qrsolve( ) — Solve AX=B for X using QR decomposition

Description

`qrsolve(`*A*`,` *B*`, ...)` uses QR decomposition to solve $AX = B$ and returns *X*. When *A* is singular or nonsquare, `qrsolve()` computes a least-squares generalized solution. When *rank* is specified, in it is placed the rank of *A*.

`_qrsolve(`*A*`,` *B*`, ...)`, does the same thing, except that it destroys the contents of *A* and it overwrites *B* with the solution. Returned is the rank of *A*.

In both cases, *tol* specifies the tolerance for determining whether *A* is of full rank. *tol* is interpreted in the standard way—as a multiplier for the default if $tol > 0$ is specified and as an absolute quantity to use in place of the default if $tol \leq 0$ is specified; see [M-1] **Tolerance**.

Syntax

numeric matrix	`qrsolve(`*A*`,` *B*`)`
numeric matrix	`qrsolve(`*A*`,` *B*`,` *rank*`)`
numeric matrix	`qrsolve(`*A*`,` *B*`,` *rank*`,` *tol*`)`
real scalar	`_qrsolve(`*A*`,` *B*`)`
real scalar	`_qrsolve(`*A*`,` *B*`,` *tol*`)`

where

A:	*numeric matrix*
B:	*numeric matrix*
rank:	irrelevant; *real scalar* returned
tol:	*real scalar*

Remarks and examples

`qrsolve(`*A*`,` *B*`, ...)` is suitable for use with square and possibly rank-deficient matrix *A*, or when *A* has more rows than columns. When *A* is square and full rank, `qrsolve()` returns the same solution as `lusolve()` (see [M-5] **lusolve( )**), up to roundoff error. When *A* is singular, `qrsolve()` returns a generalized (least-squares) solution.

Remarks are presented under the following headings:

Derivation

We wish to solve for X

$$AX = B \tag{1}$$

Perform QR decomposition on A so that we have $A = QRP'$. Then (1) can be rewritten as

$$QRP'X = B$$

Premultiplying by Q' and remembering that $Q'Q = QQ' = I$, we have

$$RP'X = Q'B \tag{2}$$

Define

$$Z = P'X \tag{3}$$

Then (2) can be rewritten as

$$RZ = Q'B \tag{4}$$

It is easy to solve (4) for Z because R is upper triangular. Having Z, we can obtain X via (3), because $Z = P'X$, premultiplied by P (and if we remember that $PP' = I$), yields

$$X = PZ$$

For more information on QR decomposition, see [M-5] **qrd( )**.

Relationship to inversion

For a general discussion, see *Relationship to inversion* in [M-5] **lusolve( )**.

For an inverse based on QR decomposition, see [M-5] **qrinv( )**. `qrinv(`A`)` amounts to `qrsolve(`A`, I(rows(`A`)))`, although it is not actually implemented that way.

Tolerance

The default tolerance used is

$$eta = \texttt{1e-13} * \texttt{trace(abs(}R\texttt{))/rows(}R\texttt{)}$$

where R is the upper-triangular matrix of the QR decomposition; see *Derivation* above. When A is less than full rank, by, say, d degrees of freedom, then R is also rank deficient by d degrees of freedom and the bottom d rows of R are essentially zero. If the ith diagonal element of R is less than or equal to eta, then the ith row of Z is set to zero. Thus if the matrix is singular, `qrsolve()` provides a generalized solution.

If you specify $tol > 0$, the value you specify is used to multiply eta. You may instead specify $tol \le 0$, and then the negative of the value you specify is used in place of eta; see [M-1] **Tolerance**.

Conformability

`qrsolve(`*A*`,` *B*`,` *rank*`,` *tol*`)`:

input:

A:	$m \times n$,	$m \geq n$
B:	$m \times k$	
tol:	1×1	(optional)

output:

rank:	1×1	(optional)
result:	$n \times k$	

`_qrsolve(`*A*`,` *B*`,` *tol*`)`:

input:

A:	$m \times n$,	$m \geq n$
B:	$m \times k$	
tol:	1×1	(optional)

output:

A:	0×0
B:	$n \times k$
result:	1×1

Diagnostics

`qrsolve(`*A*`,` *B*`, ...)` and `_qrsolve(`*A*`,` *B*`, ...)` return a result containing missing if *A* or *B* contain missing values.

`_qrsolve(`*A*`,` *B*`, ...)` aborts with error if *A* or *B* are views.

Also see

[M-5] **cholsolve( )** — Solve AX=B for X using Cholesky decomposition

[M-5] **lusolve( )** — Solve AX=B for X using LU decomposition

[M-5] **qrd( )** — QR decomposition

[M-5] **qrinv( )** — Generalized inverse of matrix via QR decomposition

[M-5] **solvelower( )** — Solve AX=B for X, A triangular

[M-5] **solve_tol( )** — Tolerance used by solvers and inverters

[M-5] **svsolve( )** — Solve AX=B for X using singular value decomposition

[M-4] **Matrix** — Matrix functions

[M-4] **Solvers** — Functions to solve AX=B and to obtain A inverse

For suggested citations, see the FAQ on citing Stata documentation.