# Title

### churdle — Cragg hurdle regression

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# Description

churdle fits a linear or exponential hurdle model for a bounded dependent variable. The hurdle model combines a selection model that determines the boundary points of the dependent variable with an outcome model that determines its nonbounded values. Separate independent covariates are permitted for each model.

# Quick start

Linear hurdle model of y1 on x1 and x2, specifying that y1 is truncated at 0 with x1 and x3 predicting selection

churdle linear y1 x1 x2, select(x1 x3) 11(0)

Add an upper truncation limit of 40 churdle linear y1 x1 x2, select(x1 x3) ll(0) ul(40)

- Same as above, with the upper truncation limit specified in trunc churdle linear y1 x1 x2, select(x1 x3) ll(0) ul(trunc)
- Same as above, and use x3 to model the variance of the selection model churdle linear y1 x1 x2, select(x1 x3, het(x3)) ll(0) ul(trunc)

Exponential hurdle model of y2 on x1 and x2, specifying that y2 is truncated at 4 with x1 and x3 predicting selection

churdle exponential y2 x1 x2, select(x1 x3) 11(4)

## Menu

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# Syntax

```
Basic syntax
churdle linear depvar, select(varlist<sub>s</sub>) {ll(...)|ul(...)}
churdle exponential depvar, select(varlist<sub>s</sub>) ll(...)
```

Full syntax for churdle linear

```
churdle linear depvar [indepvars] [if] [in] [weight],
    select(varlist<sub>s</sub>[, noconstant het(varlist<sub>o</sub>)])
    {ll(#|varname) | ul(#|varname) } [options]
```

Full syntax for churdle exponential

```
churdle <u>exp</u>onential depvar [indepvars] [if] [in] [weight],
<u>sel</u>ect(varlist<sub>s</sub>[, <u>nocons</u>tant het(varlist<sub>o</sub>)]) ll(#|varname) [options]
```

options	Description
Model	
* <u>sel</u> ect()	specify independent variables and options for selection model
<sup>‡</sup> 11(#  <i>varname</i> )	lower truncation limit
<sup>‡</sup> ul(#  <i>varname</i> )	upper truncation limit
<u>nocons</u> tant	suppress constant term
<u>const</u> raints( <i>constraints</i> )	apply specified linear constraints
het( <i>varlist</i> )	specify variables to model the variance
SE/Robust	
vce(vcetype)	<i>vcetype</i> may be oim, <u>r</u> obust, <u>cl</u> uster <i>clustvar</i> , <u>boot</u> strap, or <u>jackknife</u>
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>nocnsr</u> eport	do not display constraints
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
maximize_options	control the maximization process; seldom used
<u>coefl</u> egend	display legend instead of statistics

\*select() is required.

The full specification is <u>sel</u>ect(varlist<sub>s</sub> [, <u>noconstant</u> het(varlist<sub>o</sub>) ]).

noconstant specifies that the constant be excluded from the selection model.

het(varlist<sub>o</sub>) specifies the variables in the error-variance function of the selection model.

<sup>‡</sup>You must specify at least one of ul(# | varname) or ll(# | varname) for the linear model and must specify ll(# | varname) for the exponential model.

indepvars, varlist<sub>s</sub>, and varlist<sub>o</sub> may contain factor variables; see [U] 11.4.3 Factor variables.

bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] **11.1.10 Prefix commands**. Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# Options

#### Model

- select(varlist<sub>s</sub>[, noconstant het(varlist<sub>o</sub>)]) specifies the variables and options for the selection
  model. select() is required.
- ll(# | varname) and ul(# | varname) indicate the lower and upper limits, respectively, for the dependent variable. You must specify one or both for the linear model and must specify a lower limit for the exponential model. Observations with  $depvar \leq ll()$  have a lower bound; observations with  $depvar \geq ul()$  have an upper bound; and the remaining observations are in the continuous region.

noconstant, constraints(constraints); see [R] Estimation options.

het (varlist) specifies the variables in the error-variance function of the outcome model.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster, *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce\_option.

Reporting

level(#), nocnsreport; see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, notvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize\_options: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init\_specs); see [R] Maximize. These options are seldom used. The following option is available with churdle but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## **Remarks and examples**

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churdle fits a linear or an exponential hurdle model. It combines a selection model that determines the boundary points of the dependent variable with an outcome model that determines its nonbounded values. Hurdle models treat these boundary values as observed instead of censored. That is to say, observations where the dependent variable is equal to one of the boundary values are not the result of our inability to observe the distribution above or below a certain point; see Wooldridge (2010) chapter 17 for a thorough discussion of this point.

These models were proposed by Cragg (1971) to explain the demand for durable goods. In the Cragg model, individuals purchase zero or a positive amount of the durable good, with different factors determining each of these choices. This may be generalized to other individual decisions, such as money donated to charity, cigarette consumption, and time spent volunteering.

Hurdle models are characterized by the relationship  $y_i = s_i h_i^*$ , where  $y_i$  is the observed value of the dependent variable.

The selection variable,  $s_i$ , is 1 if the dependent variable is not bounded and 0 otherwise. In the Cragg model, the lower limit that binds the dependent variable is 0 so the selection model is

$$s_i = \begin{cases} 1 & \text{if } \mathbf{z}_i \boldsymbol{\gamma} + \epsilon_i > 0\\ 0 & \text{otherwise} \end{cases}$$

where  $z_i$  is a vector of explanatory variables,  $\gamma$  is a vector of coefficients, and  $\epsilon_i$  is a standard normal error term. churdle allows a different lower limit to be specified in 11() and, for the linear model, an upper limit in ul(). Conditional heteroskedasticity of the random error  $\epsilon_i$  is allowed if suboption het() is specified in select().

The continuous latent variable  $h_i^*$  is observed only if  $s_i = 1$ . The outcome model can be either the linear model or the exponential model, as proposed in Cragg (1971):

$$h_i^* = \mathbf{x}_i \boldsymbol{\beta} + \nu_i \qquad \text{(linear)}$$
  
$$h_i^* = \exp\left(\mathbf{x}_i \boldsymbol{\beta} + \nu_i\right) \qquad \text{(exponential)}$$

where  $\mathbf{x}_i$  is a vector of explanatory variables,  $\boldsymbol{\beta}$  is a vector of coefficients, and  $\nu_i$  is an error term.

For the linear model,  $\nu_i$  has a truncated normal distribution with lower truncation point  $-\mathbf{x}_i\beta$ . For the exponential model,  $\nu_i$  has a normal distribution. churdle extends the Cragg hurdle models to allow for conditional heteroskedasticity of the random error  $\nu_i$  if the user specifies the het() option.

The parameters and regressors in the models for  $h_i^*$  and for  $s_i$  may differ.

### Example 1: Linear hurdle model

Consider a dataset that contains the number of hours an individual exercises per day (hours), their age (age), whether they are single (single), hours they work per day (whours), whether they smoke (smoke), their weight in kilograms (weight), their distance from the nearest gym (distance), and their average commute from work (commute).

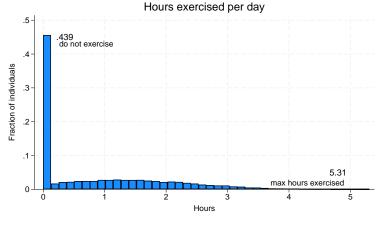




Figure 1 shows that 43.9% of the individuals in the sample do not exercise and that the hours exercised varies among individuals that decide to exercise.

We model the decision to exercise or not as a function of commute, whours, and age. These variables are written in select(). Once a decision to exercise is made, the time an individual exercises is modeled as a linear function of age, smoke, distance, and single.

. use https://www.stata-press.com/data/r18/fitness
(Fictional fitness data)
. churdle linear hours age i.smoke distance i.single,
> select(commute whours age) 11(0)
Iteration 0: Log likelihood = -23657.236
Iteration 1: Log likelihood = -23344.182
Iteration 2: Log likelihood = -23340.051
Iteration 3: Log likelihood = -23340.044
Iteration 4: Log likelihood = -23340.044
Cragg hurdle regression
Number of obs = 19,831
LR chi2(4) = 9059.26
Prob > chi2 = 0.0000
Dsg likelihood = -23340.044

hours	Coefficient	Std. err.	Z	P> z	[95% conf.	interval]
hours						
age	.0015116	.000763	1.98	0.048	.0000162	.003007
smoke						
Smoking	-1.06646	.0460578	-23.15	0.000	-1.156731	9761879
distance	1333868	.0126344	-10.56	0.000	1581497	1086238
single						
Single	.9940893	.0258775	38.42	0.000	.9433703	1.044808
_cons	.9138855	.0396227	23.06	0.000	.8362264	.9915447
selection_ll						
commute	2953345	.0624665	-4.73	0.000	4177666	1729024
whours	.0022974	.0069306	0.33	0.740	0112864	.0158811
age	0485347	.0006501	-74.65	0.000	049809	0472604
cons	2,649945	.0499795	53.02	0.000	2.551987	2.747903
	2.010010					
lnsigma						
cons	.0083199	.0099648	0.83	0.404	0112107	.0278506
			0.00	0.101	.0112101	
/sigma	1.008355	.010048			.9888519	1.028242
/ Signa	1.000000	.010040			.5000515	1.020242

The coefficients in the outcome model for the latent variable appear under hours. Because we only specified a lower limit to bind the dependent variable, the output shows parameter estimates for a single selection model under selection\_ll. Information about the estimated standard deviation of the error term in the outcome model appears under lnsigma and /sigma.

The coefficient estimates are not directly interpretable. To obtain the effect of a covariate on the model, we need to use the margins command; see [R] churdle postestimation. Consider the effect of age:

. margins, dyo	lx(age)				
Average margin Model VCE: OIM	5 5				Number of obs = 19,831
Expression: Co dy/dx wrt: ag		n estimates	of depen	ndent var	iable, predict()
	_	)elta-method	-		
	dy/dx	std. err.	Z	P> z	[95% conf. interval]
age	0216855	.000289	-75.03	0.000	022252021119

Each additional year of age is associated with about -0.02 fewer hours, or 1.2 minutes, of exercise.

### Example 2: Linear hurdle with models for the outcome and selection variances

In this example, we illustrate the possibility of fitting a heteroskedastic probit for the selection and latent model. In both cases, this is done by specifying age and single as the variables that affect the conditional variance. As in example 1, we have separate parameters for the outcome model and lower-limit selection model.

<pre>. churdle linear hours age i.smoke distance i.single, &gt; select(commute whours age, het(age single)) ll(0) het(age single) nolog</pre>						
Cragg hurdle regression					Number of obs LR chi2(4) Prob > chi2 Pseudo R2	= 19,831 = 9060.63 = 0.0000 = 0.1626
hours	Coefficient	Std. err.	Z	P> z	[95% conf.	interval]
hours						
age	.0012559	.0008198	1.53	0.126	0003508	.0028626
smoke						
Smoking	-1.065564	.0457657	-23.28	0.000	-1.155263	9758649
distance	1332939	.0126102	-10.57	0.000	1580094	1085783
single						
Single	1.002511	.032535	30.81	0.000	.9387436	1.066278
_cons	.9166356	.0388318	23.61	0.000	.8405268	.9927445
selection_11						
commute	2959986	.0641594	-4.61	0.000	4217488	1702484
whours	.0024514	.0069769	0.35	0.725	0112231	.0161259
age	048886	.0021405	-22.84	0.000	0530814	0446906
_cons	2.669613	.1139478	23.43	0.000	2.44628	2.892947
lnsigma						
age	.0003537	.0004026	0.88	0.380	0004354	.0011427
single	0080667	.019253	-0.42	0.675	0458019	.0296685
lnsigma_ll						
age	0002035	.0008424	-0.24	0.809	0018546	.0014475
single	.0268271	.0270133	0.99	0.321	0261179	.0797721

The coefficients on age and single have no effect on the conditional variance of the outcome model or on the conditional variance of the selection model. Thus, there is no evidence that the variance depends on age and marital status.

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= 0.2166

### Example 3: Exponential hurdle model

Returning to example 1, if we believe that the conditional mean of the latent variable has an exponential form instead of a linear form, we use churdle exponential.

```
. churdle exponential hours age i.smoke distance i.single,
> select(commute whours age) 11(0) nolog
Cragg hurdle regression
                                                    Number of obs = 19,831
                                                    LR chi2(4) = 8663.21
                                                    Prob > chi2 = 0.0000
Log likelihood = -15666.195
                                                    Pseudo R2
      Coofficient Std
                                              DNIAL
                                                        [QEV conf interval]
```

hours	Coefficient	Std. err.	Z	P> z	L95% conf.	interval
hours age	.0008368	.0005341	1.57	0.117	00021	.0018836
smoke Smoking distance	6431348 0772879	.0258509 .0079132	-24.88 -9.77	0.000	6938016 0927976	592468 0617783
single Single _cons	.5975111 0770619	.016108 .0254833	37.09 -3.02	0.000	.5659401 1270082	.6290821 0271157
selection_ll commute whours age _cons	2953345 .0022974 0485347 2.649945	.0624665 .0069306 .0006501 .0499795	-4.73 0.33 -74.65 53.02	0.000 0.740 0.000 0.000	4177666 0112864 049809 2.551987	1729024 .0158811 0472604 2.747903
lnsigma _cons	186917	.0067067	-27.87	0.000	200062	1737721
/sigma	.8295126	.0055633			.81868	.8404884

What was said previously regarding the interpretation of the effects of the different regressors also holds true for churdle exponential. We again use margins to estimate the effect of age on time spent exercising.

```
. margins, dydx(age)
Average marginal effects
                                                         Number of obs = 19,831
Model VCE: OIM
Expression: Conditional mean estimates of dependent variable, predict()
dy/dx wrt: age
```

	1	Delta-method	L			
	dy/dx	std. err.	z	P> z	[95% conf.	interval]
age	0245582	.0004805	-51.11	0.000	0255	0236164

With the exponential outcome model of the latent variable, our estimate is that each additional year of age decreases exercise time by about 0.025 hours, or 1.5 minutes.

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# **Stored results**

churdle stores the following in e():

Scalars	
e(N)	number of observations
e(k_eq_model)	number of equations in overall model test
e(df_m)	model degrees of freedom
e(r2_p)	pseudo- $R^2$
e(chi2)	$\chi^2$
e(11)	log likelihood
e(11_0)	log likelihood, constant-only model
e(N_clust)	number of clusters
e(p)	<i>p</i> -value for model test
e(rank)	rank of e(v)
e(converged)	1 if converged, 0 otherwise
e(converged)	1 il convergeu, o oliciwise
Macros	
e(cmd)	churdle
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(estimator)	linear or exponential
e(model)	Linear or Exponential
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(clustvar)	name of cluster variable
	Wald or LR; type of model $\chi^2$ test
e(chi2type)	ward of LR, type of model $\chi$ test vcetype specified in vce()
e(vce)	
e(vcetype)	title used to label Std. err.
e(opt)	type of optimization
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

In addition to the above, the following is stored in r():

Matrices

r(table) matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

### Methods and formulas

Let  $\ell\ell$  refer to the lower limit and  $u\ell$  to the upper limit. Also let the probabilities of being at these limits be given by

$$\begin{aligned} &\Pr\left(y_i = \ell \ell | \mathbf{z}_i\right) = \Phi\left(\ell \ell - \mathbf{z}_i' \boldsymbol{\gamma}_{\ell \ell}\right) \\ &\Pr\left(y_i = u \ell | \mathbf{z}_i\right) = \Phi\left(\mathbf{z}_i' \boldsymbol{\gamma}_{u \ell} - u \ell\right) \end{aligned}$$

where  $\mathbf{z}_i$  are the covariates of the selection model for individual *i*, which may be distinct from the covariates  $\mathbf{x}_i$  for the latent model;  $\boldsymbol{\Phi}$  corresponds to the standard normal cumulative distribution function;  $\gamma_{\ell\ell}$  is the parameter vector for the lower-limit selection model; and  $\gamma_{u\ell}$  is the parameter vector for the upper-limit selection model.

Under the assumptions that  $\nu_i$  has a truncated normal distribution with lower truncation point  $\ell \ell - \mathbf{x}'_i \boldsymbol{\beta}$  and upper truncation point  $u\ell - \mathbf{x}'_i \boldsymbol{\beta}$  and has a homoskedastic variance, the log-likelihood function is given by

$$\begin{split} \ln \mathbf{L} &= \sum_{i=1}^{n} \left( y_{i} \leq \ell \ell \right) \log \Phi \left( \ell \ell - \mathbf{z}_{i}^{\prime} \gamma_{\ell \ell} \right) + \left( y_{i} \geq u \ell \right) \log \left\{ 1 - \Phi \left( u \ell - \mathbf{z}_{i}^{\prime} \gamma_{u \ell} \right) \right\} \\ &+ \left( u \ell > y_{i} > \ell \ell \right) \left[ \log \left\{ \Phi \left( u \ell - \mathbf{z}_{i}^{\prime} \gamma_{u \ell} \right) - \Phi \left( \ell \ell - \mathbf{z}_{i}^{\prime} \gamma_{\ell \ell} \right) \right\} \right] \\ &- \left( u \ell > y_{i} > \ell \ell \right) \left[ \log \left\{ \Phi \left( \frac{u \ell - \mathbf{x}_{i}^{\prime} \beta}{\sigma} \right) - \Phi \left( \frac{\ell \ell - \mathbf{x}_{i}^{\prime} \beta}{\sigma} \right) \right\} \right] \\ &+ \left( u \ell > y_{i} > \ell \ell \right) \left[ \log \left\{ \phi \left( \frac{y_{i} - \mathbf{x}_{i}^{\prime} \beta}{\sigma} \right) \right\} - \log(\sigma) \right] \end{split}$$

Without the homoskedasticity assumption, the heteroskedasticity can be modeled using the form  $\sigma^2(\mathbf{w}_i) = \exp(2\mathbf{w}'_i\theta)$ , where  $\mathbf{w}_i$  are the variables that affect the conditional variance of  $\nu_i$ . The log-likelihood function is obtained by replacing  $\sigma$  with  $\exp(\mathbf{w}'_i\theta)$ .

The log-likelihood function for the exponential model is given by

$$\begin{aligned} \ln \mathbf{L} &= \sum_{i=1}^{n} \left( y_i \leq \ell \ell \right) \, \log \Phi \left( \ell \ell - \mathbf{z}'_i \boldsymbol{\gamma} \right) + \left( y_i > \ell \ell \right) \left[ \log \left\{ 1 - \Phi \left( \ell \ell - \mathbf{z}'_i \boldsymbol{\gamma} \right) \right\} \right] \\ &+ \left( y_i > \ell \ell \right) \left\{ \log \left\{ \phi \left[ \log(y_i - \ell \ell) - \mathbf{x}'_i \boldsymbol{\beta} \right) / \sigma \right] \right\} - \log(\sigma) - \log(y_i - \ell \ell) \right\} \end{aligned}$$

Analogous to the linear case, we can model heteroskedasticity by  $\sigma^2(\mathbf{w}_i) = \exp(2\mathbf{w}'_i\boldsymbol{\theta})$ .

Estimation of both of the aforementioned likelihood functions is done by maximum likelihood.

# References

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### Also see

- [R] churdle postestimation Postestimation tools for churdle
- [R] **intreg** Interval regression
- [R] **tobit** Tobit regression
- [SVY] svy estimation Estimation commands for survey data

[U] 20 Estimation and postestimation commands

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