Title

> **Example 18 —** Latent growth model

Description

To demonstrate a latent growth model, we use the following data:

```
. use https://www.stata-press.com/data/r18/sem_lcm

. describe

Contains data from https://www.stata-press.com/data/r18/sem_lcm.dta
 Observations:            359
    Variables:              4                    25 May 2022 11:08
                                                 (_dta has notes)
```

Variable name	Storage type	Display format	Value label	Variable label
lncrime0	float	%9.0g		ln(crime rate) in Jan & Feb
lncrime1	float	%9.0g		ln(crime rate) in Mar & Apr
lncrime2	float	%9.0g		ln(crime rate) in May & Jun
lncrime3	float	%9.0g		ln(crime rate) in Jul & Aug

```
Sorted by:

. notes

_dta:
  1. Data used in Bollen, Kenneth A. and Patrick J. Curran, 2006, _Latent
     Curve Models: A Structural Equation Perspective_. Hoboken, New Jersey:
     John Wiley & Sons
  2. Data from 1995 Uniform Crime Reports for 359 communities in New York
     state.
```

See *Latent growth models* in [SEM] **Intro 5** for background.
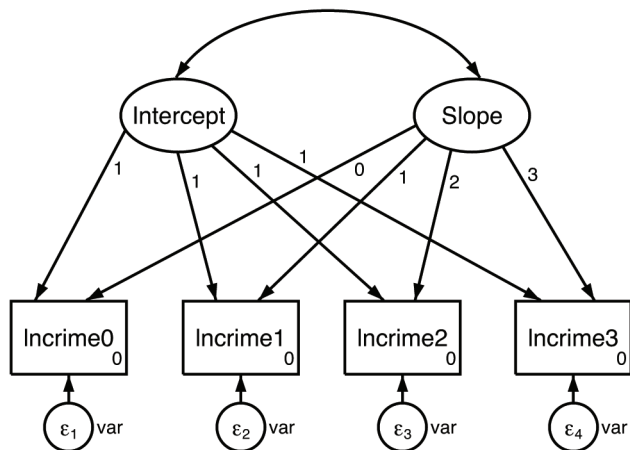
Remarks and examples

Remarks are presented under the following headings:

> *Fitting the model*
> *Fitting the model with the Builder*

Fitting the model

We fit the following model:


```
. sem (lncrime0 <- Intercept@1 Slope@0 _cons@0)
>     (lncrime1 <- Intercept@1 Slope@1 _cons@0)
>     (lncrime2 <- Intercept@1 Slope@2 _cons@0)
>     (lncrime3 <- Intercept@1 Slope@3 _cons@0),
>           latent(Intercept Slope)
>           var(e.lncrime0@var e.lncrime1@var
>               e.lncrime2@var e.lncrime3@var)
>           means(Intercept Slope)
Endogenous variables
  Measurement: lncrime0 lncrime1 lncrime2 lncrime3

Exogenous variables
  Latent: Intercept Slope

Fitting target model:
Iteration 0:   Log likelihood = -1034.1038
Iteration 1:   Log likelihood = -1033.9044
Iteration 2:   Log likelihood = -1033.9037
Iteration 3:   Log likelihood = -1033.9037

Structural equation model                          Number of obs = 359
Estimation method: ml

Log likelihood = -1033.9037
 ( 1)   [lncrime0]Intercept = 1
 ( 2)   [lncrime1]Intercept = 1
 ( 3)   [lncrime1]Slope = 1
 ( 4)   [lncrime2]Intercept = 1
 ( 5)   [lncrime2]Slope = 2
 ( 6)   [lncrime3]Intercept = 1
 ( 7)   [lncrime3]Slope = 3
 ( 8)   [/]var(e.lncrime0) - [/]var(e.lncrime3) = 0
 ( 9)   [/]var(e.lncrime1) - [/]var(e.lncrime3) = 0
 (10)   [/]var(e.lncrime2) - [/]var(e.lncrime3) = 0
 (11)   [lncrime0]_cons = 0
 (12)   [lncrime1]_cons = 0
 (13)   [lncrime2]_cons = 0
 (14)   [lncrime3]_cons = 0
```

Example 18 — Latent growth model 3

	Coefficient	OIM std. err.	z	P>|z|	[95% conf. interval]
Measurement					
lncrime0					
Intercept	1	(constrained)			
_cons	0	(constrained)			
lncrime1					
Intercept	1	(constrained)			
Slope	1	(constrained)			
_cons	0	(constrained)			
lncrime2					
Intercept	1	(constrained)			
Slope	2	(constrained)			
_cons	0	(constrained)			
lncrime3					
Intercept	1	(constrained)			
Slope	3	(constrained)			
_cons	0	(constrained)			
mean(Inter~t)	5.337915	.0407501	130.99	0.000	5.258047 5.417784
mean(Slope)	.1426952	.0104574	13.65	0.000	.1221992 .1631912
var(e.lncr~0)	.0981956	.0051826			.0885457 .1088972
var(e.lncr~1)	.0981956	.0051826			.0885457 .1088972
var(e.lncr~2)	.0981956	.0051826			.0885457 .1088972
var(e.lncr~3)	.0981956	.0051826			.0885457 .1088972
var(Interc~t)	.527409	.0446436			.4467822 .6225858
var(Slope)	.0196198	.0031082			.0143829 .0267635
cov(Interc~t, Slope)	-.034316	.0088848	-3.86	0.000	-.0517298 -.0169022

LR test of model vs. saturated: chi2(8) = 16.25   Prob > chi2 = 0.0390

Notes:

1. In this example, we have repeated measures of the crime rate in 1995. We will assume that the underlying rate grows linearly.

2. As explained in *Latent growth models* in [SEM] **Intro 5**, we assume

$$\texttt{lncrime}_i = \texttt{Intercept} + i \times \texttt{Slope}$$

3. sem does not usually report the means of latent exogenous variables because sem automatically includes the identifying constraint that the means are 0; see *How sem (gsem) solves the problem for you* in [SEM] **Intro 4** and see *Default normalization constraints* in [SEM] **sem**.

   In this case, sem did not constrain the means to be 0 because we specified sem's means() option. In particular, we specified means(Intercept Slope), which said not to constrain the means of those two exogenous latent variables and to report the estimated result.

   Our model was identified even without the usual 0 constraints on Intercept and Slope because we specified enough other constraints.

4. We estimate the Intercept to have mean 5.34 and the mean Slope to be 0.14 per two months. Remember, we have measured crime rates as log base e crime rates.

5. It might help some to think of this as a mixed model:

```
. generate id = _n
. reshape long lncrime, i(id) j(year)
. mixed lncrime year || id:year, cov(unstructured) mle var
```

The mean Intercept and Slope are what mixed would refer to as the coefficients in the fixed-effects part of the model.

Acock (2013, chap. 4) discusses the use of sem to fit latent growth-curve models in more detail. Acock demonstrates extensions to the basic model we fit here, such as including time-varying and time-invariant covariates in the model.

Fitting the model with the Builder

Use the diagram above for reference.

1. Open the dataset.

   In the Command window, type

   ```
   . use https://www.stata-press.com/data/r18/sem_lcm
   ```

2. Open a new Builder diagram.

   Select menu item **Statistics > SEM (structural equation modeling) > Model building and estimation**.

3. Create the four repeated measurements for log crime rate.

   Select the Add observed variables set tool, ⁱⁱ, and then click in the diagram about one-fourth of the way in from the left and about one-third of the way up from the bottom.

   In the resulting dialog box,

   a. select the *Select variables* radio button (it may already be selected);

   b. use the *Variables* control to select the variables in this order: lncrime0, lncrime1, lncrime2, and lncrime3;

   c. select Horizontal in the *Orientation* control;

   d. select the **Distances** tab;

   e. select .25 (inch) from the *Distance between variables* control;

   f. click on **OK**.

   If you wish, move the set of variables by clicking on any variable and dragging it.

4. Create the two latent variables.

   Select the Add latent variables set tool, ᵒᵒᵒ, and then click in the diagram about one-third of the way down from the top and above the left side of the lncrime0 rectangle.

   In the resulting dialog box,

   a. select the *Select variables* radio button (it may already be selected);

   b. type the latent variable names Intercept and Slope into the *Variables* control;

   c. select Horizontal in the *Orientation* control;

   d. select the **Distances** tab;

Example 18 — Latent growth model 5

  e. select 1 (inch) from the *Distance between variables* control;

  f. click on **OK**.

If you wish, move the set of variables by clicking on any variable and dragging it.

We could have forgone specifying the distances between variables and simply dragged each variable where we wished after it was created.

5. Create the factor-loading paths.

   a. Select the Add path tool, ⁻.

   b. Click in the bottom-left quadrant of the `Intercept` oval (it will highlight when you hover over it), and drag a path to the top of the `lncrime0` rectangle (it will highlight when you can release to connect the path).

   c. Continuing with the ⁻ tool, create the following paths by clicking first in the bottom of the latent variable and dragging it to the top of the observed (measurement) variable:

```
Intercept -> lncrime1
Intercept -> lncrime2
Intercept -> lncrime3
Slope -> lncrime0
Slope -> lncrime1
Slope -> lncrime2
Slope -> lncrime3
```

6. Clean up the direction of the errors.

   We want all the errors to be below the measurement variables.

   a. Choose the Select tool, ▸.

   b. Click on the rectangle of any measurement variable whose associated error is not below it.

   c. Click on one of the **Error rotation** buttons, ↺↻, in the Contextual Toolbar until the error is below the measurement variable.

   Repeat this for all errors that are not below the measurement variables.

7. Create the covariance between `Intercept` and `Slope`.

   a. Select the Add covariance tool, ⌒.

   b. Click in the top-right quadrant of the `Intercept` oval, and drag a covariance to the top left of the `Slope` oval.

8. Clean up paths and covariance.

   If you do not like where a path has been connected to its variables, use the Select tool, ▸, to click on the path, and then simply click on where it connects to a rectangle and drag the endpoint. Similarly, you can change where the covariance connects to the latent variables by clicking on the covariance and dragging the endpoint. You can also change the bow of the covariance by clicking on the covariance and dragging the control point that extends from one end of the selected covariance.

9. Constrain the intercepts of the measurements to 0.

   a. Choose the Select tool, ▸.

    b. Click on the rectangle for `lncrime0`. In the Contextual Toolbar, type 0 in the $^{\text{α}}$ box and press *Enter*.

    c. Repeat this process to add the 0 constraint on the intercept for `lncrime1`, `lncrime2`, and `lncrime3`.

10. Set constraints on the paths from `Intercept` to the measurements.

    a. Continue with the Select tool, ▶.

    b. Click on the path from `Intercept` to `lncrime0`. In the Contextual Toolbar, type 1 in the $^{\text{β}}$ box and press *Enter*.

    c. Repeat this process to add the 1 constraint on the following paths:

```
Intercept -> lncrime1
Intercept -> lncrime2
Intercept -> lncrime3
```

11. Set constraints on the paths from `Slope` to the measurements.

    a. Continue with the Select tool, ▶.

    b. Click on the path from `Slope` to `lncrime0`. In the Contextual Toolbar, type 0 in the $^{\text{β}}$ box and press *Enter*.

    c. Click on the path from `Slope` to `lncrime1`. In the Contextual Toolbar, type 1 in the $^{\text{β}}$ box and press *Enter*.

    d. Click on the path from `Slope` to `lncrime2`. In the Contextual Toolbar, type 2 in the $^{\text{β}}$ box and press *Enter*.

    e. Click on the path from `Slope` to `lncrime3`. In the Contextual Toolbar, type 3 in the $^{\text{β}}$ box and press *Enter*.

12. Set equality constraints on the error variances.

    a. Continue with the Select tool, ▶.

    b. Click in the ϵ_1 circle, which is the error term for `lncrime0`. In the Contextual Toolbar, type var in the $^{\text{σ}^2}$ box and press *Enter*.

    c. Repeat this process to add the var constraint on the three remaining error variances: ϵ_2, ϵ_3, and ϵ_4.

13. Clean up placement of the constraints.

From the SEM Builder menu, select **Settings > Connections > Paths...**.

In the resulting dialog box, do the following:

    a. Click on the **Results** tab.

    b. Click on the **Result 1...** button at the bottom left.

    c. In the *Appearance of result 1 - paths* dialog box that opens, choose 20 (%) in the *Distance between nodes* control.

    d. Click on **OK** on the *Appearance of result 1 - paths* dialog box.

    e. Click on **OK** on the *Connection settings - paths* dialog box.

Example 18 — **Latent growth model** 7

14. Specify that the means of `Intercept` and `Slope` are to be estimated.

    a. Choose the Select tool, ▸.

    b. Double-click on the `Intercept` oval.

    c. In the resulting dialog box, check the *Estimate mean* box.

    d. Click on **OK**.

    e. In the same manner, double-click on `Slope` and check the *Estimate mean* box.

15. Estimate.

Click on the **Estimate** button, ▸, in the Standard Toolbar, and then click on **OK** in the resulting *SEM estimation options* dialog box.

You can open a completed diagram in the Builder by typing

```
. webgetsem sem_lcm
```

Reference

Acock, A. C. 2013. *Discovering Structural Equation Modeling Using Stata*. Rev. ed. College Station, TX: Stata Press.

Also see

[SEM] **sem** — Structural equation model estimation command