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**hetprobit** — Heteroskedastic probit model

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# **Description**

hetprobit fits a maximum-likelihood heteroskedastic probit model.

## **Quick start**

Heteroskedastic probit model of y on x1, using x2 to model the variance hetprobit y x1, het(x2)

With robust standard errors

hetprobit y x1, het(x2) vce(robust)

After fitting a model, reprint the table as a coefficient legend hetprobit, coeflegend

## Menu

Statistics > Binary outcomes > Heteroskedastic probit regression

# Syntax

```
hetprobit depvar [indepvars] [if] [in] [weight], het(varlist[, offset(varname<sub>o</sub>)]) [options]
```

options	Description					
Model						
* het( <i>varlist</i> [])	independent variables to model the variance and optional offset variable					
<u>nocons</u> tant	suppress constant term					
<pre>offset(varname)</pre>	include varname in model with coefficient constrained to 1					
asis	retain perfect predictor variables					
<pre>constraints(constraints)</pre>	apply specified linear constraints					
SE/Robust						
vce(vcetype)	vcetype may be oim, <u>r</u> obust, <u>cl</u> uster $clustvar$ , opg, <u>boot</u> strap, or <u>jack</u> knife					
Reporting						
<u>l</u> evel(#)	set confidence level; default is level(95)					
lrmodel	perform the likelihood-ratio model test instead of the default Wald test					
waldhet	perform Wald test on variance instead of LR test					
<u>nocnsr</u> eport	do not display constraints					
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling					
Maximization						
maximize_options	control the maximization process; seldom used					
<u>col</u> linear	keep collinear variables					
<u>coefl</u> egend	display legend instead of statistics					

```
*het() is required. The full specification is het(varlist \mid , offset(varname_o) \mid).
```

indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.

depvar, indepvars, and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: hetprobit.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce(), 1rmode1, and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# **Options**

```
het(varlist [, offset(varname<sub>o</sub>)]) specifies the independent variables and, optionally, the offset variable in the variance function. het() is required.
```

offset (varname<sub>0</sub>) specifies that selection offset varname<sub>0</sub> be included in the model with the coefficient constrained to be 1.

noconstant, offset(varname); see [R] Estimation options.

asis forces the retention of perfect predictor variables and their associated perfectly predicted observations and may produce instabilities in maximization; see [R] probit.

constraints (constraints); see [R] Estimation options.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce\_option.

Reporting

level(#), lrmodel; see [R] Estimation options.

waldhet specifies that a Wald test of whether lnsigma = 0 be performed instead of the LR test. nocnsreport; see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nof vlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize\_options: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init\_specs); see [R] Maximize. These options are seldom used.

Setting the optimization type to technique(bhhh) resets the default vcetype to vce(opg).

The following options are available with hetprobit but are not shown in the dialog box: collinear, coeflegend; see [R] Estimation options.

# Remarks and examples

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Remarks are presented under the following headings:

Introduction Robust standard errors

#### Introduction

hetprobit fits a maximum-likelihood heteroskedastic probit model, which is a generalization of the probit model. Let  $y_i, j = 1, \dots, N$ , be a binary outcome variable taking on the value 0 (failure) or 1 (success). In the probit model, the probability that  $y_i$  takes on the value 1 is modeled as a nonlinear function of a linear combination of the k independent variables  $\mathbf{x}_i = (x_{1i}, x_{2i}, \dots, x_{ki})$ ,

$$\Pr(y_j = 1) = \Phi(\mathbf{x}_j \mathbf{b})$$

in which  $\Phi(\cdot)$  is the cumulative distribution function (CDF) of a standard normal random variable, that is, a normally distributed (Gaussian) random variable with mean 0 and variance 1. The linear combination of the independent variables,  $\mathbf{x}_j\mathbf{b}$ , is commonly called the *index function*, or *index*. Heteroskedastic probit generalizes the probit model by generalizing  $\Phi(\cdot)$  to a normal CDF with a variance that is no longer fixed at 1 but can vary as a function of the independent variables. hetprobit models the variance as a multiplicative function of these m variables  $\mathbf{z}_j = (z_{1j}, z_{2j}, \dots, z_{mj})$ , following Harvey (1976):

$$\sigma_i^2 = \left\{ \exp(\mathbf{z}_j \boldsymbol{\gamma}) \right\}^2$$

Thus, the probability of success as a function of all the independent variables is

$$\Pr(y_j = 1) = \Phi \Big\{ \mathbf{x}_j \mathbf{b} / \exp(\mathbf{z}_j \boldsymbol{\gamma}) \Big\}$$

From this expression, it is clear that, unlike the index  $\mathbf{x}_j \mathbf{b}$ , no constant term can be present in  $\mathbf{z}_j \gamma$  if the model is to be identifiable.

Suppose that the binary outcomes  $y_j$  are generated by thresholding an unobserved random variable, w, which is normally distributed with mean  $\mathbf{x}_j \mathbf{b}$  and variance 1 such that

$$y_j = \begin{cases} 1 & \text{if } w_j > 0 \\ 0 & \text{if } w_j \le 0 \end{cases}$$

This process gives the probit model:

$$Pr(y_j = 1) = Pr(w_j > 0) = \Phi(\mathbf{x}_j \mathbf{b})$$

Now, suppose that the unobserved  $w_i$  are heteroskedastic with variance

$$\sigma_i^2 = \left\{ \exp(\mathbf{z}_i \boldsymbol{\gamma}) \right\}^2$$

Relaxing the homoskedastic assumption of the probit model in this manner yields our multiplicative heteroskedastic probit model:

$$\Pr(y_j = 1) = \Phi \left\{ \mathbf{x}_j \mathbf{b} / \exp(\mathbf{z}_j \boldsymbol{\gamma}) \right\}$$

#### Example 1

For this example, we generate simulated data for a simple heteroskedastic probit model and then estimate the coefficients with hetprobit:

```
. set obs 1000
Number of observations (_N) was 0, now 1,000.
. set seed 1234567
. generate x = 1-2*runiform()
. generate xhet = runiform()
. generate sigma = exp(1.5*xhet)
. generate p = normal((0.3+2*x)/sigma)
. generate y = cond(runiform()<=p,1,0)</pre>
. hetprobit y x, het(xhet)
Fitting probit model:
Iteration 0: Log likelihood = -688.33746
              Log likelihood = -610.48362
Iteration 1:
Iteration 2:
              Log likelihood =
                                -610.3626
Iteration 3: Log likelihood =
                                -610.3626
Fitting full model:
Iteration 0: Log likelihood = -610.3626
Iteration 1: Log likelihood = -600.8767
Iteration 2: Log likelihood = -600.10154
Iteration 3: Log likelihood = -600.01544
Iteration 4: Log likelihood = -600.01521
Iteration 5: Log likelihood = -600.01521
Heteroskedastic probit model
                                                 Number of obs
                                                                           1,000
                                                 Zero outcomes
                                                                             451
                                                 Nonzero outcomes
                                                                             549
                                                 Wald chi2(1)
                                                                           54.20
Log likelihood = -600.0152
                                                 Prob > chi2
                                                                          0.0000
               Coefficient
                            Std. err.
                                                 P>|z|
                                                            [95% conf. interval]
           У
У
                 1.782479
                             .2421117
                                          7.36
                                                 0.000
                                                           1.307949
                                                                        2.257009
           х
       cons
                  .3140616
                             .0871121
                                          3.61
                                                 0.000
                                                            .1433249
                                                                        .4847982
lnsigma
        xhet
                  1.31152
                             .3011689
                                          4.35
                                                 0.000
                                                            .7212402
                                                                        1.901801
LR test of lnsigma=0: chi2(1) = 20.69
                                                           Prob > chi2 = 0.0000
```

Above, we created two variables, x and xhet, and then simulated the model

$$\Pr(y = 1) = F\{(\beta_0 + \beta_1 x) / \exp(\gamma_1 x \text{het})\}$$

for  $\beta_0=0.3$ ,  $\beta_1=2$ , and  $\gamma_1=1.5$ . According to hetprobit's output, all coefficients are significant, and, as we would expect, the Wald test of the full model versus the constant-only model—for example, the index consisting of  $\beta_0+\beta_1 x$  versus that of just  $\beta_0$ —is significant with  $\chi^2(1)=54$ . Likewise, the likelihood-ratio test of heteroskedasticity, which tests the full model with heteroskedasticity against the full model without, is significant with  $\chi^2(1)=21$ . See [R] Maximize for more explanation of the output. For this simple model, hetprobit took five iterations to converge. As stated elsewhere (Greene 2018, 764), this is a difficult model to fit, and it is not uncommon for it to require many iterations or for the optimizer to print out warnings and informative messages during the optimization. Slow convergence is especially common for models in which one or more of the independent variables appear in both the index and variance functions.

#### □ Technical note

Stata interprets a value of 0 as a negative outcome (failure) and treats all other values (except missing) as positive outcomes (successes). Thus if your dependent variable takes on the values 0 and 1, then 0 is interpreted as failure and 1 as success. If your dependent variable takes on the values 0, 1, and 2, then 0 is still interpreted as failure, but both 1 and 2 are treated as successes.

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#### Robust standard errors

If you specify the vce(robust) option, hetprobit reports robust standard errors as described in [U] **20.22 Obtaining robust variance estimates**. To illustrate the effect of this option, we will reestimate our coefficients by using the same model and data in our example, this time adding vce(robust) to our hetprobit command.

## Example 2

. hetprobit y	x, het(xhet)	vce(robust)	nolog				
Heteroskedastic probit model				Number of obs Zero outcomes Nonzero outcomes		= = =	1,000 451 549
Log pseudolikelihood = -600.0152				Wald chi2(1) Prob > chi2		=	50.49 0.0000
у	Coefficient	Robust std. err.	z	P> z	[95% co	nf.	interval]
y x _cons	1.782479 .3140616	.2508447 .087195	7.11 3.60	0.000	1.29083 .143162	_	2.274126 .4849607
lnsigma xhet	1.31152	.3059137	4.29	0.000	.711940	6	1.9111
Wald test of lnsigma=0: chi2(1) = 18.38					Prob >	chi	2 = 0.0000

The vce(robust) standard errors for two of the three parameters are larger than the previously reported conventional standard errors. This is to be expected, even though (by construction) we have perfect model specification because this option trades off efficient estimation of the coefficient variance—covariance matrix for robustness against misspecification.

Specifying the vce(cluster *clustvar*) option relaxes the usual assumption of independence between observations to the weaker assumption of independence just between clusters; that is, hetprobit, vce(cluster *clustvar*) is robust with respect to within-cluster correlation. This option is less efficient than the xtgee population-averaged models because hetprobit inefficiently sums within cluster for the standard error calculation rather than attempting to exploit what might be assumed about the within-cluster correlation.

#### Stored results

hetprobit stores the following in e():

```
Scalars
    e(N)
                              number of observations
    e(N_f)
                              number of zero outcomes
                              number of nonzero outcomes
    e(N_s)
                              number of parameters
    e(k)
    e(k_eq)
                              number of equations in e(b)
                              number of equations in overall model test
    e(k_eq_model)
    e(k_dv)
                              number of dependent variables
    e(df_m)
                              model degrees of freedom
    e(11)
                              log likelihood
    e(11_0)
                              log likelihood, constant-only model
    e(11_c)
                              log likelihood, comparison model
    e(N_clust)
                              number of clusters
    e(chi2)
                              \chi^2
                              \chi^2 for heteroskedasticity test
    e(chi2_c)
                              p-value for heteroskedasticity test
    e(p_c)
                              degrees of freedom for heteroskedasticity test
    e(df_m_c)
                              p-value for model test
    e(p)
                              rank of e(V)
    e(rank)
                              rank of e(V) for constant-only model
    e(rank0)
                              number of iterations
    e(ic)
    e(rc)
                              return code
    e(converged)
                              1 if converged, 0 otherwise
Macros
    e(cmd)
                              hetprobit
    e(cmdline)
                              command as typed
    e(depvar)
                              name of dependent variable
    e(wtype)
                              weight type
                              weight expression
    e(wexp)
    e(title)
                              title in estimation output
    e(clustvar)
                              name of cluster variable
    e(offset1)
                              offset for probit equation
    e(offset2)
                              offset for variance equation
                              Wald or LR; type of model \chi^2 test
    e(chi2type)
    e(chi2_ct)
                              Wald or LR; type of model \chi^2 test corresponding to e(chi2_c)
                              vcetype specified in vce()
    e(vce)
    e(vcetype)
                              title used to label Std. err.
    e(opt)
                              type of optimization
                              max or min; whether optimizer is to perform maximization or minimization
    e(which)
    e(method)
    e(ml_method)
                              type of ml method
                              name of likelihood-evaluator program
    e(user)
    e(technique)
                              maximization technique
    e(properties)
    e(predict)
                              program used to implement predict
    e(asbalanced)
                              factor variables fyset as asbalanced
    e(asobserved)
                              factor variables fyset as asobserved
Matrices
    e(b)
                              coefficient vector
    e(Cns)
                              constraints matrix
                              iteration log (up to 20 iterations)
    e(ilog)
    e(gradient)
                              gradient vector
    e(V)
                              variance-covariance matrix of the estimators
    e(V_modelbased)
                              model-based variance
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

Matrices

r(table) matrix containing the coefficients with their standard errors, test statistics, p-values,

and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

#### Methods and formulas

The heteroskedastic probit model is a generalization of the probit model because it allows the scale of the inverse link function to vary from observation to observation as a function of the independent variables.

The log-likelihood function for the heteroskedastic probit model is

$$\mathrm{ln}L = \sum_{j \in S} w_j \ln\!\Phi\{\mathbf{x}_j \boldsymbol{\beta}/\exp(\mathbf{z}\boldsymbol{\gamma})\} + \sum_{j \not\in S} w_j \ln\!\left[1 - \Phi\{\mathbf{x}_j \boldsymbol{\beta}/\exp(\mathbf{z}\boldsymbol{\gamma})\}\right]$$

where S is the set of all observations j such that  $y_i \neq 0$  and  $w_i$  denotes the optional weights.  $\ln L$ is maximized as described in [R] Maximize.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] \_robust, particularly Maximum likelihood estimators and Methods and formulas.

hetprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] Variance estimation.

### References

Blevins, J. R., and S. Khan. 2013. Distribution-free estimation of heteroskedastic binary response models in Stata. Stata Journal 13: 588-602.

Greene, W. H. 2018. Econometric Analysis. 8th ed. New York: Pearson.

Harvey, A. C. 1976. Estimating regression models with multiplicative heteroscedasticity. Econometrica 44: 461-465. https://doi.org/10.2307/1913974.

# Also see

- [R] **hetprobit postestimation** Postestimation tools for hetprobit
- [R] **hetoprobit** Heteroskedastic ordered probit regression
- [R] **logistic** Logistic regression, reporting odds ratios
- [R] **probit** Probit regression

[BAYES] bayes: hetprobit — Bayesian heteroskedastic probit regression

[SVY] svy estimation — Estimation commands for survey data

[XT] **xtprobit** — Random-effects and population-averaged probit models

[U] 20 Estimation and postestimation commands

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