Title

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meprobit — Multilevel mixed-effects probit regression

Description

meprobit fits mixed-effects models for binary or binomial responses. The conditional distribution of the response given the random effects is assumed to be Bernoulli, with success probability determined by the standard normal cumulative distribution function.

Quick start

Two-level probit model of y and covariate x and random intercepts by lev2

    meprobit y x || lev2:

Add random coefficients for x

    meprobit y x || lev2: x

Same as above, but specify that y records the number of successes from 10 trials

    meprobit y x || lev2: x, binomial(10)

Same as above, but with the number of trials stored in variable n

    meprobit y x || lev2: x, binomial(n)

Three-level random-intercept model of y and covariate x with lev2 nested within lev3

    meprobit y x || lev3: || lev2:

Two-way crossed random effects by factors a and b

    meprobit y x || _all:R.a || b:

Menu

Statistics > Multilevel mixed-effects models > Probit regression

Syntax

> meprobit *[depvar](#) fe_equation* $\left[\;||\; \textit{re\_equation}\;\right]$ $\left[\;||\; \textit{re\_equation}\;...\;\right]$ $\left[\;,\; \textit{[options](#)}\;\right]$

where the syntax of *fe_equation* is

> $\left[\textit{[indepvars](#)}\right]$ $\left[\textit{[if](#)}\right]$ $\left[\textit{[in](#)}\right]$ $\left[\textit{[weight](#)}\right]$ $\left[\;,\; \textit{fe\_options}\;\right]$

and the syntax of *re_equation* is one of the following:

for random coefficients and intercepts

> *levelvar*: $\left[\textit{[varlist](#)}\right]$ $\left[\;,\; \textit{re\_options}\;\right]$

for random effects among the values of a factor variable in a crossed-effects model

> *levelvar*: R.*[varname](#)*

levelvar is a variable identifying the group structure for the random effects at that level or is _all representing one group comprising all observations.

fe_options	Description
Model	
<u>nocon</u>stant	suppress constant term from the fixed-effects equation
<u>off</u>set(*[varname](#)*)	include *varname* in model with coefficient constrained to 1
asis	retain perfect predictor variables

re_options	Description
Model	
<u>cov</u>ariance(*[vartype](#)*)	variance–covariance structure of the random effects
<u>nocon</u>stant	suppress constant term from the random-effects equation
<u>fw</u>eight(*[varname](#)*)	frequency weights at higher levels
<u>iw</u>eight(*[varname](#)*)	importance weights at higher levels
<u>pw</u>eight(*[varname](#)*)	sampling weights at higher levels

options	Description
Model	
<u>bin</u>omial(*varname* │ #)	set binomial trials if data are in binomial form
constraints(*constraints*)	apply specified linear constraints
SE/Robust	
vce(*vcetype*)	*vcetype* may be oim, opg, <u>r</u>obust, or <u>cl</u>uster *clustvar*
Reporting	
<u>l</u>evel(#)	set confidence level; default is level(95)
nocnsreport	do not display constraints
notable	suppress coefficient table
noheader	suppress output header
nogroup	suppress table summarizing groups
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Integration	
<u>intm</u>ethod(*intmethod*)	integration method
intpoints(#)	set the number of integration (quadrature) points for all levels; default is intpoints(7)
Maximization	
maximize_options	control the maximization process; seldom used
startvalues(*svmethod*)	method for obtaining starting values
startgrid⌈(*gridspec*)⌉	perform a grid search to improve starting values
noestimate	do not fit the model; show starting values instead
dnumerical	use numerical derivative techniques
collinear	keep collinear variables
coeflegend	display legend instead of statistics

vartype	Description
<u>ind</u>ependent	one unique variance parameter per random effect and all covariances 0; the default unless the R. notation is used
<u>ex</u>changeable	equal variances for random effects and one common pairwise covariance
<u>id</u>entity	equal variances for random effects and all covariances 0; the default if the R. notation is used
<u>un</u>structured	all variances and covariances to be distinctly estimated
<u>f</u>ixed(*matname*)	user-selected variances and covariances constrained to specified values; the remaining variances and covariances unrestricted
<u>pa</u>ttern(*matname*)	user-selected variances and covariances constrained to be equal; the remaining variances and covariances unrestricted

intmethod	Description
<u>mvaghermite</u>	mean–variance adaptive Gauss–Hermite quadrature; the default unless a crossed random-effects model is fit
<u>mcaghermite</u>	mode-curvature adaptive Gauss–Hermite quadrature
<u>ghermite</u>	nonadaptive Gauss–Hermite quadrature
<u>laplace</u>	Laplacian approximation; the default for crossed random-effects models

indepvars and *varlist* may contain factor variables; see [U] **11.4.3 Factor variables**.

depvar, *indepvars*, and *varlist* may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

bayes, by, collect, and svy are allowed; see [U] **11.1.10 Prefix commands**. For more details, see [BAYES] **bayes: meprobit**.

vce() and weights are not allowed with the svy prefix; see [SVY] **svy**.

fweights, iweights, and pweights are allowed; see [U] **11.1.6 weight**. Only one type of weight may be specified. Weights are not supported under the Laplacian approximation or for crossed models.

startvalues(), startgrid, noestimate, dnumerical, collinear, and coeflegend do not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

> [Model]

noconstant suppresses the constant (intercept) term and may be specified for the fixed-effects equation and for any of or all the random-effects equations.

offset(*varname*) specifies that *varname* be included in the fixed-effects portion of the model with the coefficient constrained to be 1.

asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**.

covariance(*vartype*) specifies the structure of the covariance matrix for the random effects and may be specified for each random-effects equation. *vartype* is one of the following: independent, exchangeable, identity, unstructured, fixed(*matname*), or pattern(*matname*).

covariance(independent) covariance structure allows for a distinct variance for each random effect within a random-effects equation and assumes that all covariances are 0. The default is covariance(independent) unless a crossed random-effects model is fit, in which case the default is covariance(identity).

covariance(exchangeable) structure specifies one common variance for all random effects and one common pairwise covariance.

covariance(identity) is short for "multiple of the identity"; that is, all variances are equal and all covariances are 0.

covariance(unstructured) allows for all variances and covariances to be distinct. If an equation consists of p random-effects terms, the unstructured covariance matrix will have $p(p+1)/2$ unique parameters.

covariance(fixed(*matname*)) and covariance(pattern(*matname*)) covariance structures provide a convenient way to impose constraints on variances and covariances of random effects. Each specification requires a *matname* that defines the restrictions placed on variances and covariances. Only elements in the lower triangle of *matname* are used, and row and column names

of *matname* are ignored. A missing value in *matname* means that a given element is unrestricted. In a fixed(*matname*) covariance structure, (co)variance (i, j) is constrained to equal the value specified in the i, jth entry of *matname*. In a pattern(*matname*) covariance structure, (co)variances (i, j) and (k, l) are constrained to be equal if $matname[i, j] = matname[k, l]$.

fweight(*varname*) specifies frequency weights at higher levels in a multilevel model, whereas frequency weights at the first level (the observation level) are specified in the usual manner, for example, [fw=*fwtvar1*]. *varname* can be any valid Stata variable name, and you can specify fweight() at levels two and higher of a multilevel model. For example, in the two-level model

. *mecmd fixed_portion* [fw = wt1] || school: ... , fweight(wt2) ...

the variable wt1 would hold the first-level (the observation-level) frequency weights, and wt2 would hold the second-level (the school-level) frequency weights.

iweight(*varname*) specifies importance weights at higher levels in a multilevel model, whereas importance weights at the first level (the observation level) are specified in the usual manner, for example, [iw=*iwtvar1*]. *varname* can be any valid Stata variable name, and you can specify iweight() at levels two and higher of a multilevel model. For example, in the two-level model

. *mecmd fixed_portion* [iw = wt1] || school: ... , iweight(wt2) ...

the variable wt1 would hold the first-level (the observation-level) importance weights, and wt2 would hold the second-level (the school-level) importance weights.

pweight(*varname*) specifies sampling weights at higher levels in a multilevel model, whereas sampling weights at the first level (the observation level) are specified in the usual manner, for example, [pw=*pwtvar1*]. *varname* can be any valid Stata variable name, and you can specify pweight() at levels two and higher of a multilevel model. For example, in the two-level model

. *mecmd fixed_portion* [pw = wt1] || school: ... , pweight(wt2) ...

variable wt1 would hold the first-level (the observation-level) sampling weights, and wt2 would hold the second-level (the school-level) sampling weights.

binomial(*varname* | #) specifies that the data are in binomial form; that is, *depvar* records the number of successes from a series of binomial trials. This number of trials is given either as *varname*, which allows this number to vary over the observations, or as the constant #. If binomial() is not specified (the default), *depvar* is treated as Bernoulli, with any nonzero, nonmissing values indicating positive responses.

constraints(*constraints*); see [R] **Estimation options**.

    ⌐‾SE/Robust‾

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), and that allow for intragroup correlation (cluster *clustvar*); see [R] *vce_option*. If vce(robust) is specified, robust variances are clustered at the highest level in the multilevel model.

    ⌐‾Reporting‾

level(#), nocnsreport; see [R] **Estimation options**.

notable suppresses the estimation table, either at estimation or upon replay.

noheader suppresses the output header, either at estimation or upon replay.

nogroup suppresses the display of group summary information (number of groups, average group size, minimum, and maximum) from the output header.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **Estimation options**.

┌─ Integration ───

intmethod(*intmethod*) specifies the integration method to be used for the random-effects model. mvaghermite performs mean–variance adaptive Gauss–Hermite quadrature; mcaghermite performs mode-curvature adaptive Gauss–Hermite quadrature; ghermite performs nonadaptive Gauss–Hermite quadrature; and laplace performs the Laplacian approximation, equivalent to mode-curvature adaptive Gaussian quadrature with one integration point.

The default integration method is mvaghermite unless a crossed random-effects model is fit, in which case the default integration method is laplace. The Laplacian approximation has been known to produce biased parameter estimates; however, the bias tends to be more prominent in the estimates of the variance components rather than in the estimates of the fixed effects.

For crossed random-effects models, estimation with more than one quadrature point may be prohibitively intensive even for a small number of levels. For this reason, the integration method defaults to the Laplacian approximation. You may override this behavior by specifying a different integration method.

intpoints(#) sets the number of integration points for quadrature. The default is intpoints(7), which means that seven quadrature points are used for each level of random effects. This option is not allowed with intmethod(laplace).

The more integration points, the more accurate the approximation to the log likelihood. However, computation time increases as a function of the number of quadrature points raised to a power equaling the dimension of the random-effects specification. In crossed random-effects models and in models with many levels or many random coefficients, this increase can be substantial.

┌─ Maximization ───

maximize_options: difficult, technique(*algorithm_spec*), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(*init_specs*); see [R] **Maximize**. Those that require special mention for meprobit are listed below.

from() accepts a properly labeled vector of initial values or a list of coefficient names with values. A list of values is not allowed.

The following options are available with meprobit but are not shown in the dialog box:

startvalues(*svmethod*), startgrid[(*gridspec*)], noestimate, and dnumerical; see [ME] **meglm**.

collinear, coeflegend; see [R] **Estimation options**.

Remarks and examples

Mixed-effects probit regression is probit regression containing both fixed effects and random effects. In longitudinal data and panel data, random effects are useful for modeling intracluster correlation; that is, observations in the same cluster are correlated because they share common cluster-level random effects.

meprobit allows for many levels of random effects. However, for simplicity, we here consider the two-level model, where for a series of M independent clusters, and conditional on a set of fixed effects $\mathbf{x}_{ij}$ and a set of random effects $\mathbf{u}_j$,

$$\Pr(y_{ij} = 1 | \mathbf{x}_{ij}, \mathbf{u}_j) = H(\mathbf{x}_{ij}\boldsymbol{\beta} + \mathbf{z}_{ij}\mathbf{u}_j) \tag{1}$$

for $j = 1, \ldots, M$ clusters, with cluster j consisting of $i = 1, \ldots, n_j$ observations. The responses are the binary-valued y_{ij}, and we follow the standard Stata convention of treating $y_{ij} = 1$ if $depvar_{ij} \neq 0$ and treating $y_{ij} = 0$ otherwise. The $1 \times p$ row vector $\mathbf{x}_{ij}$ are the covariates for the fixed effects, analogous to the covariates you would find in a standard probit regression model, with regression coefficients (fixed effects) $\boldsymbol{\beta}$. For notational convenience here and throughout this manual entry, we suppress the dependence of y_{ij} on $\mathbf{x}_{ij}$.

The $1 \times q$ vector $\mathbf{z}_{ij}$ are the covariates corresponding to the random effects and can be used to represent both random intercepts and random coefficients. For example, in a random-intercept model, $\mathbf{z}_{ij}$ is simply the scalar 1. The random effects $\mathbf{u}_j$ are M realizations from a multivariate normal distribution with mean $\mathbf{0}$ and $q \times q$ variance matrix $\boldsymbol{\Sigma}$. The random effects are not directly estimated as model parameters but are instead summarized according to the unique elements of $\boldsymbol{\Sigma}$, known as variance components. One special case of (1) places $\mathbf{z}_{ij} = \mathbf{x}_{ij}$, so that all covariate effects are essentially random and distributed as multivariate normal with mean $\boldsymbol{\beta}$ and variance $\boldsymbol{\Sigma}$.

Finally, because this is probit regression, $H(\cdot)$ is the standard normal cumulative distribution function, which maps the linear predictor to the probability of a success ($y_{ij} = 1$) with $H(v) = \Phi(v)$.

Model (1) may also be stated in terms of a latent linear response, where only $y_{ij} = I(y_{ij}^* > 0)$ is observed for the latent

$$y_{ij}^* = \mathbf{x}_{ij}\boldsymbol{\beta} + \mathbf{z}_{ij}\mathbf{u}_j + \epsilon_{ij}$$

The errors ϵ_{ij} are distributed as a standard normal with mean 0 and variance 1 and are independent of $\mathbf{u}_j$.

Below we present two short examples of mixed-effects probit regression; refer to [ME] **me** and [ME] **meglm** for examples of other random-effects models. A two-level probit model can also be fit using xtprobit with the re option; see [XT] **xtprobit**. In the absence of random effects, mixed-effects probit regression reduces to standard probit regression; see [R] **probit**.

▷ Example 1: Two-level random-intercept model

Ng et al. (2006) analyzed a subsample of data from the 1989 Bangladesh fertility survey (Huq and Cleland 1990), which polled 1,934 Bangladeshi women on their use of contraception. The women sampled were from 60 districts, identified by the variable district. Each district contained either urban or rural areas (variable urban) or both. The variable c_use is the binary response, with a value of 1 indicating contraceptive use. Other covariates include mean-centered age and three indicator variables recording number of children.

```
. use https://www.stata-press.com/data/r18/bangladesh
(Bangladesh Fertility Survey, 1989)

. meprobit c_use i.urban age i.children || district:

Fitting fixed-effects model:

Iteration 0:   Log likelihood = -1228.8313
Iteration 1:   Log likelihood = -1228.2466
Iteration 2:   Log likelihood = -1228.2466

Refining starting values:

Grid node 0:   Log likelihood = -1237.3973

Fitting full model:

Iteration 0:   Log likelihood = -1237.3973  (not concave)
Iteration 1:   Log likelihood = -1221.2111  (not concave)
Iteration 2:   Log likelihood = -1207.4451
Iteration 3:   Log likelihood = -1206.7002
Iteration 4:   Log likelihood = -1206.5346
Iteration 5:   Log likelihood = -1206.5336
Iteration 6:   Log likelihood = -1206.5336
```

Mixed-effects probit regression   Number of obs   =   1,934
Group variable: district   Number of groups =   60

                                                    Obs per group:
                                                                 min =   2
                                                                 avg =   32.2
                                                                 max =   118

Integration method: mvaghermite   Integration pts. =   7

                                                    Wald chi2(5)   =   115.36
Log likelihood = -1206.5336   Prob > chi2   =   0.0000

c_use	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
urban						
Urban	.4490191	.0727176	6.17	0.000	.3064953	.5915429
age	-.0162203	.0048005	-3.38	0.001	-.0256291	-.0068114
children						
1 child	.674377	.0947829	7.11	0.000	.488606	.8601481
2 children	.8281581	.1048136	7.90	0.000	.6227272	1.033589
3 or more..	.8137876	.1073951	7.58	0.000	.6032972	1.024278
_cons	-1.02799	.0870307	-11.81	0.000	-1.198567	-.8574132
district						
var(_cons)	.0798719	.026886			.0412921	.1544972

LR test vs. probit model: chibar2(01) = 43.43   Prob >= chibar2 = 0.0000

Probit regression coefficients are most commonly interpreted in terms of partial effects, as we demonstrate in example 1 of [ME] **meprobit postestimation**. For now, we only note that urban women and women with more children are more likely to use contraceptives and that contraceptive use decreases with age. The estimated variance of the random intercept at the district level, $\widehat{\sigma}^2$, is 0.08 with standard error 0.03. The reported likelihood-ratio test shows that there is enough variability between districts to favor a mixed-effects probit regression over an ordinary probit regression; see *Distribution theory for likelihood-ratio test* in [ME] **me** for a discussion of likelihood-ratio testing of variance components.

◁

▷ Example 2: Three-level random-intercept model

Rabe-Hesketh, Touloupoulou, and Murray (2001) analyzed data from a study that measured the cognitive ability of patients with schizophrenia compared with their relatives and control subjects. Cognitive ability was measured as the successful completion of the "Tower of London", a computerized task, measured at three levels of difficulty. For all but one of the 226 subjects, there were three measurements (one for each difficulty level). Because patients' relatives were also tested, a family identifier, family, was also recorded.

We fit a probit model with response dtlm, the indicator of cognitive function, and with covariates difficulty and a set of indicator variables for group, with the controls (group==1) being the base category. We also allow for random effects due to families and due to subjects within families. The first is a random intercept (constant only) at the family level, and the second is a random intercept at the subject level. The order in which these are specified (from left to right) is significant—meprobit assumes that subject is nested within family. The equations are separated by ||.

```
. use https://www.stata-press.com/data/r18/towerlondon
(Tower of London data)

. meprobit dtlm difficulty i.group || family: || subject:

Fitting fixed-effects model:

Iteration 0:  Log likelihood = -317.11238
Iteration 1:  Log likelihood = -314.50535
Iteration 2:  Log likelihood = -314.50121
Iteration 3:  Log likelihood = -314.50121

Refining starting values:

Grid node 0:  Log likelihood = -326.18533

Fitting full model:

Iteration 0:  Log likelihood = -326.18533  (not concave)
Iteration 1:  Log likelihood = -313.16256  (not concave)
Iteration 2:  Log likelihood = -308.47528
Iteration 3:  Log likelihood = -305.02228
Iteration 4:  Log likelihood = -304.88972
Iteration 5:  Log likelihood = -304.88845
Iteration 6:  Log likelihood = -304.88845
```

Mixed-effects probit regression   Number of obs   =   677

       Grouping information

Group variable	No. of groups	Observations per group Minimum	Average	Maximum
family	118	2	5.7	27
subject	226	2	3.0	3

Integration method: mvaghermite			Integration pts.	=	7
			Wald chi2(3)	=	83.28
Log likelihood = -304.88845			Prob > chi2	=	0.0000

dtlm	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
difficulty	-.9329891	.1037376	-8.99	0.000	-1.136311	-.7296672
group						
2	-.1632243	.204265	-0.80	0.424	-.5635763	.2371276
3	-.6220196	.228063	-2.73	0.006	-1.069015	-.1750244
_cons	-.8405154	.1597223	-5.26	0.000	-1.153565	-.5274654
family						
var(_cons)	.2120948	.1736281			.0426292	1.055244
family>subject						
var(_cons)	.3559141	.219331			.106364	1.190956

LR test vs. probit model: chi2(2) = 19.23   Prob > chi2 = 0.0001

Note: LR test is conservative and provided only for reference.

We see that we have 226 subjects from 118 families. After adjusting for the random-effects structure, the probability of successful completion of the Tower of London decreases dramatically as the level of difficulty increases. Also, people with schizophrenia (group==3) tended not to perform as well as the control subjects.

◁

The above extends to models with more than two levels of nesting by adding more random-effects equations, each separated by ||.

Stored results

meprobit stores the following in e():

Scalars
e(N)	number of observations
e(k)	number of parameters
e(k_dv)	number of dependent variables
e(k_eq)	number of equations in e(b)
e(k_eq_model)	number of equations in overall model test
e(k_f)	number of fixed-effects parameters
e(k_r)	number of random-effects parameters
e(k_rs)	number of variances
e(k_rc)	number of covariances
e(df_m)	model degrees of freedom
e(ll)	log likelihood
e(N_clust)	number of clusters
e(chi2)	χ^2
e(p)	p-value for model test
e(ll_c)	log likelihood, comparison model
e(chi2_c)	χ^2, comparison test
e(df_c)	degrees of freedom, comparison test
e(p_c)	p-value for comparison test
e(rank)	rank of e(V)
e(ic)	number of iterations
e(rc)	return code
e(converged)	1 if converged, 0 otherwise

Macros
e(cmd)	meglm
e(cmd2)	meprobit
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(wtype)	weight type
e(wexp)	weight expression (first-level weights)
e(fweight$k$)	fweight variable for kth highest level, if specified
e(iweight$k$)	iweight variable for kth highest level, if specified
e(pweight$k$)	pweight variable for kth highest level, if specified
e(covariates)	list of covariates
e(ivars)	grouping variables
e(model)	probit
e(title)	title in estimation output
e(link)	probit
e(family)	bernoulli or binomial
e(clustvar)	name of cluster variable
e(offset)	offset
e(binomial)	binomial number of trials
e(intmethod)	integration method
e(n_quad)	number of integration points
e(chi2type)	Wald; type of model χ^2
e(vce)	*vcetype* specified in vce()
e(vcetype)	title used to label Std. err.
e(opt)	type of optimization
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(datasignature)	the checksum
e(datasignaturevars)	variables used in calculation of checksum

e(properties)	b V
e(estat_cmd)	program used to implement estat
e(predict)	program used to implement predict
e(marginsnotok)	predictions disallowed by margins
e(marginswtype)	weight type for margins
e(marginswexp)	weight expression for margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(N_g)	group counts
e(g_min)	group-size minimums
e(g_avg)	group-size averages
e(g_max)	group-size maximums
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions

e(sample)	marks estimation sample

In addition to the above, the following is stored in r():

Matrices

r(table)	matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

meprobit is a convenience command for meglm with a probit link and a bernoulli or binomial family; see [ME] **meglm**.

Model (1) assumes Bernoulli data, a special case of the binomial. Because binomial data are also supported by meprobit (option binomial()), the methods presented below are in terms of the more general binomial mixed-effects model.

For a two-level binomial model, consider the response y_{ij} as the number of successes from a series of r_{ij} Bernoulli trials (replications). For cluster j, $j = 1, \ldots, M$, the conditional distribution of $\mathbf{y}_j = (y_{j1}, \ldots, y_{jn_j})'$, given a set of cluster-level random effects $\mathbf{u}_j$, is

$$
\begin{aligned}
f(\mathbf{y}_j | \mathbf{u}_j) &= \prod_{i=1}^{n_j} \left[\binom{r_{ij}}{y_{ij}} \left\{ \Phi(\boldsymbol{\eta}_{ij}) \right\}^{y_{ij}} \left\{ 1 - \Phi(\boldsymbol{\eta}_{ij}) \right\}^{r_{ij} - y_{ij}} \right] \\
&= \exp \left(\sum_{i=1}^{n_j} \left[y_{ij} \log \left\{ \Phi(\boldsymbol{\eta}_{ij}) \right\} - (r_{ij} - y_{ij}) \log \left\{ \Phi(-\boldsymbol{\eta}_{ij}) \right\} + \log \binom{r_{ij}}{y_{ij}} \right] \right)
\end{aligned}
$$

for $\boldsymbol{\eta}_{ij} = \mathbf{x}_{ij}\boldsymbol{\beta} + \mathbf{z}_{ij}\mathbf{u}_j + \text{offset}_{ij}$.

Defining $\mathbf{r}_j = (r_{j1}, \ldots, r_{jn_j})'$ and

$$
c(\mathbf{y}_j, \mathbf{r}_j) = \sum_{i=1}^{n_j} \log \binom{r_{ij}}{y_{ij}}
$$

where $c(\mathbf{y}_j, \mathbf{r}_j)$ does not depend on the model parameters, we can express the above compactly in matrix notation,

$$f(\mathbf{y}_j|\mathbf{u}_j) = \exp\left[\mathbf{y}_j' \log\left\{\Phi(\boldsymbol{\eta}_j)\right\} - (\mathbf{r}_j - \mathbf{y}_j)' \log\left\{\Phi(-\boldsymbol{\eta}_j)\right\} + c\left(\mathbf{y}_j, \mathbf{r}_j\right)\right]$$

where $\boldsymbol{\eta}_j$ is formed by stacking the row vectors $\boldsymbol{\eta}_{ij}$. We extend the definitions of $\Phi(\cdot)$, $\log(\cdot)$, and $\exp(\cdot)$ to be vector functions where necessary.

Because the prior distribution of $\mathbf{u}_j$ is multivariate normal with mean $\mathbf{0}$ and $q \times q$ variance matrix $\boldsymbol{\Sigma}$, the likelihood contribution for the jth cluster is obtained by integrating $\mathbf{u}_j$ out of the joint density $f(\mathbf{y}_j, \mathbf{u}_j)$,

$$\begin{aligned}
\mathcal{L}_j(\boldsymbol{\beta}, \boldsymbol{\Sigma}) &= (2\pi)^{-q/2} |\boldsymbol{\Sigma}|^{-1/2} \int f(\mathbf{y}_j|\mathbf{u}_j) \exp\left(-\mathbf{u}_j' \boldsymbol{\Sigma}^{-1} \mathbf{u}_j/2\right) d\mathbf{u}_j \\
&= \exp\left\{c\left(\mathbf{y}_j, \mathbf{r}_j\right)\right\} (2\pi)^{-q/2} |\boldsymbol{\Sigma}|^{-1/2} \int \exp\left\{h\left(\boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{u}_j\right)\right\} d\mathbf{u}_j
\end{aligned} \tag{2}$$

where

$$h\left(\boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{u}_j\right) = \mathbf{y}_j' \log\left\{\Phi(\boldsymbol{\eta}_j)\right\} - (\mathbf{r}_j - \mathbf{y}_j)' \log\left\{\Phi(-\boldsymbol{\eta}_j)\right\} - \mathbf{u}_j' \boldsymbol{\Sigma}^{-1} \mathbf{u}_j/2$$

and for convenience, in the arguments of $h(\cdot)$ we suppress the dependence on the observable data $(\mathbf{y}_j, \mathbf{r}_j, \mathbf{X}_j, \mathbf{Z}_j)$.

The integration in (2) has no closed form and thus must be approximated; see *Methods and formulas* in [ME] **meglm** for details.

meprobit supports multilevel weights and survey data; see *Methods and formulas* in [ME] **meglm** for details.

References

Huq, N. M., and J. Cleland. 1990. *Bangladesh Fertility Survey 1989 (Main Report)*. National Institute of Population Research and Training.

Ng, E. S.-W., J. R. Carpenter, H. Goldstein, and J. Rasbash. 2006. Estimation in generalised linear mixed models with binary outcomes by simulated maximum likelihood. *Statistical Modelling* 6: 23–42. https://doi.org/10.1191/1471082X06st106oa.

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Also see

[ME] **meprobit postestimation** — Postestimation tools for meprobit

[ME] **mecloglog** — Multilevel mixed-effects complementary log–log regression

[ME] **melogit** — Multilevel mixed-effects logistic regression

[ME] **me** — Introduction to multilevel mixed-effects models

[BAYES] **bayes: meprobit** — Bayesian multilevel probit regression

[SEM] **Intro 5** — Tour of models (*Multilevel mixed-effects models*)

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtprobit** — Random-effects and population-averaged probit models

[U] **20 Estimation and postestimation commands**