons | . 0388021 | . 1608489 | 0.24 | 0.809 | -. 276456 | . 3540602 |
| q2 |  |  |  |  |  |  |
| M1 [school] | . 6673925 | . 3058328 | 2.18 | 0.029 | . 0679712 | 1.266814 |
| MathAb | 1 | (constrain |  |  |  |  |
| _cons | -. 4631159 | . 1201227 | -3.86 | 0.000 | -. 698552 | -. 2276798 |
| q3 |  |  |  |  |  |  |
| M1 [school] | . 3555867 | . 3043548 | 1.17 | 0.243 | -. 2409377 | . 9521111 |
| MathAb | 1.455529 | . 5187786 | 2.81 | 0.005 | . 4387416 | 2.472316 |
| _cons | . 1537831 | . 1070288 | 1.44 | 0.151 | -. 0559894 | . 3635556 |
| q4 |  |  |  |  |  |  |
| M1 [school] | . 7073241 | . 3419273 | 2.07 | 0.039 | . 037159 | 1.377489 |
| MathAb | . 8420897 | . 3528195 | 2.39 | 0.017 | . 1505762 | 1.533603 |
| _cons | -. 3252735 | . 1202088 | -2.71 | 0.007 | -. 5608784 | -. 0896686 |
| q5 |  |  |  |  |  |  |
| M1 [school] | . 7295553 | . 3330652 | 2.19 | 0.028 | . 0767595 | 1.382351 |
| MathAb | 2.399529 | . 8110973 | 2.96 | 0.003 | . 8098079 | 3.989251 |
| _cons | -. 0488674 | . 1378015 | -0.35 | 0.723 | -. 3189533 | . 2212185 |
| q6 |  |  |  |  |  |  |
| M1 [school] | . 484903 | . 2844447 | 1.70 | 0.088 | -. 0725983 | 1.042404 |
| MathAb | 1.840627 | . 5934017 | 3.10 | 0.002 | . 6775813 | 3.003673 |
| _cons | -. 3139302 | . 1186624 | -2.65 | 0.008 | -. 5465042 | -. 0813563 |
|  |  |  |  |  |  |  |
| M1 [school] | . 3677241 | . 2735779 | 1.34 | 0.179 | -. 1684787 | . 903927 |
| MathAb | 2.444023 | . 8016872 | 3.05 | 0.002 | . 8727449 | 4.015301 |
| _cons | . 1062164 | . 1220796 | 0.87 | 0.384 | -. 1330552 | . 3454881 |
| q8 |  |  |  |  |  |  |
| M1 [school] | . 5851299 | . 3449508 | 1.70 | 0.090 | -. 0909612 | 1.261221 |
| MathAb | 1.606287 | . 5367614 | 2.99 | 0.003 | . 5542541 | 2.65832 |
| _cons | -. 0261962 | . 1189835 | -0.22 | 0.826 | -. 2593995 | . 2070071 |
| var ( |  |  |  |  |  |  |
| M1 [school]) | . 2121216 | . 1510032 |  |  | . 052558 | . 8561121 |
| $\operatorname{var}($ MathAb) | . 2461246 | . 1372513 |  |  | . 0825055 | . 7342217 |

## Notes:

1. The variance of M1 [school] is estimated to be 0.21 .
2. So how important is M1 [school]? The variance of MathAb is estimated to be 0.25 , so math ability and school have roughly the same variance, and both of course have mean 0 . The math ability coefficients, meanwhile, are larger-often much larger-than the school coefficients in every case, so math ability is certainly more important than school in explaining whether questions were answered correctly. At this point, we are merely exploring the magnitude of effect.
3. You could also include a school-level latent variable for each question. For instance, you could type
```
. gsem (MathAb M1[school] N1[school] -> q1) ///
    (MathAb M1[school] N2[school] -> q2) ///
    (MathAb M1[school] N3[school] -> q3) ///
    (MathAb M1[school] N4[school] -> q4) ///
    (MathAb M1[school] N5[school] -> q5) ///
    (MathAb M1[school] N6[school] -> q6) ///
    (MathAb M1[school] N7[school] -> q7) ///
    (MathAb M1[school] N8[school] -> q8), logit
```

You will sometimes see such effects included in multilevel measurement models in theoretical discussions of models. Be aware that estimation of models with many latent variables is problematic, requiring both time and luck.

## Fitting the variance-components model

In a variance-components model, school would affect math ability which would affect correctness of answers to questions. The model might be drawn like this:


The above is a great way to draw the model. Sadly, gsem cannot understand it. The problem from gsem's perspective is that one latent variable is affecting another and the two latent variables are at different levels.

So we have to draw the model differently:


The models may look different, but constraining the coefficients along the paths from math ability and from school to each question is identical in effect to the model above.

The result of fitting the model is

```
. gsem (MathAb M1 [school] ->
> q1@1 q2@c2 q3@c3 q4@c4 q5@c5 q6@c6 q7@c7 q8@c8), logit
Fitting fixed-effects model:
Iteration 0: Log likelihood = -2750.3114
Iteration 1: Log likelihood = -2749.3709
Iteration 2: Log likelihood = -2749.3708
Refining starting values:
Grid node 0: Log likelihood = -2642.8248
Fitting full model:
Iteration 0: Log likelihood = -2651.7239 (not concave)
Iteration 1: Log likelihood = -2644.4937
Iteration 2: Log likelihood = -2634.92
Iteration 3: Log likelihood = -2633.9336
Iteration 4: Log likelihood = -2633.5924
Iteration 5: Log likelihood = -2633.5922
Generalized structural equation model Number of obs = 500
    (output omitted)
```

Log likelihood = -2633.5922
( 1) [q1]M1[school] = 1
( 2) [q1]MathAb $=1$
( 3) [q2]M1[school] - [q2]MathAb $=0$
( 4) [q3]M1[school] - [q3]MathAb $=0$
( 5) [q4]M1[school] - [q4]MathAb $=0$
( 6) [q5]M1[school] - [q5]MathAb $=0$
( 7) [q6]M1[school] - [q6]MathAb $=0$
( 8) [q7]M1[school] - [q7]MathAb $=0$
( 9) [q8]M1[school] - [q8]MathAb $=0$

|  | Coefficient | Std. err. | z | $P>\|z\|$ | [95\% conf | interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q1 |  |  |  |  |  |  |
| MathAb _cons | $\begin{array}{r} 1 \\ .0385522 \end{array}$ | $\begin{aligned} & \text { (constrain } \\ & .1556214 \end{aligned}$ | $0.25$ | 0.804 | -. 2664601 | . 3435646 |
| q2 |  |  |  |  |  |  |
| MathAb | . 3876281 | . 1156823 | 3.35 | 0.001 | . 1608951 | . 6143612 |
| _cons | -. 4633143 | . 1055062 | -4.39 | 0.000 | -. 6701028 | -. 2565259 |
| q3 |  |  |  |  |  |  |
| M1 [school] | . 4871164 | . 1295515 | 3.76 | 0.000 | . 2332001 | . 7410328 |
| MathAb | . 4871164 | . 1295515 | 3.76 | 0.000 | . 2332001 | . 7410328 |
| _cons | . 1533212 | . 1098068 | 1.40 | 0.163 | -. 0618962 | . 3685386 |
| q4 |  |  |  |  |  |  |
| M1 [school] | . 3407151 | . 1058542 | 3.22 | 0.001 | . 1332446 | . 5481856 |
| MathAb | . 3407151 | . 1058542 | 3.22 | 0.001 | . 1332446 | . 5481856 |
| _cons | -. 3246936 | . 1011841 | -3.21 | 0.001 | -. 5230108 | -. 1263763 |

Example 30g - Two-level measurement model (multilevel, generalized response)

| q5 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| M1 [school] | . 8327426 | . 1950955 | 4.27 | 0.000 | . 4503624 | 1.215123 |
| MathAb | . 8327426 | . 1950955 | 4.27 | 0.000 | . 4503624 | 1.215123 |
| _cons | -. 0490579 | . 1391324 | -0.35 | 0.724 | -. 3217524 | . 2236365 |
| q6 |  |  |  |  |  |  |
| M1 [school] | . 6267415 | . 1572247 | 3.99 | 0.000 | . 3185868 | . 9348962 |
| MathAb | . 6267415 | . 1572247 | 3.99 | 0.000 | . 3185868 | . 9348962 |
| _cons | -. 3135398 | . 1220389 | -2.57 | 0.010 | -. 5527317 | -. 074348 |
| q7 |  |  |  |  |  |  |
| M1 [school] | . 7660343 | . 187918 | 4.08 | 0.000 | . 3977219 | 1.134347 |
| MathAb | . 7660343 | . 187918 | 4.08 | 0.000 | . 3977219 | 1.134347 |
| _cons | . 1039102 | . 1330652 | 0.78 | 0.435 | -. 1568927 | . 3647131 |
| q8 |  |  |  |  |  |  |
| M1 [school] | . 5600833 | . 1416542 | 3.95 | 0.000 | . 2824462 | . 8377203 |
| MathAb | . 5600833 | . 1416542 | 3.95 | 0.000 | . 2824462 | . 8377203 |
| _cons | -. 0264193 | . 1150408 | -0.23 | 0.818 | -. 2518951 | . 1990565 |
| var ( |  |  |  |  |  |  |
| M1 [school]) | . 1719347 | . 1150138 |  |  | . 0463406 | . 6379187 |
| var (MathAb) | 2.062489 | . 6900045 |  |  | 1.070589 | 3.973385 |

1. Note that for each question, the coefficient on MathAb is identical to the coefficient on M1 [school].
2. We estimate separate variances for M1 [school] and MathAb. They are 0.17 and 2.06. Now that the coefficients are the same on school and ability, we can directly compare these variances. We see that math ability has a much larger affect than does school.

## Fitting the model with the Builder

Use the diagram in Fitting the two-level model above for reference.

1. Open the dataset.

In the Command window, type
. use https://www.stata-press.com/data/r18/gsem_cfa
2. Open a new Builder diagram.

Select menu item Statistics > SEM (structural equation modeling) > Model building and estimation.
3. Put the Builder in gsem mode by clicking on the $\mathrm{sem}^{\mathrm{G}}$ button.
4. Create the measurement component for MathAb.

Select the Add measurement component tool, 양, and then click in the diagram about one-fourth of the way down from the top and slightly left of the center.

In the resulting dialog box,
a. change the Latent variable name to MathAb;
b. select q1, q2, q3, q4, q5, q6, q7, and q8 by using the Measurement variables control;
c. check Make measurements generalized;
d. select Bernoulli, Logit in the Family/Link control;
e. select Down in the Measurement direction control;
f. click on OK.

If you wish, move the component by clicking on any variable and dragging it.
5. Create the school-level latent variable.
a. Select the Add multilevel latent variable tool, ${ }^{\circ}$, and click about one-fourth of the way up from the bottom and slightly left of the center.
b. In the Contextual Toolbar, click on the ${ }^{-7}$ button.
c. Select the nesting level and nesting variable by selecting 2 from the Nesting depth control and selecting school $>$ Observations in the next control.
d. Specify M1 as the Base name.
e. Click on OK.
6. Create the factor-loading paths for the multilevel latent variable.
a. Select the Add path tool, $\rightarrow$.
b. Click in the top-left quadrant of the double oval for $\mathrm{school}_{1}$ (it will highlight when you hover over it), and drag a path to the bottom of the q1 rectangle (it will highlight when you can release to connect the path).
c. Continuing with the $\rightarrow$ tool, draw paths from $\mathrm{school}_{1}$ to each of the remaining rectangles.
7. Clean up paths.

If you do not like where a path has been connected to its variables, use the Select tool, ${ }^{\wedge}$, to click on the path, and then simply click on where it connects to a rectangle or oval and drag the endpoint.
8. Estimate.

Click on the Estimate button, in the Standard Toolbar, and then click on OK in the resulting GSEM estimation options dialog box.
9. To fit the model in Fitting the variance-components model, add constraints to the diagram created above.
a. From the SEM Builder menu, select Estimation > Clear estimates to clear results from the previous model.
b. Choose the Select tool,
c. Click on the path from MathAb to q1. In the Contextual Toolbar, type 1 in the ${ }^{\boldsymbol{\varepsilon} \beta}$ box and press Enter.
d. Click on the path from $\operatorname{school}_{1}$ to q1. In the Contextual Toolbar, type 1 in the ${ }^{\boldsymbol{\beta} \beta}$ box and press Enter.
c. Click on the path from MathAb to q2. In the Contextual Toolbar, type c2 in the ${ }^{\boldsymbol{\beta} \beta}$ box and press Enter.
d. Click on the path from $\operatorname{school}_{1}$ to $q 2$. In the Contextual Toolbar, type $c 2$ in the ${ }^{\boldsymbol{\beta} \beta}$ box and press Enter.
e. Repeat this process to add the c 3 constraint on both paths to q 3 , the c 4 constraint on both paths to $\mathrm{q} 4, \ldots$, and the c 8 constraint on both paths to q 8 .
10. Estimate again.

Click on the Estimate button, in the Standard Toolbar, and then click on OK in the resulting GSEM estimation options dialog box.

You can open a completed diagram in the Builder for the first model by typing
. webgetsem gsem_mlcfa1
You can open a completed diagram in the Builder for the second model by typing
. webgetsem gsem_mlcfa2

## References

Mehta, P. D., and M. C. Neale. 2005. People are variables too: Multilevel structural equations modeling. Psychological Methods 10: 259-284. https://doi.org/10.1037/1082-989X.10.3.259.

Skrondal, A., and S. Rabe-Hesketh. 2004. Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models. Boca Raton, FL: Chapman and Hall/CRC.

## Also see

[SEM] Example 27 g - Single-factor measurement model (generalized response)
[SEM] Example 29g - Two-parameter logistic IRT model
[SEM] Intro 5 - Tour of models
[SEM] gsem - Generalized structural equation model estimation command

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