centile - Report centile and confidence interval

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## Description

centile estimates specified centiles and calculates confidence intervals. If no varlist is specified, centile calculates centiles for all the variables in the dataset. If no centiles are specified, medians are reported.

By default, centile uses a binomial method for obtaining confidence intervals that makes no assumptions about the underlying distribution of the variable.

## Quick start

50th percentile with $95 \%$ confidence intervals for v1 and v2

```
    centile v1 v2
```

For all variables in the dataset
centile
25th, 50 th, and 75 th percentiles of $v 1$
centile v1, centile (25 50 75)
10th, 20th, 30th, ..., 90th percentiles of v1
centile v1, centile(10(10)90)
Force confidence limits to fall on sample values
centile v1 v2, cci
Confidence intervals based on standard errors for a normal-distribution quantile
centile v1 v2, normal
Centile and confidence intervals based on mean and standard deviation
centile v1 v2, meansd
Replace data in memory with centiles for groups defined by categorical variable cvar statsby, by (cvar) clear: centile v1, centile(25 50 75)

## Menu

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## Syntax

$$
\text { centile }[\text { varlist }][\text { if }][\text { in }][, \text { options }]
$$

options

Main

Options
normal
meansd
level(\#)
centile(numlist) report specified centiles; default is centile(50)
cci binomial exact; conservative confidence interval
Description normal, based on observed centiles
normal, based on mean and standard deviation
set confidence level; default is level (95)
by, collect, and statsby are allowed; see [U] 11.1.10 Prefix commands.

## Options

$\xrightarrow{\text { Main }}$
centile(numlist) specifies the centiles to be reported. The default is to display the 50th centile. Specifying centile(5) requests that the fifth centile be reported. Specifying centile (5 50 95 ) requests that the 5 th, 50 th, and 95 th centiles be reported. Specifying centile (10 (10) 90) requests that the 10 th, 20 th, $\ldots, 90$ th centiles be reported; see [U] 11.1.8 numlist.

Options
cci (conservative confidence interval) forces the confidence limits to fall exactly on sample values. Confidence intervals displayed with the cci option are slightly wider than those with the default (nocci) option.
normal causes the confidence interval to be calculated by using a formula for the standard error of a normal-distribution quantile given by Kendall and Stuart (1969, 237). The normal option is useful when you want empirical centiles - that is, centiles based on sample order statistics rather than on the mean and standard deviation - and are willing to assume normality.
meansd causes the centile and confidence interval to be calculated based on the sample mean and standard deviation, and it assumes normality.
level(\#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [R] level.

## Remarks and examples

stata.com
The $q$ th centile of a continuous random variable, $X$, is defined as the value of $C_{q}$, which fulfills the condition $\operatorname{Pr}\left(X \leq C_{q}\right)=q / 100$. The value of $q$ must be in the range $0<q<100$, though $q$ is not necessarily an integer. By default, centile estimates $C_{q}$ for the variables in varlist and for the values of $q$ given in centile (numlist). It makes no assumptions about the distribution of $X$ and, if necessary, uses linear interpolation between neighboring sample values. Extreme centiles (for example, the 99th centile in samples smaller than 100) are fixed at the minimum or maximum sample value. An "exact" confidence interval for $C_{q}$ is also given, using the binomial-based method described below in Methods and formulas and in Conover (1999, 143-148). Again linear interpolation is used to improve the accuracy of the estimated confidence limits, but extremes are fixed at the minimum or maximum sample value.

You can prevent centile from interpolating when calculating binomial-based confidence intervals by specifying cci. The resulting intervals are generally wider than with the default; that is, the coverage (confidence level) tends to be greater than the nominal value (given as usual by level(\#), by default $95 \%$ ).

If the data are believed to be normally distributed (a common case), there are two alternative methods for estimating centiles. If normal is specified, $C_{q}$ is calculated, as just described, but its confidence interval is based on a formula for the standard error (se) of a normal-distribution quantile given by Kendall and Stuart (1969, 237). If meansd is alternatively specified, $C_{q}$ is estimated as $\bar{x}+z_{q} \times s$, where $\bar{x}$ and $s$ are the sample mean and standard deviation, respectively, and $z_{q}$ is the $q$ th centile of the standard normal distribution (for example, $z_{95}=1.645$ ). The confidence interval is derived from the se of the estimate of $C_{q}$.

## Example 1

Using auto.dta, we estimate the 5th, 50th, and 95th centiles of the price variable:

```
. use https://www.stata-press.com/data/r18/auto
(1978 automobile data)
. format price %8.2fc
. centile price, centile(5 50 95)
```

| Variable | Obs | Percentile | Centile | Binom. interp. |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
|  | [95\% conf. interval] |  |  |  |  |
| price | 74 | 5 | $3,727.75$ | $3,291.23$ | $3,914.16$ |
|  |  | 50 | $5,006.50$ | $4,593.57$ | $5,717.90$ |
|  |  | 95 | $13,498.00$ | $11,061.53$ | $15,865.30$ |

summarize produces somewhat different results from centile; see Methods and formulas.
. summarize price, detail
Price

|  | Percentiles | Smallest |  |  |
| ---: | ---: | ---: | :--- | ---: |
| $1 \%$ | 3291 | 3291 |  | 74 |
| $5 \%$ | 3748 | 3299 |  | 74 |
| $10 \%$ | 3895 | 3667 | Obs |  |
| $25 \%$ | 4195 | 3748 | Sum of wgt. |  |
| $50 \%$ | 5006.5 |  | Mean | 6165.257 |
|  |  | Largest | Std. dev. | 2949.496 |
| $75 \%$ | 6342 | 13466 |  | 8699526 |
| $90 \%$ | 11385 | 13594 | Variance | 1.653434 |
| $95 \%$ | 13466 | 14500 | Skewness | 1.3 |
| $99 \%$ | 15906 | 15906 | Kurtosis | 4.819188 |

The confidence limits produced by using the cci option are slightly wider than those produced without this option:

```
. centile price, c(5 50 95) cci
```

| Variable | Obs | Percentile | Centile | Binomial exact <br> [95\% conf. interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| price | 74 | 5 | $3,727.75$ | $3,291.00$ | $3,955.00$ |
|  |  | 50 | $5,006.50$ | $4,589.00$ | $5,719.00$ |
|  |  | 95 | $13,498.00$ | $10,372.00$ | $15,906.00$ |

If we are willing to assume that price is normally distributed, we could include either the normal or the meansd option:

```
. centile price, c(5 50 95) normal
```

| Variable | - Normal, based on observed centiles - |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
|  | Obs | Percentile | Centile | [95\% conf. interval] |  |
| price | 74 | 5 | $3,727.75$ | $3,211.19$ | $4,244.31$ |
|  |  | 50 | $5,006.50$ | $4,096.68$ | $5,916.32$ |
|  |  | 95 | $13,498.00$ | $5,426.81$ | $21,569.19$ |

. centile price, c(5 50 95) meansd

| Variable | Obs | Percentile | Normal, based on mean and std. dev. Centile [95\% conf. interval] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| price | 74 | 5 | 1,313.77 | 278.93 | 2,348.61 |
|  |  | 50 | 6,165.26 | 5,493.24 | 6,837.27 |
|  |  | 95 | 11,016.75 | 9,981.90 | 12,051.59 |

With the normal option, the centile estimates are, by definition, the same as before. The confidence intervals for the 5th and 50th centiles are similar to the previous ones, but the interval for the 95th centile is different. The results using the meansd option also differ from both previous sets of estimates.

We can use sktest (see [R] sktest) to check the correctness of the normality assumption:
. sktest price
Skewness and kurtosis tests for normality

| Variable | Obs | $\operatorname{Pr}$ (skewness) | $\operatorname{Pr}($ kurtosis) | Adj chi2(2) | Prob>chi2 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| price | 74 | 0.0000 | 0.0127 | 21.77 | 0.0000 |

sktest reveals that price is definitely not normally distributed, so the normal assumption is not reasonable, and the normal and meansd options are not appropriate for these data. We should rely on the results from the default choice, which does not assume normality. If the data are normally distributed, however, the precision of the estimated centiles and their confidence intervals will be ordered (best) meansd $>$ normal $>$ [default] (worst). The normal option is useful when we really do want empirical centiles (that is, centiles based on sample order statistics rather than on the mean and standard deviation) but are willing to assume normality.

## Stored results

centile stores the following in $r()$ :
Scalars
$r(N) \quad$ number of observations
$r$ ( $n$ _cent) number of centiles requested
$r$ (c_\#) value of \# centile
r(lb_\#) \#-requested centile lower confidence bound
r(ub_\#) \#-requested centile upper confidence bound
Macros
$r$ (centiles) centiles requested

## Methods and formulas

Methods and formulas are presented under the following headings:
Default case
Normal case
meansd case

## Default case

The calculation is based on the method of Mood and Graybill $(1963,408)$. Let $x_{1} \leq x_{2} \leq \cdots \leq x_{n}$ be a sample of size $n$ arranged in ascending order. Denote the estimated $q$ th centile of the $x$ 's as $c_{q}$. We require that $0<q<100$. Let $R=(n+1) q / 100$ have integer part $r$ and fractional part $f$; that is, $r=\operatorname{int}(R)$ and $f=R-r$. (If $R$ is itself an integer, then $r=R$ and $f=0$.) Note that $0 \leq r \leq n$. For convenience, define $x_{0}=x_{1}$ and $x_{n+1}=x_{n} . C_{q}$ is estimated by

$$
c_{q}=x_{r}+f \times\left(x_{r+1}-x_{r}\right)
$$

that is, $c_{q}$ is a weighted average of $x_{r}$ and $x_{r+1}$. Loosely speaking, a (conservative) $p \%$ confidence interval for $C_{q}$ involves finding the observations ranked $t$ and $u$, which correspond, respectively, to the $\alpha=(100-p) / 200$ and $1-\alpha$ quantiles of a binomial distribution with parameters $n$ and $q / 100$, that is, $\mathrm{B}(n, q / 100)$. More precisely, define the $i$ th value $(i=0, \ldots, n$ ) of the cumulative binomial distribution function as $F_{i}=\operatorname{Pr}(S \leq i)$, where $S$ has distribution $B(n, q / 100)$. For convenience, let $F_{-1}=0$ and $F_{n+1}=1 . t$ is found such that $F_{t} \leq \alpha$ and $F_{t+1}>\alpha$, and $u$ is found such that $1-F_{u} \leq \alpha$ and $1-F_{u-1}>\alpha$.

With the cci option in force, the (conservative) confidence interval is $\left(x_{t+1}, x_{u+1}\right)$, and its actual coverage probability is $F_{u}-F_{t}$.

The default case uses linear interpolation on the $F_{i}$ as follows. Let

$$
\begin{aligned}
g & =\left(\alpha-F_{t}\right) /\left(F_{t+1}-F_{t}\right) \\
h & =\left\{\alpha-\left(1-F_{u}\right)\right\} /\left\{\left(1-F_{u-1}\right)-\left(1-F_{u}\right)\right\} \\
& =\left(\alpha-1+F_{u}\right) /\left(F_{u}-F_{u-1}\right)
\end{aligned}
$$

The interpolated lower and upper confidence limits $\left(c_{q L}, c_{q U}\right)$ for $C_{q}$ are

$$
\begin{aligned}
c_{q L} & =x_{t+1}+g \times\left(x_{t+2}-x_{t+1}\right) \\
c_{q U} & =x_{u+1}-h \times\left(x_{u+1}-x_{u}\right)
\end{aligned}
$$

Suppose that we want a $95 \%$ confidence interval for the median of a sample of size $13 . n=13$, $q=50, p=95, \alpha=0.025, R=14 \times 50 / 100=7$, and $f=0$. Therefore, the median is the 7th observation. Some example data, $x_{i}$, and the values of $F_{i}$ are as follows:

| $i$ | $F_{i}$ | $1-F_{i}$ | $x_{i}$ | $i$ | $F_{i}$ | $1-F_{i}$ | $x_{i}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0.0001 | 0.9999 | - | 7 | 0.7095 | 0.2905 | 33 |
| 1 | 0.0017 | 0.9983 | 5 | 8 | 0.8666 | 0.1334 | 37 |
| 2 | 0.0112 | 0.9888 | 7 | 9 | 0.9539 | 0.0461 | 45 |
| 3 | 0.0461 | 0.9539 | 10 | 10 | 0.9888 | 0.0112 | 59 |
| 4 | 0.1334 | 0.8666 | 15 | 11 | 0.9983 | 0.0017 | 77 |
| 5 | 0.2905 | 0.7095 | 23 | 12 | 0.9999 | 0.0001 | 104 |
| 6 | 0.5000 | 0.5000 | 28 | 13 | 1.0000 | 0.0000 | 211 |

The median is $x_{7}=33$. Also, $F_{2} \leq 0.025$ and $F_{3}>0.025$, so $t=2 ; 1-F_{10} \leq 0.025$ and $1-F_{9}>0.025$, so $u=10$. The conservative confidence interval is therefore

$$
\left(c_{50 L}, c_{50 U}\right)=\left(x_{t+1}, x_{u+1}\right)=\left(x_{3}, x_{11}\right)=(10,77)
$$

with actual coverage $F_{10}-F_{2}=0.9888-0.0112=0.9776$ ( $97.8 \%$ confidence). For the interpolation calculation, we have

$$
\begin{aligned}
& g=(0.025-0.0112) /(0.0461-0.0112)=0.395 \\
& h=(0.025-1+0.9888) /(0.9888-0.9539)=0.395
\end{aligned}
$$

So,

$$
\begin{array}{r}
c_{50 L}=x_{3}+0.395 \times\left(x_{4}-x_{3}\right)=10+0.395 \times 5=11.98 \\
c_{50 U}=x_{11}-0.395 \times\left(x_{11}-x_{10}\right)=77-0.395 \times 18=69.89
\end{array}
$$

## Normal case

The value of $c_{q}$ is as above. Its se is given by the formula

$$
s_{q}=\sqrt{q(100-q)} /\left\{100 \sqrt{n} Z\left(c_{q} ; \bar{x}, s\right)\right\}
$$

where $\bar{x}$ and $s$ are the mean and standard deviation of the $x_{i}$, and

$$
Z(Y ; \mu, \sigma)=\left(1 / \sqrt{2 \pi \sigma^{2}}\right) e^{-(Y-\mu)^{2} / 2 \sigma^{2}}
$$

is the density function of a normally distributed variable $Y$ with mean $\mu$ and standard deviation $\sigma$. The confidence interval for $C_{q}$ is $\left(c_{q}-z_{100(1-\alpha)} s_{q}, c_{q}+z_{100(1-\alpha)} s_{q}\right)$.

## meansd case

The value of $c_{q}$ is $\bar{x}+z_{q} \times s$. Its se is given by the formula

$$
s_{q}^{\star}=s \sqrt{1 / n+z_{q}^{2} /(2 n-2)}
$$

The confidence interval for $C_{q}$ is $\left(c_{q}-z_{100(1-\alpha)} \times s_{q}^{\star}, c_{q}+z_{100(1-\alpha)} \times s_{q}^{\star}\right)$.

## Acknowledgment

centile was written by Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model.

## References

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Kendall, M. G., and A. Stuart. 1969. The Advanced Theory of Statistics, Vol. 1: Distribution Theory. 3rd ed. London: Griffin.

Mood, A. M., and F. A. Graybill. 1963. Introduction to the Theory of Statistics. 2nd ed. New York: McGraw-Hill.
Stuart, A., and J. K. Ord. 1994. Kendall's Advanced Theory of Statistics: Distribution Theory, Vol. 1. 6th ed. London: Arnold.

## Also see

$[\mathrm{R}]$ ci - Confidence intervals for means, proportions, and variances
[R] summarize - Summary statistics
[D] petile - Create variable containing percentiles

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