Title

ivregress — Single-equation instrumental-variables regression

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References Also see

Description

ivregress fits linear models where one or more of the regressors are endogenously determined. ivregress supports estimation via two-stage least squares (2SLS), limited-information maximum likelihood (LIML), and generalized method of moments (GMM).

Quick start

2SLS estimation of a linear regression of y1 on x1 and endogenous regressor y2 that is instrumented by z1

```
ivregress 2sls y1 x1 (y2 = z1)
```

Same as above, but with two endogenous regressors, y2 and y3 instrumented by z1 and z2 ivregress 2sls y1 x1 (y2 y3 = z1 z2)

With robust standard errors

```
ivregress 2sls y1 x1 (y2 y3 = z1 z2), vce(robust)
```

Report small-sample statistics

```
ivregress 2sls y1 x1 (y2 y3 = z1 z2), small
```

Use LIML estimation

```
ivregress liml y1 x1 (y2 y3 = z1 z2)
```

Use GMM estimation

```
ivregress gmm y1 x1 (y2 y3 = z1 z2)
```

Also specify a weighting matrix that allows for correlation within clusters identified by cvar ivregress gmm y1 x1 (y2 y3 = z1 z2), wmatrix(cluster cvar)

Menu

Statistics > Endogenous covariates > Linear regression with endogenous covariates

Syntax

```
ivregress estimator depvar [varlist_1] (varlist_2 = varlist_{iv}) [if] [in] [weight] [, options]
```

 $varlist_1$ is the list of exogenous variables.

varlist2 is the list of endogenous variables.

 $varlist_{iv}$ is the list of exogenous variables used with $varlist_1$ as instruments for $varlist_2$.

estimator	Description					
2sls	two-stage least squares (2SLS)					
liml	mited-information maximum likelihood (LIML)					
gmm	generalized method of moments (GMM)					
options	Description					
Model						
<u>nocons</u> tant	suppress constant term					
$\underline{\mathtt{h}}\mathtt{ascons}$	has user-supplied constant					
GMM^1						
<pre>wmatrix(wmtype)</pre>	wmtype may be <u>r</u> obust, <u>cl</u> uster <u>clustvar</u> , hac <u>kernel</u> , or <u>un</u> adjusted					
<u>c</u> enter	center moments in weight matrix computation					
<u>i</u> gmm	use iterative instead of two-step GMM estimator					
$eps(\#)^2$	specify # for parameter convergence criterion; default is eps(1e-6)					
$weps(#)^2$	specify # for weight matrix convergence criterion; default is weps(1e-6)					
optimization_options ²	control the optimization process; seldom used					
SE/Robust						
vce(vcetype)						
Reporting						
<u>l</u> evel(#)	set confidence level; default is level(95)					
first	report first-stage regression					
small	make degrees-of-freedom adjustments and report small-sample statistics					
<u>nohe</u> ader	display only the coefficient table					
$\underline{\text{dep}}_{\text{name}}(\underline{depname})$	substitute dependent variable name					
<pre>eform(string)</pre>	report exponentiated coefficients and use string to label them					
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling					

perfect	do 1	ot	check	for co	ollinearity	between	endogenous	regressors	and
	e	xclu	ided i	nstrum	nents				
		_	_						

<u>coefl</u>egend

display legend instead of statistics

varlist₁, varlist₂, and varlist_{iv} may contain factor variables; see [U] 11.4.3 Factor variables.

depvar, varlist₁, varlist₂, and varlist₁, may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bootstrap, by, collect, fmm, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [FMM] fmm: ivregress.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

aweights are not allowed with the jackknife prefix; see [R] jackknife.

hascons, vce(), noheader, depname(), and weights are not allowed with the svy prefix; see [SVY] svy.

aweights, fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

perfect and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

noconstant; see [R] Estimation options.

has cons indicates that a user-defined constant or its equivalent is specified among the independent variables.

GMM

wmatrix(wmtype) specifies the type of weighting matrix to be used in conjunction with the GMM estimator.

Specifying wmatrix(robust) requests a weighting matrix that is optimal when the error term is heteroskedastic. wmatrix(robust) is the default.

Specifying wmatrix(cluster *clustvar*) requests a weighting matrix that accounts for arbitrary correlation among observations within clusters identified by *clustvar*.

Specifying wmatrix(hac *kernel* #) requests a heteroskedasticity- and autocorrelation-consistent (HAC) weighting matrix using the specified kernel (see below) with # lags. The bandwidth of a kernel is equal to # + 1.

Specifying wmatrix(hac *kernel* opt [#]) requests an HAC weighting matrix using the specified kernel, and the lag order is selected using Newey and West's (1994) optimal lag-selection algorithm. # is an optional tuning parameter that affects the lag order selected; see the discussion in *Methods* and formulas.

Specifying wmatrix(hac kernel) requests an HAC weighting matrix using the specified kernel and N-2 lags, where N is the sample size.

There are three kernels available for HAC weighting matrices, and you may request each one by using the name used by statisticians or the name perhaps more familiar to economists:

bartlett or nwest requests the Bartlett (Newey-West) kernel;

parzen or gallant requests the Parzen (Gallant 1987) kernel; and

quadraticspectral or andrews requests the quadratic spectral (Andrews 1991) kernel.

¹These options may be specified only when gmm is specified.

²These options may be specified only when igmm is specified.

Specifying wmatrix(unadjusted) requests a weighting matrix that is suitable when the errors are homoskedastic. The GMM estimator with this weighting matrix is equivalent to the 2SLS estimator.

- center requests that the sample moments be centered (demeaned) when computing GMM weight matrices. By default, centering is not done.
- igmm requests that the iterative GMM estimator be used instead of the default two-step GMM estimator. Convergence is declared when the relative change in the parameter vector from one iteration to the next is less than eps() or the relative change in the weight matrix is less than weps().
- eps(#) specifies the convergence criterion for successive parameter estimates when the iterative GMM estimator is used. The default is eps(1e-6). Convergence is declared when the relative difference between successive parameter estimates is less than eps() and the relative difference between successive estimates of the weighting matrix is less than weps().
- weps(#) specifies the convergence criterion for successive estimates of the weighting matrix when the iterative GMM estimator is used. The default is weps(1e-6). Convergence is declared when the relative difference between successive parameter estimates is less than eps() and the relative difference between successive estimates of the weighting matrix is less than weps().
- optimization_options: <u>iter</u>ate(#), [no]log. iterate() specifies the maximum number of iterations to perform in conjunction with the iterative GMM estimator. The default is the number set using set maxiter, which is 300 by default. log/nolog specifies whether to show the iteration log; see set iterlog in [R] set iter. These options are seldom used.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are robust to
some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar),
and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

vce(unadjusted), the default for 2sls and liml, specifies that an unadjusted (nonrobust) VCE
matrix be used. The default for gmm is based on the wmtype specified in the wmatrix() option;
see wmatrix() above. If wmatrix() is specified with gmm but vce() is not, then vcetype is
set equal to wmtype. To override this behavior and obtain an unadjusted (nonrobust) VCE matrix,
specify vce(unadjusted).

ivregress also allows the following:

vce(hac kernel [#|opt [#]]) specifies that an HAC covariance matrix be used. The syntax
used with vce(hac kernel ...) is identical to that used with wmatrix(hac kernel ...); see
wmatrix() above.

Reporting

level(#); see [R] Estimation options.

first requests that the first-stage regression results be displayed.

- small requests that the degrees-of-freedom adjustment N/(N-k) be made to the variance-covariance matrix of parameters and that small-sample F and t statistics be reported, where N is the sample size and k is the number of parameters estimated. By default, no degrees-of-freedom adjustment is made, and Wald and t statistics are reported. Even with this option, no degrees-of-freedom adjustment is made to the weighting matrix when the GMM estimator is used.
- noheader suppresses the display of the summary statistics at the top of the output, displaying only the coefficient table.
- depname (*depname*) is used only in programs and ado-files that use ivregress to fit models other than instrumental-variables regression. depname() may be specified only at estimation time. *depname*

is recorded as the identity of the dependent variable, even though the estimates are calculated using *depvar*. This method affects the labeling of the output—not the results calculated—but could affect later calculations made by predict, where the residual would be calculated as deviations from *depname* rather than *depvar*. depname() is most typically used when *depvar* is a temporary variable (see [P] macro) used as a proxy for *depname*.

eform(string) is used only in programs and ado-files that use ivregress to fit models other than instrumental-variables regression. eform() specifies that the coefficient table be displayed in "exponentiated form", as defined in [R] Maximize, and that string be used to label the exponentiated coefficients in the table.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

The following options are available with ivregress but are not shown in the dialog box:

perfect requests that ivregress not check for collinearity between the endogenous regressors and excluded instruments, allowing one to specify "perfect" instruments. This option cannot be used with the LIML estimator. This option may be required when using ivregress to implement other estimators.

coeflegend; see [R] Estimation options.

Remarks and examples

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ivregress performs instrumental-variables regression and weighted instrumental-variables regression. For a general discussion of instrumental variables, see Baum (2006), Cameron and Trivedi (2005; 2022, chap. 7) Davidson and MacKinnon (1993), Greene (2018, chap. 8), and Wooldridge (2010, 2020). See Hall (2005) for a lucid presentation of GMM estimation. Angrist and Pischke (2009, chap. 4) offer a casual yet thorough introduction to instrumental-variables estimators, including their use in estimating treatment effects. Some of the earliest work on simultaneous systems can be found in Cowles Commission monographs—Koopmans and Marschak (1950) and Koopmans and Hood (1953)—with the first developments of 2SLS appearing in Theil (1953) and Basmann (1957). However, Stock and Watson (2019, 401–402) present an example of the method of instrumental variables that was first published in 1928 by Philip Wright.

The syntax for ivregress assumes that you want to fit one equation from a system of equations or an equation for which you do not want to specify the functional form for the remaining equations of the system. To fit a full system of equations, using either 2SLS equation-by-equation or three-stage least squares, see [R] reg3. An advantage of ivregress is that you can fit one equation of a multiple-equation system without specifying the functional form of the remaining equations.

Formally, the model fit by ivregress is

$$y_i = \mathbf{y}_i \boldsymbol{\beta}_1 + \mathbf{x}_{1i} \boldsymbol{\beta}_2 + u_i \tag{1}$$

$$\mathbf{y}_i = \mathbf{x}_{1i} \mathbf{\Pi}_1 + \mathbf{x}_{2i} \mathbf{\Pi}_2 + \mathbf{v}_i \tag{2}$$

Here y_i is the dependent variable for the ith observation, \mathbf{y}_i represents the endogenous regressors ($varlist_2$ in the syntax diagram), \mathbf{x}_{1i} represents the included exogenous regressors ($varlist_1$ in the syntax diagram), and \mathbf{x}_{2i} represents the excluded exogenous regressors ($varlist_{iv}$ in the syntax diagram). \mathbf{x}_{1i} and \mathbf{x}_{2i} are collectively called the instruments. u_i and \mathbf{v}_i are zero-mean error terms, and the correlations between u_i and the elements of \mathbf{v}_i are presumably nonzero.

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The rest of the discussion is presented under the following headings:

2SLS and LIML estimators GMM estimator Video example

2SLS and LIML estimators

The most common instrumental-variables estimator is 2SLS.

Example 1: 2SLS estimator

We have state data from the 1980 census on the median dollar value of owner-occupied housing (hsngval) and the median monthly gross rent (rent). We want to model rent as a function of hsngval and the percentage of the population living in urban areas (pcturban):

$$rent_i = \beta_0 + \beta_1 hsngval_i + \beta_2 pcturban_i + u_i$$

where i indexes states and u_i is an error term.

Because random shocks that affect rental rates in a state probably also affect housing values, we treat hsngval as endogenous. We believe that the correlation between hsngval and u is not equal to zero. On the other hand, we have no reason to believe that the correlation between pcturban and u is nonzero, so we assume that pcturban is exogenous.

Because we are treating hsngval as an endogenous regressor, we must have one or more additional variables available that are correlated with hsngval but uncorrelated with u. Moreover, these excluded exogenous variables must not affect rent directly, because if they do then they should be included in the regression equation we specified above. In our dataset, we have a variable for family income (faminc) and for region of the country (region) that we believe are correlated with hsngval but not the error term. Together, pcturban, faminc, and factor variables 2.region, 3.region, and 4.region constitute our set of instruments.

To fit the equation in Stata, we specify the dependent variable and the list of included exogenous variables. In parentheses, we specify the endogenous regressors, an equal sign, and the excluded exogenous variables. Only the additional exogenous variables must be specified to the right of the equal sign; the exogenous variables that appear in the regression equation are automatically included as instruments.

Here we fit our model with the 2SLS estimator:

. use https://www.stata-press.com/data/r18/hsng
(1980 Census housing data)

. ivregress 2sls rent pcturban (hsngval = faminc i.region)

Instrumental variables 2SLS regression

Number of obs = 50 Wald chi2(2) = 90.76 Prob > chi2 = 0.0000 R-squared = 0.5989 Root MSE = 22.166

rent	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
hsngval	.0022398	.0003284	6.82	0.000	.0015961	.0028836
pcturban	.081516	.2987652	0.27	0.785	504053	.667085
_cons	120.7065	15.22839	7.93	0.000	90.85942	150.5536

Endogenous: hsngval

Exogenous: pcturban faminc 2.region 3.region 4.region

1

As we would expect, states with higher housing values have higher rental rates. The proportion of a state's population that is urban does not have a significant effect on rents.

□ Technical note

In a simultaneous-equations framework, we could write the model we just fit as

$$\begin{split} \mathtt{hsngval}_i &= \pi_0 + \pi_1 \mathtt{faminc}_i + \pi_2 \mathtt{2.region}_i + \pi_3 \mathtt{3.region}_i + \pi_4 \mathtt{4.region}_i + v_i \\ \mathtt{rent}_i &= \beta_0 + \beta_1 \mathtt{hsngval}_i + \beta_2 \mathtt{pcturban}_i + u_i \end{split}$$

which here happens to be recursive (triangular), because hsngval appears in the equation for rent but rent does not appear in the equation for hsngval. In general, however, systems of simultaneous equations are not recursive. Because this system is recursive, we could fit the two equations individually via OLS if we were willing to assume that u and v were independent. For a more detailed discussion of triangular systems, see Kmenta (1997, 719–720).

Historically, instrumental-variables estimation and systems of simultaneous equations were taught concurrently, and older textbooks describe instrumental-variables estimation solely in the context of simultaneous equations. However, in recent decades, the treatment of endogeneity and instrumentalvariables estimation has taken on a much broader scope, while interest in the specification of complete systems of simultaneous equations has waned. Most recent textbooks, such as Cameron and Trivedi (2005), Davidson and MacKinnon (1993), and Wooldridge (2010, 2020), treat instrumentalvariables estimation as an integral part of the modern economists' toolkit and introduce it long before shorter discussions on simultaneous equations.

In addition to the 2SLS member of the κ -class estimators, ivregress implements the LIML estimator. Both theoretical and Monte Carlo exercises indicate that the LIML estimator may yield less bias and confidence intervals with better coverage rates than the 2SLS estimator. See Poi (2006) and Stock, Wright, and Yogo (2002) (and the papers cited therein) for Monte Carlo evidence.

Example 2: LIML estimator

Here we refit our model with the LIML estimator:

. ivregress liml rent pcturban (hsngval = faminc i.region)

Number of obs Instrumental variables LIML regression

50 Wald chi2(2) 75.71 0.0000 Prob > chi2 R-squared 0.4901 Root MSE 24.992

rent	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
hsngval	.0026686	.0004173	6.39	0.000	.0018507	.0034865
pcturban	1827391	.3571132	-0.51	0.609	8826681	.5171899
_cons	117.6087	17.22625	6.83	0.000	83.84587	151.3715

Endogenous: hsngval

Exogenous: pcturban faminc 2.region 3.region 4.region

These results are qualitatively similar to the 2SLS results, although the coefficient on hsngval is about 19% higher.

R

GMM estimator

Since the celebrated paper of Hansen (1982), the GMM has been a popular method of estimation in economics and finance, and it lends itself well to instrumental-variables estimation. The basic principle is that we have some *moment* or *orthogonality* conditions of the form

$$E(\mathbf{z}_i u_i) = \mathbf{0} \tag{3}$$

From (1), we have $u_i = y_i - \mathbf{y}_i \beta_1 - \mathbf{x}_{1i} \beta_2$. What are the elements of the instrument vector \mathbf{z}_i ? By assumption, \mathbf{x}_{1i} is uncorrelated with u_i , as are the excluded exogenous variables \mathbf{x}_{2i} , and so we use $\mathbf{z}_i = [\mathbf{x}_{1i} \ \mathbf{x}_{2i}]$. The moment conditions are simply the mathematical representation of the assumption that the instruments are exogenous—that is, the instruments are orthogonal to (uncorrelated with) u_i .

If the number of elements in \mathbf{z}_i is just equal to the number of unknown parameters, then we can apply the analogy principle to (3) and solve

$$\frac{1}{N} \sum_{i} \mathbf{z}_{i} u_{i} = \frac{1}{N} \sum_{i} \mathbf{z}_{i} \left(y_{i} - \mathbf{y}_{i} \boldsymbol{\beta}_{1} - \mathbf{x}_{1i} \boldsymbol{\beta}_{2} \right) = \mathbf{0}$$

$$\tag{4}$$

This equation is known as the method of moments estimator. Here, where the number of instruments equals the number of parameters, the method of moments estimator coincides with the 2SLS estimator, which also coincides with what has historically been called the indirect least-squares estimator (Judge et al. 1985, 595).

The "generalized" in GMM addresses the case in which the number of instruments (columns of \mathbf{z}_i) exceeds the number of parameters to be estimated. Here there is no unique solution to the population moment conditions defined in (3), so we cannot use (4). Instead, we define the objective function

$$Q(\beta_1, \beta_2) = \left(\frac{1}{N} \sum_{i} \mathbf{z}_i u_i\right)' \mathbf{W} \left(\frac{1}{N} \sum_{i} \mathbf{z}_i u_i\right)$$
 (5)

where \mathbf{W} is a positive-definite matrix with the same number of rows and columns as the number of columns of \mathbf{z}_i . \mathbf{W} is known as the weighting matrix, and we specify its structure with the wmatrix() option. The GMM estimator of (β_1, β_2) minimizes $Q(\beta_1, \beta_2)$; that is, the GMM estimator chooses β_1 and β_2 to make the moment conditions as close to zero as possible for a given \mathbf{W} . For a more general GMM estimator, see [R] gmm. gmm does not restrict you to fitting a single linear equation, though the syntax is more complex.

A well-known result is that if we define the matrix S_0 to be the covariance of $z_i u_i$ and set $W = S_0^{-1}$, then we obtain the optimal two-step GMM estimator, where by optimal estimator we mean the one that results in the smallest variance given the moment conditions defined in (3).

Suppose that the errors u_i are heteroskedastic but independent among observations. Then

$$\mathbf{S}_0 = E(\mathbf{z}_i u_i u_i \mathbf{z}_i') = E(u_i^2 \mathbf{z}_i \mathbf{z}_i')$$

and the sample analogue is

$$\widehat{\mathbf{S}} = \frac{1}{N} \sum_{i} \widehat{u}_{i}^{2} \mathbf{z}_{i} \mathbf{z}_{i}^{\prime} \tag{6}$$

To implement this estimator, we need estimates of the sample residuals \widehat{u}_i . ivregress gmm obtains the residuals by estimating β_1 and β_2 by 2SLS and then evaluates (6) and sets $\mathbf{W} = \widehat{\mathbf{S}}^{-1}$. Equation (6) is the same as the center term of the "sandwich" robust covariance matrix available from most Stata estimation commands through the vce(robust) option.

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Example 3: GMM estimator

Here we refit our model of rents by using the GMM estimator, allowing for heteroskedasticity in u_i :

. ivregress gmm rent pcturban (hsngval = faminc i.region), wmatrix(robust)

rent	Coefficient	Robust std. err.	z	P> z	[95% conf.	interval]
hsngval	.0014643	.0004473	3.27	0.001	.0005877	.002341
pcturban	.7615482	.2895105	2.63	0.009	.1941181	1.328978
_cons	112.1227	10.80234	10.38	0.000	90.95052	133.2949

Endogenous: hsngval

Exogenous: pcturban faminc 2.region 3.region 4.region

Because we requested that a heteroskedasticity-consistent weighting matrix be used during estimation but did not specify the vce() option, ivregress reported standard errors that are robust to heteroskedasticity. Had we specified vce(unadjusted), we would have obtained standard errors that would be correct only if the weighting matrix \mathbf{W} does in fact converge to \mathbf{S}_0^{-1} .

□ Technical note

Many software packages that implement GMM estimation use the same heteroskedasticity-consistent weighting matrix we used in the previous example to obtain the optimal two-step estimates but do not use a heteroskedasticity-consistent VCE, even though they may label the standard errors as being "robust". To replicate results obtained from other packages, you may have to use the vce(unadjusted) option. See *Methods and formulas* below for a discussion of robust covariance matrix estimation in the GMM framework.

By changing our definition of S_0 , we can obtain GMM estimators suitable for use with other types of data that violate the assumption that the errors are independent and identically distributed. For example, you may have a dataset that consists of multiple observations for each person in a sample. The observations that correspond to the same person are likely to be correlated, and the estimation technique should account for that lack of independence. Say that in your dataset, people are identified by the variable personid and you type

. ivregress gmm ..., wmatrix(cluster personid)

Here ivregress estimates S_0 as

$$\widehat{\mathbf{S}} = \frac{1}{N} \sum_{c \in C} \mathbf{q}_c \mathbf{q}_c'$$

where C denotes the set of clusters and

$$\mathbf{q}_c = \sum_{i \in c_j} \widehat{u}_i \mathbf{z}_i$$

where c_j denotes the jth cluster. This weighting matrix accounts for the within-person correlation among observations, so the GMM estimator that uses this version of S_0 will be more efficient than the estimator that ignores this correlation.

Example 4: GMM estimator with clustering

We have data from the National Longitudinal Survey on young women's wages as reported in a series of interviews from 1968 through 1988, and we want to fit a model of wages as a function of each woman's age and age squared, job tenure, birth year, and level of education. We believe that random shocks that affect a woman's wage also affect her job tenure, so we treat tenure as endogenous. As additional instruments, we use her union status, number of weeks worked in the past year, and a dummy indicating whether she lives in a metropolitan area. Because we have several observations for each woman (corresponding to interviews done over several years), we want to control for clustering on each person.

```
. use https://www.stata-press.com/data/r18/nlswork
(National Longitudinal Survey of Young Women, 14-24 years old in 1968)
```

- . ivregress gmm ln_wage age c.age#c.age birth_yr grade
- > (tenure = union wks_work msp), wmatrix(cluster idcode)

Instrumental variables GMM regression	Number of obs	=	18,625
·	Wald chi2(5)	=	1807.17
	Prob > chi2	=	0.0000
	R-squared	=	
GMM weight matrix: Cluster (idcode)	Root MSE	=	.46951
(Std. err. adjusted	for 4,110 clus	ters in	idcode)

Robi					
Kobi	ıst				
st.d.	err.	7.	P> z	[95% conf.	intervall

ln_wage	Coefficient	Robust std. err.	z	P> z	[95% conf.	interval]
tenure age	.099221 .0171146	.0037764	26.27 2.56	0.000 0.011	.0918194	.1066227
c.age#c.age	0005191	.000111	-4.68	0.000	0007366	0003016
birth_yr grade _cons	0085994 .071574 .8575071	.0021932 .0029938 .1616274	-3.92 23.91 5.31	0.000 0.000 0.000	012898 .0657062 .5407231	0043008 .0774417 1.174291

Endogenous: tenure

age c.age#c.age birth_yr grade union wks_work msp Exogenous:

Both job tenure and years of schooling have significant positive effects on wages.

Time-series data are often plagued by serial correlation. In these cases, we can construct a weighting matrix to account for the fact that the error in period t is probably correlated with the errors in periods t-1, t-2, etc. An HAC weighting matrix can be used to account for both serial correlation and potential heteroskedasticity.

To request an HAC weighting matrix, you specify the wmatrix(hac kernel | # | opt |) option. kernel specifies which of three kernels to use: bartlett, parzen, or quadraticspectral. kernel determines the amount of weight given to lagged values when computing the HAC matrix, and # denotes the maximum number of lags to use. Many texts refer to the bandwidth of the kernel instead of the number of lags; the bandwidth is equal to the number of lags plus one. If neither opt nor # is specified, then N-2 lags are used, where N is the sample size.

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If you specify wmatrix(hac kernel opt), then ivregress uses Newey and West's (1994) algorithm for automatically selecting the number of lags to use. Although the authors' Monte Carlo simulations do show that the procedure may result in size distortions of hypothesis tests, the procedure is still useful when little other information is available to help choose the number of lags.

For more on GMM estimation, see Baum (2006); Baum, Schaffer, and Stillman (2003, 2007); Cameron and Trivedi (2005); Davidson and MacKinnon (1993); Hayashi (2000); or Wooldridge (2010). See Newey and West (1987) and Wang and Wu (2012) for an introduction to HAC covariance matrix estimation.

Video example

Instrumental variables regression using Stata

Stored results

ivregress stores the following in e():

```
Scalars
                             number of observations
    e(N)
                             model sum of squares
    e(mss)
    e(df_m)
                             model degrees of freedom
    e(rss)
                             residual sum of squares
    e(df_r)
                             residual degrees of freedom
                             \mathbb{R}^2
    e(r2)
                             adjusted R^2
    e(r2_a)
    e(F)
                             F statistic
    e(rmse)
                             root mean squared error
    e(N_clust)
                             number of clusters
                             \chi^2
    e(chi2)
    e(kappa)
                             \kappa used in LIML estimator
                             value of GMM objective function
    e(J)
    e(wlagopt)
                             lags used in HAC weight matrix (if Newey-West algorithm used)
                             lags used in HAC VCE matrix (if Newey-West algorithm used)
    e(vcelagopt)
    e(hac_lag)
                             HAC lag
    e(rank)
                             rank of e(V)
    e(iterations)
                             number of GMM iterations (0 if not applicable)
Macros
    e(cmd)
                             ivregress
    e(cmdline)
                             command as typed
    e(depvar)
                             name of dependent variable
    e(endog)
                             names of endogenous variables
    e(exog)
                             names of exogenous variables
    e(constant)
                             noconstant or hasconstant if specified
    e(wtype)
                             weight type
    e(wexp)
                             weight expression
    e(title)
                             title in estimation output
                             name of cluster variable
    e(clustvar)
    e(hac_kernel)
                             HAC kernel
    e(vce)
                             vcetype specified in vce()
    e(vcetype)
                             title used to label Std. err.
                             2sls, liml, or gmm
    e(estimator)
    e(exogr)
                             exogenous regressors
    e(wmatrix)
                             wmtype specified in wmatrix()
    e(moments)
                             centered if center specified
    e(small)
                             small if small-sample statistics
    e(properties)
    e(estat_cmd)
                             program used to implement estat
```

```
e(predict)
                             program used to implement predict
    e(footnote)
                             program used to implement footnote display
    e(marginsok)
                             predictions allowed by margins
    e(marginsnotok)
                             predictions disallowed by margins
    e(asbalanced)
                             factor variables fyset as asbalanced
    e(asobserved)
                             factor variables fvset as asobserved
Matrices
    e(b)
                             coefficient vector
    e(W)
                             weight matrix used to compute GMM estimates
    e(S)
                             moment covariance matrix used to compute GMM variance-covariance matrix
    e(V)
                             variance-covariance matrix of the estimators
    e(V_modelbased)
                             model-based variance
Functions
    e(sample)
                             marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
    r(table)
                                 matrix containing the coefficients with their standard errors, test statistics, p-values,
                                     and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

Methods and formulas are presented under the following headings:

Notation 2SLS and LIML estimators GMM estimator

Notation

Items printed in lowercase and italicized (for example, x) are scalars. Items printed in lowercase and boldfaced (for example, x) are vectors. Items printed in uppercase and boldfaced (for example, X) are matrices.

The model is

$$\mathbf{y} = \mathbf{Y}\boldsymbol{\beta}_1 + \mathbf{X}_1\boldsymbol{\beta}_2 + \mathbf{u} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$
$$\mathbf{Y} = \mathbf{X}_1\mathbf{\Pi}_1 + \mathbf{X}_2\mathbf{\Pi}_2 + \mathbf{v} = \mathbf{Z}\mathbf{\Pi} + \mathbf{V}$$

where y is an $N \times 1$ vector of the left-hand-side variable; N is the sample size; Y is an $N \times p$ matrix of p endogenous regressors; X_1 is an $N \times k_1$ matrix of k_1 included exogenous regressors; \mathbf{X}_2 is an $N \times k_2$ matrix of k_2 excluded exogenous variables, $\mathbf{X} = [\mathbf{Y} \ \mathbf{X}_1], \ \mathbf{Z} = [\mathbf{X}_1 \ \mathbf{X}_2], \mathbf{u}$ is an $N \times 1$ vector of errors; V is an $N \times p$ matrix of errors; $\beta = [\beta_1, \beta_2]$ is a $k = (p + k_1) \times 1$ vector of parameters; and Π is a $(k_1 + k_2) \times p$ vector of parameters. If a constant term is included in the model, then one column of X_1 contains all ones.

Let v be a column vector of weights specified by the user. If no weights are specified, v = 1. Let w be a column vector of normalized weights. If no weights are specified or if the user specified fweights or iweights, $\mathbf{w} = \mathbf{v}$; otherwise, $\mathbf{w} = \{\mathbf{v}/(\mathbf{1}'\mathbf{v})\}(\mathbf{1}'\mathbf{1})$. Let \mathbf{D} denote the $N \times N$ matrix with w on the main diagonal and zeros elsewhere. If no weights are specified, D is the identity matrix.

The weighted number of observations n is defined as 1'w. For iweights, this is truncated to an integer. The sum of the weights is 1'v. Define c=1 if there is a constant in the regression and zero otherwise.

The order condition for identification requires that $k_2 \geq p$: the number of excluded exogenous variables must be at least as great as the number of endogenous regressors.

In the following formulas, if weights are specified, $X_1'X_1$, X'X, X'y, y'y, Z'Z, Z'X, and Z'yare replaced with X'_1DX_1 , X'DX, X'Dy, y'Dy, Z'DZ, Z'DX, and Z'Dy, respectively. We suppress the **D** below to simplify the notation.

2SLS and LIML estimators

Define the κ -class estimator of β as

$$\mathbf{b} = \left\{ \mathbf{X}'(\mathbf{I} - \kappa \mathbf{M}_{\mathbf{Z}}) \mathbf{X} \right\}^{-1} \mathbf{X}'(\mathbf{I} - \kappa \mathbf{M}_{\mathbf{Z}}) \mathbf{y}$$

where $M_Z = I - Z(Z'Z)^{-1}Z'$. The 2SLS estimator results from setting $\kappa = 1$. The LIML estimator results from selecting κ to be the minimum eigenvalue of $(\mathbf{Y}'\mathbf{M}_{\mathbf{Z}}\mathbf{Y})^{-1/2}\mathbf{Y}'\mathbf{M}_{\mathbf{X}_{1}}\mathbf{Y}(\mathbf{Y}'\mathbf{M}_{\mathbf{Z}}\mathbf{Y})^{-1/2}$, where $M_{X_1} = I - X_1(X_1'X_1)^{-1}X_1'$.

The total sum of squares (TSS) equals $\mathbf{y}'\mathbf{y}$ if there is no intercept and $\mathbf{y}'\mathbf{y} - \{(\mathbf{1}'\mathbf{y})^2/n\}$ otherwise. The degrees of freedom is n-c. The error sum of squares (ESS) is defined as y'y-2bX'y+b'X'Xb. The model sum of squares (MSS) equals TSS – ESS. The degrees of freedom is k-c.

The mean squared error, s^2 , is defined as ESS/(n-k) if small is specified and ESS/n otherwise. The root mean squared error is s, its square root.

If c=1 and small is not specified, a Wald statistic, W, of the joint significance of the k-1parameters of β except the constant term is calculated; $W \sim \chi^2(k-1)$. If c=1 and small is specified, then an F statistic is calculated as F = W/(k-1); $F \sim F(k-1, n-k)$.

The R^2 is defined as $R^2 = 1 - ESS/TSS$.

The adjusted R^2 is $R_a^2 = 1 - (1 - R^2)(n - c)/(n - k)$.

The unadjusted (default) variance estimate is $Var(\mathbf{b}) = s^2 \{ \mathbf{X}' (\mathbf{I} - \kappa \mathbf{M_Z}) \mathbf{X} \}^{-1}$.

For a general discussion of robust variance estimates in regression, see A general notation for the robust variance calculation in [R] regress. ivregress uses the same definitions for terms discussed in Robust calculation for regress in its robust variance calculation, except for the following.

The vector of scores is given by

$$\mathbf{u}_j = (y_j - \mathbf{x}_j \mathbf{b}) \widehat{\mathbf{x}}_j$$

where $\hat{\mathbf{x}}_j' = \mathbf{P}\mathbf{z}_j'$ and $\mathbf{P} = (\mathbf{X}'\mathbf{Z})(\mathbf{Z}'\mathbf{Z})^{-1}$. When the formulas in [R] regress are applied, q_c is given by its regressionlike definition. If small is not specified, then k=0 in the formulas given in [R] regress.

ivregress 2sls and ivregress liml also support estimation with survey data. For details on VCEs with survey data, see [SVY] Variance estimation.

GMM estimator

We obtain an initial consistent estimate of β by using the 2SLS estimator; see above. Using this estimate of β , we compute the weighting matrix **W** and calculate the GMM estimator

$$\mathbf{b}_{\mathrm{GMM}} = \left\{ \mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{X} \right\}^{-1}\mathbf{X}'\mathbf{Z}\mathbf{W}\mathbf{Z}'\mathbf{y}$$

The variance of b_{GMM} is

$$Var(\mathbf{b}_{GMM}) = n \{ \mathbf{X}' \mathbf{Z} \mathbf{W} \mathbf{Z}' \mathbf{X} \}^{-1} \mathbf{X}' \mathbf{Z} \mathbf{W} \widehat{\mathbf{S}} \mathbf{W} \mathbf{Z}' \mathbf{X} \{ \mathbf{X}' \mathbf{Z} \mathbf{W} \mathbf{Z}' \mathbf{X} \}^{-1}$$

 $Var(\mathbf{b}_{GMM})$ is of the sandwich form \mathbf{DMD} ; see [P] <u>robust</u>. If the user specifies the small option, ivregress implements a small-sample adjustment by multiplying the VCE by N/(N-k).

If vce(unadjusted) is specified, then we set $\widehat{S} = W^{-1}$ and the VCE reduces to the "optimal" GMM variance estimator

$$Var(\boldsymbol{\beta}_{GMM}) = n \{ \mathbf{X}' \mathbf{Z} \mathbf{W} \mathbf{Z}' \mathbf{X} \}^{-1}$$

However, if \mathbf{W}^{-1} is not a good estimator of $E(\mathbf{z}_i u_i u_i \mathbf{z}_i')$, then the optimal GMM estimator is inefficient, and inference based on the optimal variance estimator could be misleading.

 ${\bf W}$ is calculated using the residuals from the initial 2SLS estimates, whereas ${\bf S}$ is estimated using the residuals based on ${\bf b}_{\rm GMM}$. The wmatrix() option affects the form of ${\bf W}$, whereas the vce() option affects the form of ${\bf S}$. Except for different residuals being used, the formulas for ${\bf W}^{-1}$ and ${\bf S}$ are identical, so we focus on estimating ${\bf W}^{-1}$.

If wmatrix(unadjusted) is specified, then

$$\mathbf{W}^{-1} = \frac{s^2}{n} \sum_{i} \mathbf{z}_i \mathbf{z}_i'$$

where $s^2 = \sum_i u_i^2/n$. This weight matrix is appropriate if the errors are homoskedastic.

If wmatrix(robust) is specified, then

$$\mathbf{W}^{-1} = \frac{1}{n} \sum_{i} u_i^2 \mathbf{z}_i \mathbf{z}_i'$$

which is appropriate if the errors are heteroskedastic.

If wmatrix(cluster *clustvar*) is specified, then

$$\mathbf{W}^{-1} = \frac{1}{n} \sum_{c} \mathbf{q}_c \mathbf{q}_c'$$

where c indexes clusters,

$$\mathbf{q}_c = \sum_{i \in c_i} u_i \mathbf{z}_i$$

and c_i denotes the jth cluster.

If wmatrix(hac kernel [#]) is specified, then

$$\mathbf{W}^{-1} = \frac{1}{n} \sum_{i} u_i^2 \mathbf{z}_i \mathbf{z}_i' + \frac{1}{n} \sum_{l=1}^{l=n-1} \sum_{i=l+1}^{i=n} K(l, m) u_i u_{i-l} \left(\mathbf{z}_i \mathbf{z}_{i-l}' + \mathbf{z}_{i-l} \mathbf{z}_i' \right)$$

where m = # if # is specified and m = n - 2 otherwise. Define z = l/(m+1). If kernel is nwest, then

$$K(l,m) = \left\{ \begin{array}{ll} 1-z & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{array} \right.$$

If kernel is gallant, then

$$K(l,m) = \begin{cases} 1 - 6z^2 + 6z^3 & 0 \le z \le 0.5\\ 2(1-z)^3 & 0.5 < z \le 1\\ 0 & \text{otherwise} \end{cases}$$

If kernel is quadraticspectral, then

$$K(l,m) = \begin{cases} 1 & z = 0\\ 3\left\{\sin(\theta)/\theta - \cos(\theta)\right\}/\theta^2 & \text{otherwise} \end{cases}$$

where $\theta = 6\pi z/5$.

If wmatrix(hac kernel opt) is specified, then ivregress uses Newey and West's (1994) automatic lag-selection algorithm, which proceeds as follows. Define \mathbf{h} to be a $(k_1+k_2)\times 1$ vector containing ones in all rows except for the row corresponding to the constant term (if present); that row contains a zero. Define

$$f_{i} = (u_{i}\mathbf{z}_{i})\mathbf{h}$$

$$\widehat{\sigma}_{j} = \frac{1}{n} \sum_{i=j+1}^{n} f_{i}f_{i-j} \qquad j = 0, \dots, m^{*}$$

$$\widehat{s}^{(q)} = 2 \sum_{j=1}^{m^{*}} \widehat{\sigma}_{j}j^{q}$$

$$\widehat{s}^{(0)} = \widehat{\sigma}_{0} + 2 \sum_{j=1}^{m^{*}} \widehat{\sigma}_{j}$$

$$\widehat{\gamma} = c_{\gamma} \left\{ \left(\frac{\widehat{s}^{(q)}}{\widehat{s}^{(0)}} \right)^{2} \right\}^{1/2q+1}$$

$$m = \widehat{\gamma}n^{1/(2q+1)}$$

where q, m^* , and c_{γ} depend on the kernel specified:

Kernel	q	m^*	c_{γ}
Bartlett	1	int $\left\{20(T/100)^{2/9}\right\}$	1.1447
Parzen	2	int $\left\{20(T/100)^{4/25}\right\}$	2.6614
Quadratic spectral	2	$\inf \left\{ 20 (T/100)^{2/25} \right\}$	1.3221

where int(x) denotes the integer obtained by truncating x toward zero. For the Bartlett and Parzen kernels, the optimal lag is $min\{int(m), m^*\}$. For the quadratic spectral, the optimal lag is $min\{m, m^*\}$.

If wmatrix(hac *kernel* opt #) is specified, then ivregress uses # instead of 20 in the definition of m^* above to select the optimal lag.

If center is specified, when computing weighting matrices ivregress replaces the term $u_i z_i$ in the formulas above with $u_i \mathbf{z}_i - \overline{u}\overline{\mathbf{z}}$, where $\overline{u}\overline{\mathbf{z}} = \sum_i u_i \mathbf{z}_i / N$.

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Also see

- [R] **ivregress postestimation** Postestimation tools for ivregress
- [R] gmm Generalized method of moments estimation
- [R] **ivprobit** Probit model with continuous endogenous covariates
- [R] **ivqregress** Instrumental-variables quantile regression
- [R] **ivtobit** Tobit model with continuous endogenous covariates
- [R] reg3 Three-stage estimation for systems of simultaneous equations
- [R] regress Linear regression
- [ERM] **eregress** Extended linear regression
- [FMM] fmm: ivregress Finite mixtures of linear regression models with endogenous covariates
- [SEM] Intro 5 Tour of models
- [SP] **spivregress** Spatial autoregressive models with endogenous covariates
- [SVY] svy estimation Estimation commands for survey data
- [TS] **forecast** Econometric model forecasting
- [XT] **xtivreg** Instrumental variables and two-stage least squares for panel-data models
- [U] 20 Estimation and postestimation commands

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