biprobit - Bivariate probit regression

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## Description

biprobit fits maximum-likelihood two-equation probit models-either a bivariate probit or a seemingly unrelated probit (limited to two equations).

## Quick start

Bivariate probit regression of y 1 and y 2 on x 1
biprobit y1 y2 x1
Bivariate probit regression of y 1 and y 2 on x 1 , x 2 , and x 3
biprobit y1 y2 x1 x2 x3
Constrain the coefficients for x 1 to equality in both equations

```
constraint define 1 _b[y1:x1] = _b[y2:x1]
biprobit y1 y2 x1 x2 x3, constraints(1)
```

Seemingly unrelated bivariate probit regression
biprobit ( $\mathrm{y} 1=\mathrm{x} 1 \mathrm{x} 2 \mathrm{x} 3)(\mathrm{y} 2=\mathrm{x} 1 \mathrm{x} 2)$
With robust standard errors
biprobit ( $\mathrm{y} 1=\mathrm{x} 1 \mathrm{x} 2 \mathrm{x} 3$ ) ( $\mathrm{y} 2 \mathrm{=}$ x1 x 2 ), vce(robust)
Poirier partial observability model with difficult option
biprobit ( $\mathrm{y} 1=\mathrm{x} 1 \mathrm{x} 2$ ) ( $\mathrm{y} 2=\mathrm{x} 2 \mathrm{x} 3)$, partial difficult

## Menu

## biprobit

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## Seemingly unrelated biprobit

Statistics $>$ Binary outcomes $>$ Seemingly unrelated bivariate probit regression

## Syntax

Bivariate probit regression
biprobit depvar ${ }_{1}$ depvar 2 [indepvars $][$ if $][$ in $][$ weight $][$, options $]$

Seemingly unrelated bivariate probit regression
${\text { biprobit } \text { equation }_{1} \text { equation }_{2}[\text { if }][\text { in }][\text { weight }][\text {, su_options }] ~}_{\text {[ }}$
where equation $_{1}$ and equation ${ }_{2}$ are specified as
([eqname:] depvar $[=]$ [indepvars] [, noconstant offset(varname)])
options
Model
noconstant
partial
offset1 (varname)
offset2(varname)
constraints (constraints)
SE/Robust
vce (vcetype)

Reporting
level(\#)
lrmodel
nocnsreport
display_options

Description
suppress constant term
fit partial observability model
offset variable for first equation
offset variable for second equation
apply specified linear constraints
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
perform the likelihood-ratio model test instead of the default Wald test do not display constraints
control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Maximization
maximize_options
collinear
coeflegend
control the maximization process; seldom used
keep collinear variables
display legend instead of statistics

## su_options

Description
Model
partial
constraints (constraints)
SE/Robust
vce (vcetype)

Reporting
level(\#)
lrmodel
nocnsreport
display_options

Maximization
maximize_options
collinear
coeflegend
fit partial observability model
apply specified linear constraints
vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife
set confidence level; default is level (95)
perform the likelihood-ratio model test instead of the default Wald test do not display constraints
control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
control the maximization process; seldom used
keep collinear variables
display legend instead of statistics
indepvars may contain factor variables; see [U] 11.4.3 Factor variables.
depvar ${ }_{1}$, depvar ${ }_{2}$, indepvars, and depvar may contain time-series operators; see [U] 11.4.4 Time-series varlists.
bayes, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: biprobit.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
vce(), lrmodel, and weights are not allowed with the svy prefix; see [SVY] svy.
pweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.
collinear and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

$\qquad$ Model
noconstant; see [R] Estimation options.
partial specifies that the partial observability model be fit. This particular model commonly has poor convergence properties, so we recommend that you use the difficult option if you want to fit the Poirier partial observability model; see [R] Maximize.

This model computes the product of the two dependent variables so that you do not have to replace each with the product.
offset1(varname), offset2(varname), constraints (constraints); see [R] Estimation options.

SE/Robust
vce (vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.
$\qquad$ Reporting
level(\#), lrmodel, nocnsreport; see [R] Estimation options.
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap (\#), fvwrapon (style), cformat ( $\%$ fmt), pformat ( $\% f m t$ ), sformat ( $\% f m t$ ), and nolstretch; see [R] Estimation options.

Maximization
maximize_options: difficult, technique (algorithm_spec), iterate(\#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(\#), ltolerance (\#), nrtolerance (\#), nonrtolerance, and from(init_specs); see [R] Maximize. These options are seldom used.

Setting the optimization type to technique (bhhh) resets the default vcetype to vce (opg).

The following options are available with biprobit but are not shown in the dialog box:
collinear, coeflegend; see [R] Estimation options.

## Remarks and examples

stata.com
For a good introduction to the bivariate probit models, see Greene (2018, sec. 17.9) and Pindyck and Rubinfeld (1998). Poirier (1980) explains the partial observability model. Van de Ven and Van Pragg (1981) explain the probit model with sample selection; see $[R]$ heckprobit for details.
$>$ Example 1
We use the data from Pindyck and Rubinfeld (1998, 332). In this dataset, the variables are whether children attend private school (private), number of years the family has been at the present residence (years), log of property tax (logptax), log of income (loginc), and whether the head of the household voted for an increase in property taxes (vote).

We wish to model the bivariate outcomes of whether children attend private school and whether the head of the household voted for an increase in property tax based on the other covariates.
. use https://www.stata-press.com/data/r18/school
. biprobit private vote years logptax loginc
Fitting comparison equation 1 :

```
Iteration 0: Log likelihood = -31.967097
Iteration 1: Log likelihood = -31.452424
Iteration 2: Log likelihood = -31.448958
Iteration 3: Log likelihood = -31.448958
Fitting comparison equation 2:
Iteration 0: Log likelihood = -63.036914
Iteration 1: Log likelihood = -58.534843
Iteration 2: Log likelihood = -58.497292
Iteration 3: Log likelihood = -58.497288
Comparison: Log likelihood = -89.946246
Fitting full model:
Iteration 0: Log likelihood = -89.946246
Iteration 1: Log likelihood = -89.258897
Iteration 2: Log likelihood = -89.254028
Iteration 3: Log likelihood = -89.254028
Bivariate probit regression Number of obs = 95
Wald chi2(6) = 9.59
Prob > chi2 = 0.1431
```

|  | Coefficient | Std. err. | z | $P>\|z\|$ | [95\% conf | interval] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| private |  |  |  |  |  |  |
| years | -. 0118884 | . 0256778 | -0.46 | 0.643 | -. 0622159 | . 0384391 |
| logptax | -. 1066962 | . 6669782 | -0.16 | 0.873 | -1.413949 | 1.200557 |
| loginc | . 3762037 | . 5306484 | 0.71 | 0.478 | -. 663848 | 1.416255 |
| _cons | -4.184694 | 4.837817 | -0.86 | 0.387 | -13.66664 | 5.297253 |
| vote |  |  |  |  |  |  |
| years | -. 0168561 | . 0147834 | -1.14 | 0.254 | -. 0458309 | . 0121188 |
| logptax | -1.288707 | . 5752266 | -2.24 | 0.025 | -2.416131 | -. 1612839 |
| loginc | . 998286 | . 4403565 | 2.27 | 0.023 | . 1352031 | 1.861369 |
| _cons | -. 5360573 | 4.068509 | -0.13 | 0.895 | -8.510188 | 7.438073 |
| /athrho | -. 2764525 | . 2412099 | -1.15 | 0.252 | -. 7492153 | . 1963102 |
| rho | -. 2696186 | . 2236753 |  |  | -. 6346806 | . 1938267 |

The output shows several iteration logs. The first iteration $\log$ corresponds to running the univariate probit model for the first equation, and the second log corresponds to running the univariate probit for the second model. If $\rho=0$, the sum of the log likelihoods from these two models will equal the log likelihood of the bivariate probit model; this sum is printed in the iteration $\log$ as the comparison log likelihood.

The final iteration $\log$ is for fitting the full bivariate probit model. A likelihood-ratio test of the $\log$ likelihood for this model and the comparison $\log$ likelihood is presented at the end of the output. If we had specified the vce (robust) option, this test would be presented as a Wald test instead of as a likelihood-ratio test.

We could have fit the same model by using the seemingly unrelated syntax as

[^0]
## Stored results

biprobit stores the following in e() :
Scalars

```
e(N)
e(k)
e(k_eq)
e(k_aux)
e(k_eq_model)
e(k_dv)
e(df_m)
e(ll)
e(ll_0)
e(ll_c)
e(N_clust)
e(chi2)
e(chi2_c)
e(p)
e(rho)
e(rank)
e(rank0)
e(ic)
e(rc)
e(converged)
```

Macros
e(cmd)
e(cmdline)
e(depvar)
e(wtype)
e(wexp)
e(title)
e(clustvar)
e(offset1)
e(offset2)
e(chi2type)
e(chi2_ct)
e(vce)
e(vcetype)
e(opt)
e(which)
e(ml_method)
e(user)
e(technique)
e(properties)
d(predict)
e(marginsok)
e(marginsnotok)
e(asbalanced)
e(asobserved)
Matrices
e(b)
e(Cns)
e(ilog)
e(gradient)
e(V)
e(V_modelbased)

Functions
e(sample)
number of observations
number of parameters
number of equations in $e(b)$
number of auxiliary parameters
number of equations in overall model test
number of dependent variables
model degrees of freedom
log likelihood
log likelihood, constant-only model (lrmodel only)
log likelihood, comparison model
number of clusters
$\chi^{2}$
$\chi^{2}$ for comparison test
$p$-value for model test
$\rho$
rank of e(v)
rank of $e(V)$ for constant-only model
number of iterations
return code
1 if converged, 0 otherwise
biprobit
command as typed
names of dependent variables
weight type
weight expression
title in estimation output
name of cluster variable
offset for first equation
offset for second equation
Wald or LR; type of model $\chi^{2}$ test
Wald or LR; type of model $\chi^{2}$ test corresponding to e(chi2_c)
vcetype specified in vce()
title used to label Std. err.
type of optimization
max or min; whether optimizer is to perform maximization or minimization
type of ml method
name of likelihood-evaluator program
maximization technique
b V
program used to implement predict
predictions allowed by margins
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
constraints matrix
iteration $\log$ (up to 20 iterations)
gradient vector
variance-covariance matrix of the estimators
model-based variance
marks estimation sample

In addition to the above, the following is stored in r() :
Matrices
$r$ (table) matrix containing the coefficients with their standard errors, test statistics, $p$-values, and confidence intervals

Note that results stored in $r()$ are updated when the command is replayed and will be replaced when any r -class command is run after the estimation command.

## Methods and formulas

The $\log$ likelihood, $\ln L$, is given by

$$
\begin{aligned}
\xi_{j}^{\beta} & =x_{j} \beta+\text { offset }_{j}^{\beta} \\
\xi_{j}^{\gamma} & =z_{j} \gamma+\text { offset }_{j}^{\gamma} \\
q_{1 j} & = \begin{cases}1 & \text { if } y_{1 j} \neq 0 \\
-1 & \text { otherwise }\end{cases} \\
q_{2 j} & = \begin{cases}1 & \text { if } y_{2 j} \neq 0 \\
-1 & \text { otherwise }\end{cases} \\
\rho_{j}^{*} & =q_{1 j} q_{2 j} \rho \\
\ln L & =\sum_{j=1}^{n} w_{j} \ln \Phi_{2}\left(q_{1 j} \xi_{j}^{\beta}, q_{2 j} \xi_{j}^{\gamma}, \rho_{j}^{*}\right)
\end{aligned}
$$

where $\Phi_{2}()$ is the cumulative bivariate normal distribution function (with mean $\left[\begin{array}{ll}0 & 0\end{array}\right]^{\prime}$ ) and $w_{j}$ is an optional weight for observation $j$. This derivation assumes that

$$
\begin{aligned}
y_{1 j}^{*} & =x_{j} \beta+\epsilon_{1 j}+\operatorname{offset}_{j}^{\beta} \\
y_{2 j}^{*} & =z_{j} \gamma+\epsilon_{2 j}+\operatorname{offset}_{j}^{\gamma} \\
E\left(\epsilon_{1}\right) & =E\left(\epsilon_{2}\right)=0 \\
\operatorname{Var}\left(\epsilon_{1}\right) & =\operatorname{Var}\left(\epsilon_{2}\right)=1 \\
\operatorname{Cov}\left(\epsilon_{1}, \epsilon_{2}\right) & =\rho
\end{aligned}
$$

where $y_{1 j}^{*}$ and $y_{2 j}^{*}$ are the unobserved latent variables; instead, we observe only $y_{i j}=1$ if $y_{i j}^{*}>0$ and $y_{i j}=0$ otherwise (for $i=1,2$ ).

In the maximum likelihood estimation, $\rho$ is not directly estimated, but atanh $\rho$ is

$$
\operatorname{atanh} \rho=\frac{1}{2} \ln \left(\frac{1+\rho}{1-\rho}\right)
$$

From the form of the likelihood, if $\rho=0$, then the log likelihood for the bivariate probit models is equal to the sum of the $\log$ likelihoods of the two univariate probit models. A likelihood-ratio test may therefore be performed by comparing the likelihood of the full bivariate model with the sum of the log likelihoods for the univariate probit models.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. See [P] _robust, particularly Maximum likelihood estimators and Methods and formulas.
biprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] Variance estimation.

## References

De Luca, G. 2008. SNP and SML estimation of univariate and bivariate binary-choice models. Stata Journal 8: 190-220.

Greene, W. H. 2018. Econometric Analysis. 8th ed. New York: Pearson.
Heckman, J. J. 1979. Sample selection bias as a specification error. Econometrica 47: 153-161. https://doi.org/10.2307/1912352.

Hernández-Alava, M., and S. Pudney. 2016. bicop: A command for fitting bivariate ordinal regressions with residual dependence characterized by a copula function and normal mixture marginals. Stata Journal 16: 159-184.
Lokshin, M., and Z. Sajaia. 2011. Impact of interventions on discrete outcomes: Maximum likelihood estimation of the binary choice models with binary endogenous regressors. Stata Journal 11: 368-385.

Mullahy, J. 2016. Estimation of multivariate probit models via bivariate probit. Stata Journal 16: 37-51.
Pindyck, R. S., and D. L. Rubinfeld. 1998. Econometric Models and Economic Forecasts. 4th ed. New York: McGraw-Hill.

Poirier, D. J. 1980. Partial observability in bivariate probit models. Journal of Econometrics 12: 209-217. https://doi.org/10.1016/0304-4076(80)90007-X.

Van de Ven, W. P. M. M., and B. M. S. Van Pragg. 1981. The demand for deductibles in private health insurance: A probit model with sample selection. Journal of Econometrics 17: 229-252. https://doi.org/10.1016/0304-4076(81)90028-2.

## Also see

[R] biprobit postestimation - Postestimation tools for biprobit
[R] mprobit - Multinomial probit regression
[R] probit - Probit regression
[BAYES] bayes: biprobit - Bayesian bivariate probit regression
[SVY] svy estimation - Estimation commands for survey data
[U] 20 Estimation and postestimation commands

[^1]


[^0]:    . biprobit (private=years logptax loginc) (vote=years logptax loginc)

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